Question 1 (Use a separate booklet)

Marks

(a) If $x^7 = 2000$, find x correct to 4 significant figures. 2

(b) A store adds a GST of 10% to the selling price of all its goods. Including the GST, an item costs \$119.90. What was the selling price before the GST was added? 1

(c) If $V = \pi r^2 h$, find r correct to 1 decimal place when $V = 1000$ and $h = 22$. 1
(Use the calculator value of π.)

(d) Solve $12 - 5x < x - 3$. 2

(e) Factorise $t^3 - 8$. 1

(f) Find integers a and b such that $\dfrac{\sqrt{3}}{2+\sqrt{3}} = a + b\sqrt{3}$. 3

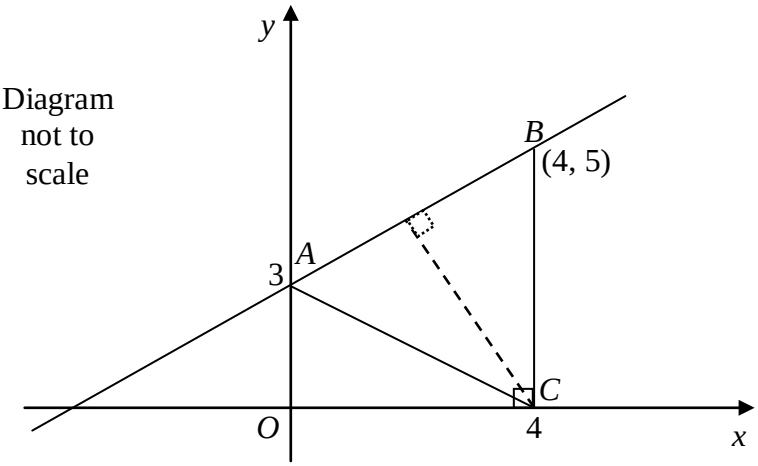
(g) Solve $15 - x^2 = 2x$. 2

Question 2 (Use a separate booklet)

In the diagram, the line AB cuts the y-axis at the point A (0, 3) and passes through the point B (4, 5). A perpendicular is dropped from B to meet the x-axis at C (4, 0). **Marks**

Copy or trace the diagram into your working booklet.

Diagram not to scale



(a) Calculate the length of the interval AB. 1

(b) Find the gradient of the line AB. 1

(c) Show that the equation of the line AB is $x - 2y + 6 = 0$ 1

(d) Find the equation of the line which is perpendicular to AB and which passes through C. 2

(e) Calculate the perpendicular distance from C to AB. 2

(f) Find the area of the triangle ABC. 1

(g) Prove that triangle ABC is isosceles. 2

(h) Let D be the point such that $ADBC$ is a rhombus. Find the coordinates of D, and give a reason for your answer. 2

Question 3 (Use a separate booklet)

Marks

(a) On a particular parabola, all the points are equidistant from a fixed point $(0,5)$ and a fixed line $y = -5$.

(i) Write down the equation of the parabola. 2

(ii) State the coordinates of the vertex of the parabola. 1

(b) A parabola has axis of symmetry $x = 3$, vertex $(3,-1)$ and focal length 2.

(i) If the parabola is concave up, what are the coordinates of the focus of the parabola? 1

(ii) If the parabola is concave up, what is the equation of the directrix? 1

(iii) If the parabola is concave down, what is the equation of the parabola? 2

(c) In a number plane are two fixed points $A(-1,4)$ and $B(2,-2)$. A variable point $P(x, y)$ moves so that $PA = 2PB$.

(i) Show that P moves on the circle $x^2 + y^2 - 6x + 8y + 5 = 0$. 3

(ii) Find the centre and radius of this circle. 2

Question 4 (Use a separate booklet)

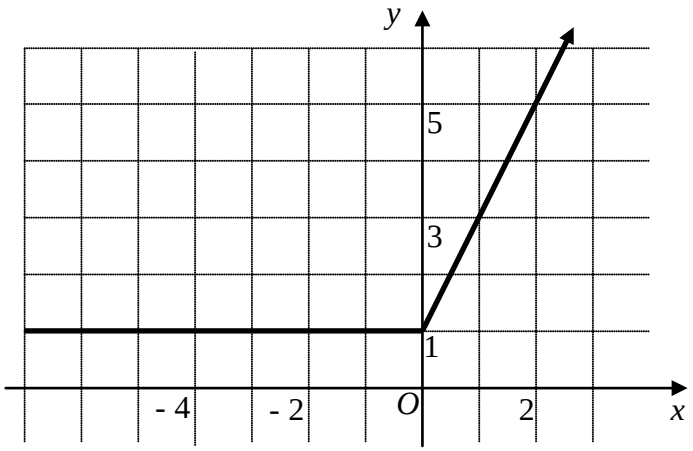
Marks

(a) Find a primitive of each of the following.

(i) $x^6 - 5$. 1

(ii) $\sqrt[3]{x}$. 1

(b) In the number plane, the dark line is the graph of $y = f(x)$. 2



Use the graph to evaluate $\int_{-4}^{2} f(x)\,dx$.

(c) Find

(i) $\int 9u^{-4}du$ 1

(ii) $\int \sqrt{6x+3}dx$. 2

(d) Consider the function $y = 2x - x^2$ defined between $x = -1$ and $x = 2$.

(i) Sketch the function. 2

(ii) Find the area of the region bounded by the curve $y = 2x - x^2$, the x-axis, and the line $x = -1$. 3

Question 5 (Use a separate booklet)

Marks

(a) For the curve $y = f(x)$, the second derivative is given by $f''(x) = 12x - 4$. The tangent at the point $(1,3)$ on the curve has gradient 1.

(i) Show that $f'(x) = 6x^2 - 4x - 1$. 2

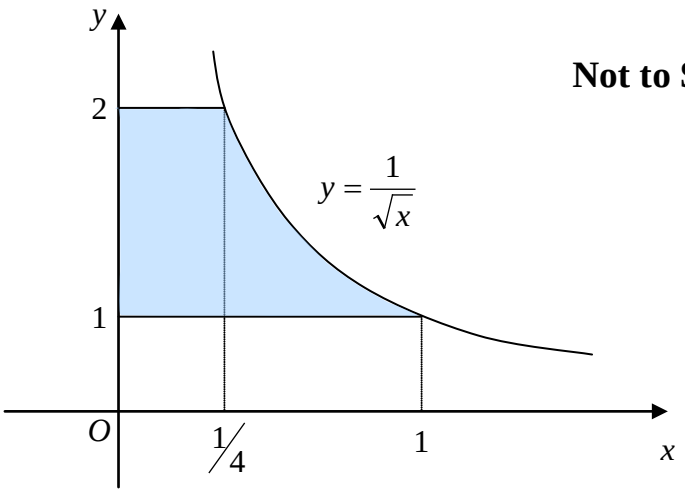
(ii) Find the equation of the curve. 2

(b) The table below shows values of t and corresponding values of $s(t)$. 2

t	0	0.5	1
$s(t)$	10	6.7	4.5

Use the trapezoidal rule and the three function values given in the table to evaluate $\int_0^1 s(t)dt$ correct to 1 decimal place.

(c) In the diagram the shaded region is bounded by the curve $y = \frac{1}{\sqrt{x}}$, the y-axis, and the lines $y = 1$ and $y = 2$.



(i) Find the area of the shaded region. 3

(ii) Find the volume of the solid obtained by rotating the shaded region about the y-axis. Leave the answer in terms of π. 3

Question 6 (Use a separate booklet)

Marks

(a) Evaluate $\int_{-2}^{3}(2x+1)^4\,dx$. 3

(b) Consider the curves $y = x^2 - 2x + 3$ and $y = 3 - x^2$.

(i) Show that the curves have the same *y*-intercept. 1

(ii) Show that the curves intersect at the point (1, 2). 1

(iii) Find the area of the region bounded by the two curves. 3

(c) A table of values for the function $y = \dfrac{6}{1+x^2}$, with one of the values missing, is shown below.

x	1	2	3	4	5
y	3	1.2	0.6		0.23

(i) Show that the missing value in the table, correct to two decimal places, is 0.35. 1

(ii) Use Simpson's rule with five function values to find an estimate of the area under the curve $y = \dfrac{6}{1+x^2}$ between $x = 1$ and $x = 5$. Give the answer correct to one decimal place. 3

Question 7 (Use a separate booklet)

Marks

(a) Find the value of

(i) $\log_3 \sqrt{27}$ 1

(ii) $(\log_5 7) \times (\log_7 5)$ 1

(iii) $e^{\ln 3}$. 1

(b) Differentiate with respect to x:

(i) $\dfrac{e^{2x}}{x}$ 2

(ii) $\log_e \left(x^2 + x\right)$. 2

(c) Find $\int e^{\left(\frac{5x}{3}\right)} dx$. 1

(d) (i) Show that $3 + \dfrac{1}{x+1} = \dfrac{3x+4}{x+1}$. 1

(ii) Hence evaluate $\int_0^2 \dfrac{3x+4}{x+1} dx$. Leave the answer in exact form. 3

Question 8 (Use a separate booklet)

Marks

(a) (i) On the same set of axes, carefully draw the graphs of $y = \frac{1}{x}$ and $y = \ln x$ for $0 < x \leq 3$. 2

(ii) By referring to where the graphs cross, find an approximate solution to the equation $\frac{1}{x} = \ln x$. 1

(iii) Using trial and error and your calculator, find the solution asked for in part (ii) correct to two decimal places. 1

(b) (i) Find the stationary point on the curve $y = \frac{1}{x} + \ln x$. 2

(ii) Determine the nature of this stationary point. 2

(c) (i) Show that $\frac{d}{dx}(x \ln x - x) = \ln x$ 2

(ii) Hence find the area under the curve $y = \ln x$ between $x = 1$ and $x = 3$. You may leave the answer in exact form. 2