SAINT IGNATIUS' COLLEGE RIVERVIEW
YEAR 12

MID-YEAR EXAMINATION

2004

MATHEMATICS

Time allowed – 2.5 hours
(plus 5 minutes reading time)

Directions to Candidates

1. Attempt **ALL** questions.
2. All questions are of equal value.
3. All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
4. Board-approved calculators may be used.
5. Each question attempted is to be returned in a **SEPARATE BOOKLET** clearly marked Question 1, Question 2, ...etc.
6. Each booklet must show your **NAME** and your **TEACHER'S NAME**.

QUESTION 1 (12 marks)
(Start a new booklet)

(a) Evaluate $\sqrt{\dfrac{a^4}{9-b^2}}$, given that $a = 4.8$ and $b = \dfrac{4}{7}$. (2)

Give your answer correct to 3 significant figures.

(b) Simplify

(i) $\dfrac{3}{a-2} \div \dfrac{a+3}{a^2-4}$ (2)

(ii) $\dfrac{125-m^3}{m-5}$ (2)

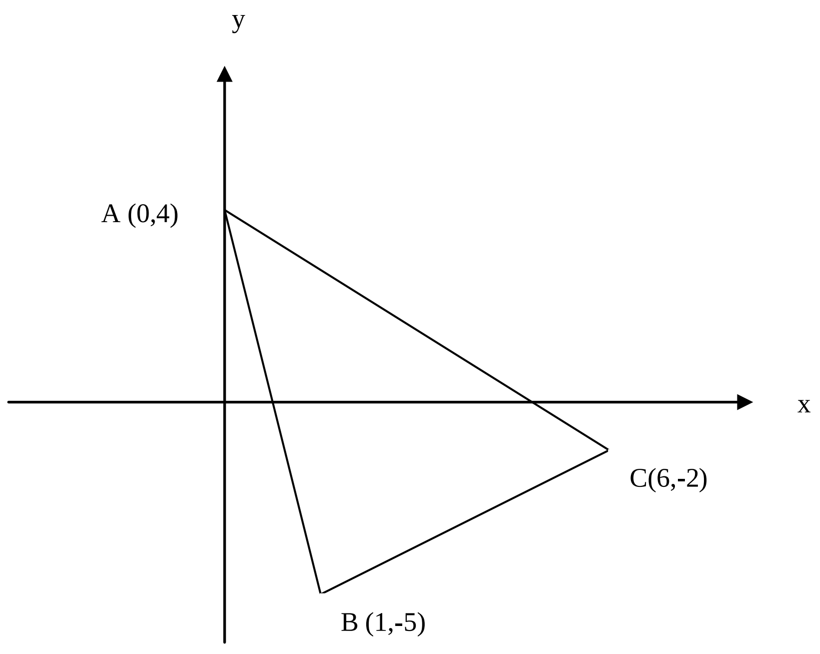
(c) If $(2\sqrt{5}-\sqrt{3})^2 = p - q\sqrt{15}$, find the value of p and q. (2)

(d) Solve $|4-2t| \geq 9$ (2)

(e) Solve for x only, (2)

$$y - 2x + 1 = 0$$
$$3y^2 - y - 2x^2 = 0$$

QUESTION 2 (12 marks)
(Start a new booklet)



The diagram shows the points A(0,4), B(1,-5) and C(6,-2).

(a) Show that the length of the interval AC is $6\sqrt{2}$ units. (1)

(b) Show that the line AC has equation $x + y - 4 = 0$. (2)

(c) Show that the perpendicular distance from from B to AC is $4\sqrt{2}$ units. (2)

(d) Hence find the area of triangle ABC. (1)

(e) Find the coordinates of the point D if C is the midpoint of the interval BD. (2)

(f) Find the equation of the line perpendicular to AC and passing through the point B. (2)

(g) Find the angle of inclination that the line AC makes with the positive direction of the X-axis. (2)

QUESTION 3 (12 marks)
(Start a new booklet)

(a) Consider the parabola with equation $(x-7)^2 = -24(y-6)$.

(i) Write down the coordinates of the vertex of the parabola. (1)

(ii) State the focal length of the parabola. (1)

(iii) Find the co-ordinates of the focus. (1)

(b) The focus of a parabola is S(4,2) and its directrix is the line $x = 6$.

(i) Sketch the parabola and indicate the coordinates of the vertex. (1)
(Note – the x and y intercepts are **not** required)

(ii) Write down the focal length of the parabola. (1)

(iii) Find the equation of the parabola. (1)

(c) Given that α and β are the roots of the quadratic equation

$$3x^2 + x - 7 = 0$$

Find the value of

(i) $\alpha + \beta$ (1)

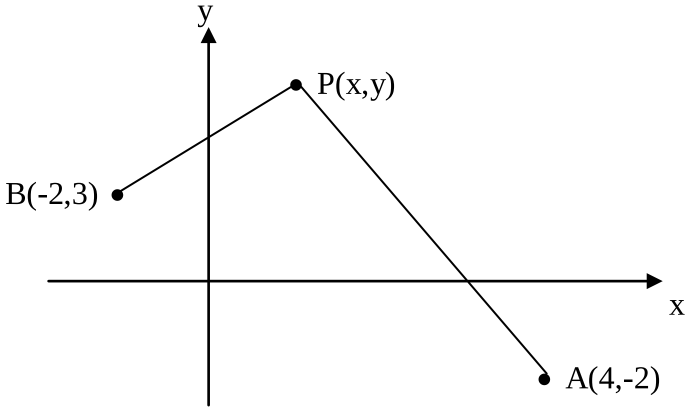
(ii) $\alpha\beta$ (1)

(iii) $\alpha^2 + \beta^2$ (2)

(iv) $\dfrac{3}{4\alpha^2} + \dfrac{3}{4\beta^2}$ (2)

QUESTION 5 (12 marks)
(Start a new booklet)

(a) Let A(4,-2) and B(-2,3) be two fixed points on the number plane. P(x,y) is a variable point that moves such that its distance from A is **twice** its distance from B.



(i) Write down the distances AP and BP in terms of x and y. (2)

(ii) Show that the locus of P is given by the equation (3)

$$3x^2 + 24x + 3y^2 - 28y + 32 = 0$$

(b) Let $Q(-2,4)$ and $R(-2,-4)$ be two fixed points on the number plane.

$P(x, y)$ is a variable point that moves such that the angle QPR is always 90°.

(i) Draw a neat sketch in your booklet indicating the information described above. (1)

(ii) Write down the gradients of PQ and PR in terms of x and y. (2)

(iii) Show that the equation of the locus of P is given by (2)

$$x^2 + 4x + y^2 - 12 = 0$$

(iv) Hence show that the locus of P is a circle with centre $(-2,0)$ and radius 4. (2)

QUESTION 6 (12 marks)
(Start a new booklet)

(a) Find a primitive function of

(i) $6-5x^4$ (1)

(ii) $\sqrt{3x-4}$ (2)

(b) Evaluate the following integral (3)

$$\int_0^2 (3x-5)^2\,dx$$

(c) The graph of $y = f(x)$ passes through the point (2,4) and (3)

$$f'(x) = 3x^2 - 1$$

Find $f(x)$.

(d) Determine the value of a given that $\int_1^a \frac{3}{\sqrt{x}}\,dx = 30$. (3)

QUESTION 4 (12 marks)
(Start a new booklet)

(a) Find the equation of the parabola with x-intercepts 6 and $-\frac{2}{3}$ and y-intercept -8. (2)

(b) Solve $3^{2x} - 8(3^x) - 9 = 0$. (3)

(c) Consider the quadratic equation

$$(m+1)x^2 + (m-1)x + (m+1) = 0$$

(i) For what values of m will the equation have roots which are opposite of each other. (2)

(hint – let the roots be α and $-\alpha$)

(ii) Show that the discriminant is given by (2)

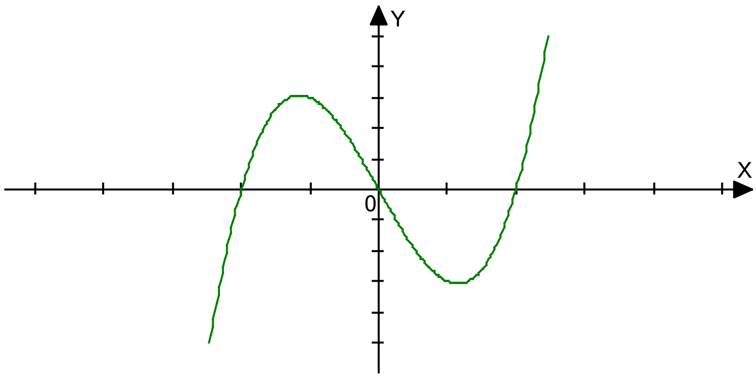
$$\Delta = -(3m+1)(m+3)$$

(iii) For what values of m does the equation have real roots. (2)

(iv) For what values of m is the quadratic positive definite. (1)

QUESTION 7 (12 marks)
(Start a new booklet)

(a) Consider the function $f(x) = x^3 - 4x$.



(i) Using the above graph of $y = f(x)$, state the x-intercepts. (1)

(ii) Calculate the area between the curve and the x-axis. (3)

(b) Consider the function $f(x) = \sqrt{x-1}$.

(i) Sketch the graph of $y = f(x)$. (1)

(ii) Calculate the area between the curve, the y-axis and the lines $y = 1$ and $y = 3$. (3)

(c) Consider the graph of the parabola $x^2 = 4ay$

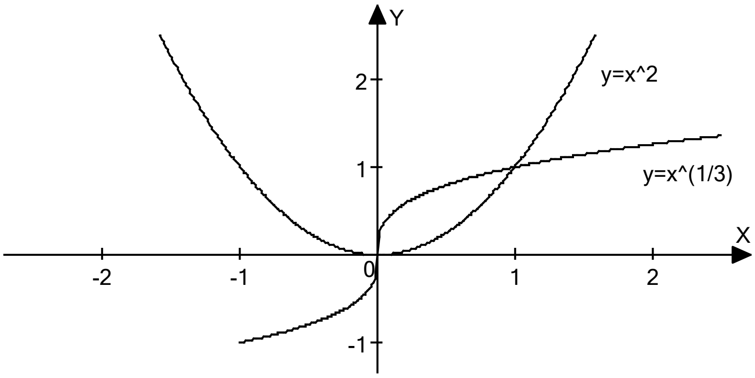
(i) Show that the volume of the solid formed by rotating the graph about the y-axis, between $y = 0$ and $y = 3$ is given by (3)

$$V = 18a\pi \ \ units^3$$

(ii) If the volume of the solid formed in part (i) is $54\pi \ units^3$, find the focal length of the parabola. (1)

QUESTION 8 (12 marks)
(Start a new booklet)

(a)



The diagram shows the graphs of $y = x^2$ and $x = y^3$.

(i) Show that the curves intersect at the points $(0,0)$ and $(1,1)$. (1)

(ii) Determine the area between the two curves. (3)

(b) Consider the circle with equation $x^2 + y^2 = r^2$. (4)

By rotating the circle about the x-axis, prove that the volume
Of the solid formed is given by the formula

$$V = \frac{4}{3}\pi r^3$$

(c) A bowl has a shape obtained by rotating part of the curve $y = \frac{x^4}{36}$ (4)
about the y-axis.

The bowl is 100 centimetres deep.

Find the volume of liquid that the bowl will hold, giving your answer to the nearest litre (use $1000 cm^3 = 1L$).