(Marks)

Question 1

(a) Simplify $4(2x-8)-(x-1)$ **(2)**

(b) Differentiate $2x^2-8x+1$ **(2)**

(c) Solve $|x-8|=2$ **(2)**

(d) Write 1 438 500 000 in scientific notation to 3 significant figures **(2)**

(e) Factorise fully: $4x^3-4x$ **(2)**

(f) Find a primitive function of x^2-2x **(2)**

(Marks)

Question 2 Start a new page

(a) Solve: $\dfrac{x-8}{5} - \dfrac{x+1}{3} = 2$ **(2)**

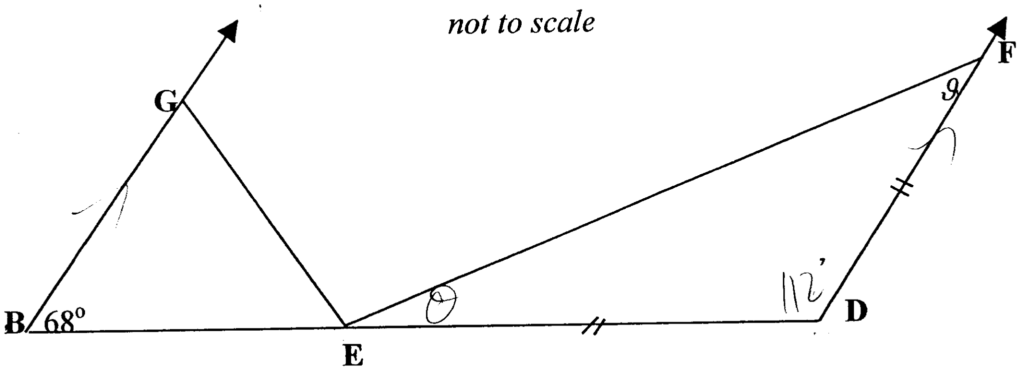
(b) Differentiate: $\dfrac{1}{5}\cos(3x)$ **(2)**

(c) Solve: $x^2 - 4x + 3 \geq 0$ **(2)**

(d) Write down the exact value of $\tan 300^\circ$ **(1)**

(e) Convert $\dfrac{5\pi}{8}$ radians to degrees **(1)**

(f)



$\theta = 34°$

BG is parallel to DF

ED = FD

$\angle GBE = 68^o$

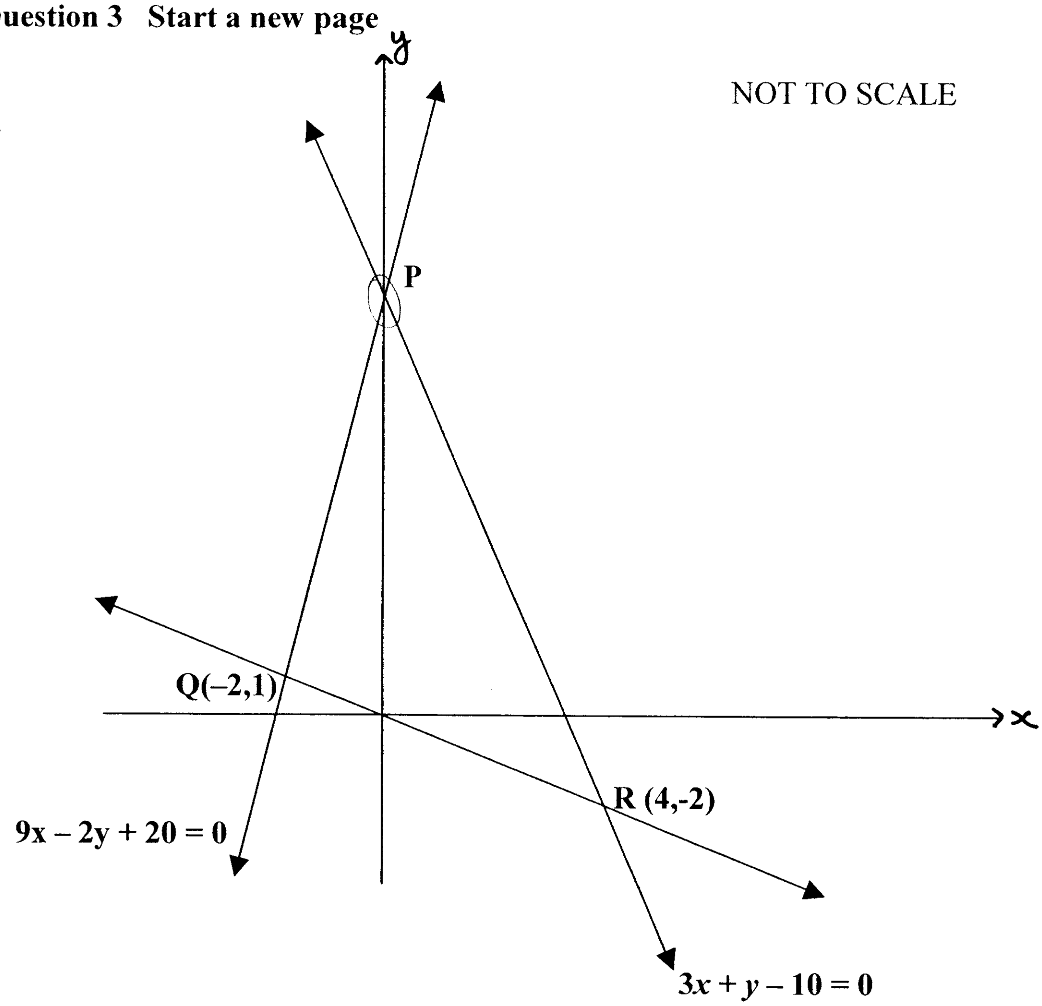
Find the value of $\angle EFD(\vartheta)$
Give reasons for your answer **(2)**

(g) Students studying at least one of the languages, French and German, attend a meeting.
Of the 30 students present 18 study French and 21 study German.
What is the probability that a student chosen at random studies only French? **(2)**

(Marks)

Question 3 Start a new page

a)



The point Q (-2, 1) lies on the line $9x - 2y + 20 = 0$
The point R (4, -2) lies on the line $3x + y - 10 = 0$

(i) Solve the equations $9x - 2y + 20 = 0$
and $3x + y - 10 = 0$
simultaneously to find the point P **(2)**

(ii) Show that the equation of the line QR is $x + 2y = 0$ **(2)**

(iii) Find the perpendicular distance of P from the line $x + 2y = 0$.
Leave your answer in surd form. **(2)**

(iv) Find the distance QR and hence the area of triangle PQR **(2)**

(b) Differentiate:

(i) $\dfrac{2x^2}{3x-1}$ **(2)**

(ii) $x^3 \sin x$ **(2)**

(Marks)

Question 4 Start a new page

(a) Solve: $2\cos x = -\sqrt{3}$ for $0^\circ \le x \le 360^\circ$ **(2)**

(b) If α and β are the roots of $2x^2 - 6x + 1 = 0$, find:

(i) $\alpha + \beta$ **(1)**

(ii) $\alpha\beta$ **(1)**

(iii) $\alpha^2 + \beta^2$ **(1)**

(iv) $\dfrac{1}{\alpha^2} + \dfrac{1}{\beta^2}$ **(1)**

(c) Find the sum of the first 15 terms of the series:

$$4 + 8 + 16 + 32 + \ldots\ldots.$$ **(2)**

(d) Find:

(i) $\displaystyle\int \frac{4x^3 - x}{4}\,dx$ **(2)**

(ii) $\displaystyle\int (2-3x)^3\,dx$ **(2)**

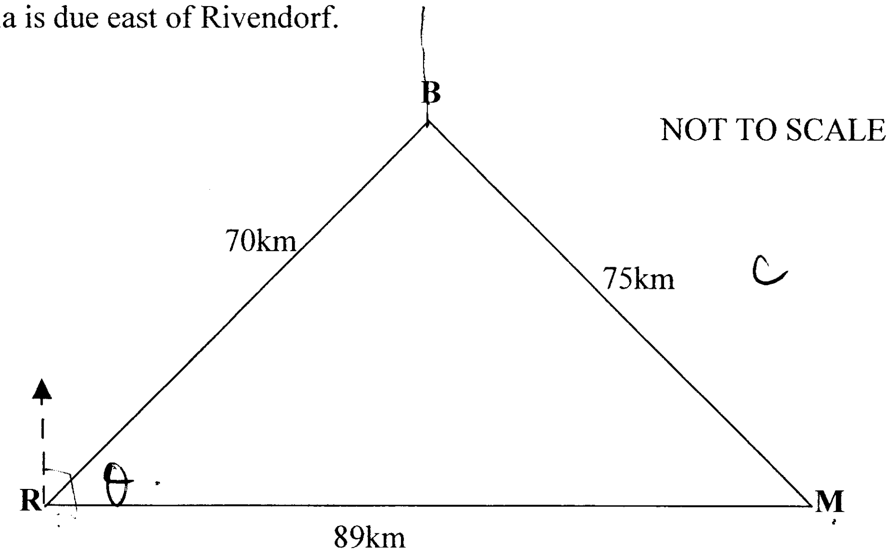
(Marks)

Question 5 Start a new page

(a) (i) Write down the discriminant of: $x^2 + (k + 2)x + 4 = 0$ **(1)**

(ii) For what values of k does the equation $x^2 + (k + 2)\,x + 4 = 0$ have no real roots. **(2)**

(b) The distances from Rivendorf (R), Moria (M) and Bree (B) are shown in the diagram below. Moria is due east of Rivendorf.



(i) Find the size of angle BRM to the nearest degree **(2)**

(ii) Find the bearing of Rivendorf (R) from Bree (B) **(1)**

(c) Jan is training for a long distance race. On the first day she runs 4km. On the second day she runs 4.5km. Each day she runs 0.5km further than the previous day.

(i) How far will she run on the 20th day? **(1)**

(ii) How many kilometres will she have run in total at the end of the 20th day? **(1)**

(iii) On which day will she run 21 km? **(1)**

(d) Find the equation of the tangent to the curve $y = \tan x$ at $x = \dfrac{\pi}{4}$ **(3)**

(Marks)

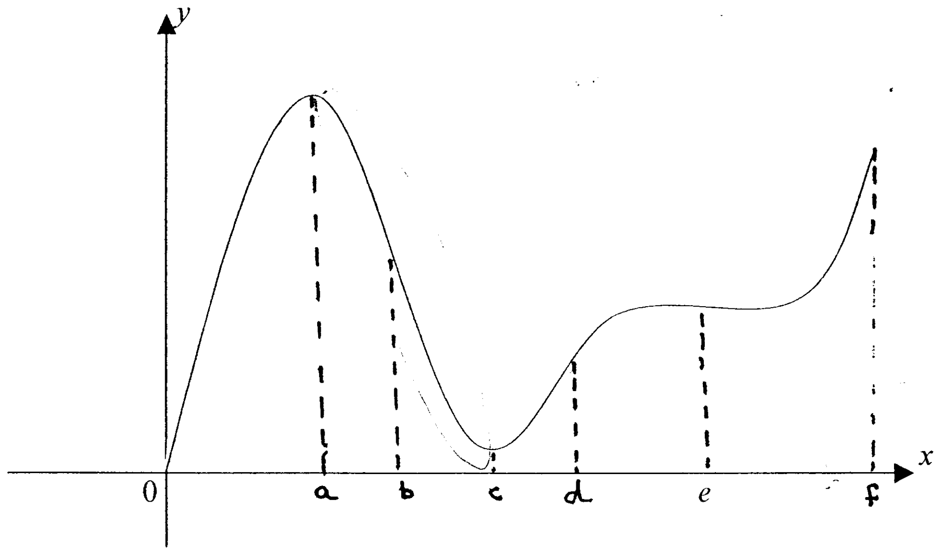
Question 6 Start a new page

(a)

(i) For the parabola $x^2 = 4\,(y-1)$ state the coordinates of the vertex and focus. **(2)**

(ii) The parabola $x^2 = 4\,(y-1)$ and the line $x - 2y + 2 = 0$ intersect at the points $(0,1)$ and $(2,2)$.
Find the area between the line $x - 2y + 2 = 0$ and the parabola $x^2 = 4\,(y-1)$ **(3)**

(b)



The curve $y = f(x)$ is drawn in the domain $0 \le x \le f$.

At $x = a$ there is a maximum turning point $y'' < 0$
At $x = b$ *and* $x = d$ there are points of inflexion
At $x = c$ there is a minimum turning point $y'' > 0$
At $x =$ e there is a horizontal point of inflexion $y'' = 0,\ y' = 0$

$< \ \le$

For what value(s) of x in the domain $0 \le x \le f$ is

(i) $f'(x) = 0$ **(1)**
(ii) $f''(x) = 0$ **(1)**
(iii) The curve $y = f(x)$ decreasing **(1)**
(iv) The curve $y = f(x)$ concave down **(1)**

(c) For the series $2 + \dfrac{2}{\sqrt{3}} + \dfrac{2}{3} + \ldots\ldots$

(i) Explain why it has a limiting sum **(1)**
(ii) Show that the limiting sum is $\sqrt{3} + 3$ **(2)**

Question 7 Start a new page

(a) In a board game, players toss a pair of dice. The players can chose to move either:

- The sum of the 2 numbers on the dice

  or

- Either of the individual numbers showing

Nicole needs a 6 to win. What is the probability she will win after she next throws the dice? **(2)**

(b)

(i) Sketch $y = \sqrt{16 - x^2}$ **(1)**

(ii) State the domain and range of $y = \sqrt{16 - x^2}$ **(2)**

(iii) Use Simpson's Rule to find an approximation for the area enclosed between $y = \sqrt{16 - x^2}$, and the x and y axes in the first quadrant using 3 functions values. Answer to 2 decimal places. **(2)**

(iv) What is the exact area of the region in (iii)? **(1)**

(c) A curve $y = f(x)$ is such that

$$\frac{dy}{dx} = 3x^2 - 4x + c$$

and it is stationary at the point (2, -5).

Find:

(i) The value of c **(2)**

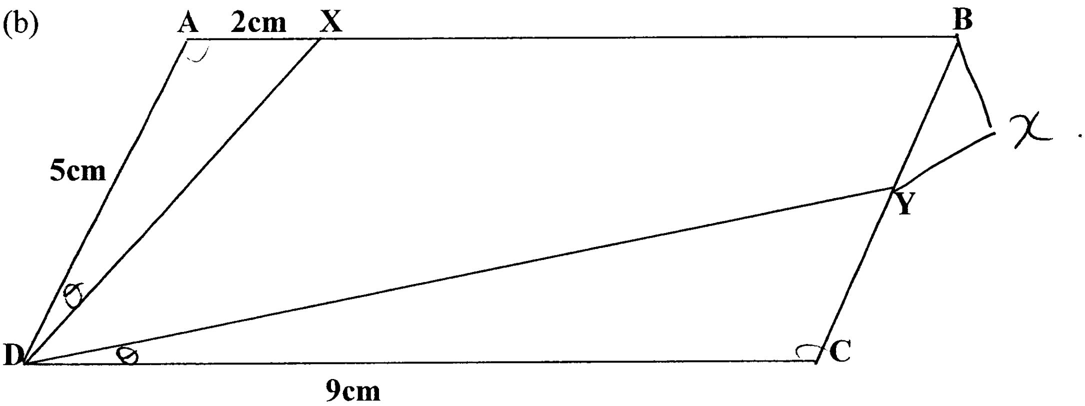
(ii) The equation of the curve $y = f(x)$ **(2)**

(Marks)

Question 8 Start a new page

(a) Simplify $1 + \dfrac{\cos^2 \vartheta}{\sin^2 \vartheta}$ **(2)**

(b)



ABCD is a parallelogram where AD = 5cm, DC = 9cm.
DX and DY are drawn so that $\angle ADX = \angle CDY$
AX = 2cm

(i) Prove ΔAXD is similar to ΔCYD **(2)**

(ii) Find the length of BY **(2)**

(c) Given $y = 3\sin 2x$

(i) What is the period of this curve? **(1)**

(ii) Sketch the curve $y = 3\sin 2x$ for $0 \leq x \leq 2\pi$ **(2)**

(d) (i) Graph $y = |2x - 3|$ for $0 \leq x \leq 4$ showing clearly the x and y intercepts **(2)**

(ii) Hence, or otherwise, find $\int_0^4 |2x - 3|\,dx$ **(1)**

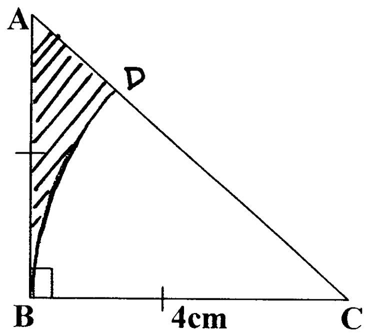
(Marks)

Question 9 Start a new page

(a) (i) Solve $2x^2 - x - 1 = 0$ **(1)**

(ii) Hence, or otherwise, solve $2\cos^2 x - \cos x - 1 = 0$ for $0 \le x \le 2\pi$ **(2)**

(b)



2 1 1 −1

$(2x+1)(x-1)$

$-2x + x$

ΔABC is a right angled isosceles triangle with AB = BC = 4cm.
An arc, centre C and radius 4cm is drawn to cut the side AC at D

(i) Explain why $\angle BCD$ is equal to 45° **(1)**

(ii) Show that the shaded area ABD is $2(4 - \pi)$ cm^2 **(2)**

(iii) Show that the perimeter of ABD is $(4\sqrt{2} + \pi)$cm **(2)**

(c) Dan is doing the following question for homework:

"The area enclosed by $y = f(x)$ and the x axis is rotated about the x axis. Find its volume."

Dan writes down the correct statement:

$$\text{Volume} = 2\pi \int_0^3 36x^2 - 12x^3 + x^4 \, dx$$

X **(i)** Find the equation of $f(x)$ **(2)**

(ii) Continue with Dan's question and find the volume in the question. Answer in terms of π **(2)**

(Marks)

Question 10 Start a new page

(a) Find the gradient of the normal to $y = \dfrac{x}{\sqrt{1-x}}$ at $x = -3$ **(3)**

(b) (i) Show that $f(x) = (x^3 - x)^3$ is an odd function **(2)**

(ii) Find $\displaystyle\int_{-2}^{2} (x^3 - x)^3 \, dx$ **(1)**

(b) A metal container of volume 1024π cm^3 is to be made in the shape of a cylinder, closed at both ends, with radius r cm and height h cm.

(i) Show that the amount A cm^2 of sheet metal used is given by

$$A = 2\pi r^2 + \frac{2048\pi}{r}$$ **(2)**

(ii) Find the minimum amount of sheet metal used. **(4)**

END OF EXAMINATION

$(x^3 - x)(x^3 - x)(x^3 - x)$

$x^{12} - x^{10} - 2x^7 + 2x^5 + x^6 - x^3$

2unit

QUESTION 1

a) $4(2x-8)-(x-1)$
$= 8x-32-x+1$
$= 7x-31$

b) $y = 2x^2-8x+1$
$y' = 4x-8$

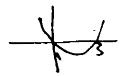
c) $|x-8| = 2$
$x-8=2 \qquad -x+8=2$
$x=10 \qquad x=6$

d) 1.44×10^9

e) $4x(x^2-1)$
$= 4x(x-1)(x+1)$

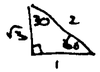
f) $\int x^2-2\cancel{x}\,dx = \frac{x^3}{3}-x^2+c$

c) $x^2-4x+3 \geq 0$



$(x-3)(x-1)\geq0$
$x\leq1 \qquad x\geq3$

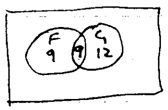
d) $\tan 300^\circ = -\sqrt3$



e) $\frac{5\pi}{8}\times\frac{180}{\pi} = 112.5^\circ$

f) $\angle EDF = 112^\circ$ (cointerior $\angle$s, $BC \parallel DF$)
$\angle FED + \angle DFE = 180-112 = 68$ (angle sum of Δ)
$\theta = 68\div2$ (isosceles Δ)
$\theta = 34^\circ$

g)



$18+21 = 39$

P(only French) $= \frac{9}{30} = \frac{3}{10}$

QUESTION 2

a) $\frac{x-8}{5} - \frac{x+1}{3} = 2$
$3(x-8)-5(x+1) = 30$
$3x-24-5x-5 = 30$
$-2x-29 = 30$
$-2x = 59$
$x = -29.5$

b) $y = \frac15\cos(3x)$
$y' = -\frac35\sin(3x)$

QUESTION 3

i) $9x-2y+20=0 \quad \ldots(1)$
$3x+y-10=0 \quad \ldots(2)$
$(2)\times2 \quad 6x+2y-20=0 \quad \ldots(3)$
$(3)+(1) \quad 15x=0$
$x=0$
$y=10 \qquad P(0,10)$

ii) $Q(-2,1) \quad R(4,-2)$
$m = \frac{y_2-y_1}{x_2-x_1} \qquad y-y_1 = m(x-x_1)$
$= \frac{-2-1}{4+2} \qquad y-1 = -\frac12(x+2)$
$= \frac{-3}{6} \qquad 2y-2 = -x-2$
$= -\frac12 \qquad x+2y=0$

iii) $P(0,10) \quad x+2y=0$
$p.d = \frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$
$= \frac{|1(0)+2(10)+0|}{\sqrt{1^2+2^2}}$
$= \frac{20}{\sqrt5}\times\frac{\sqrt5}{\sqrt5}$
$= \frac{20\sqrt5}{5}$
$= 4\sqrt5$

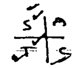
iv) $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
$= \sqrt{(4+2)^2+(-2-1)^2}$
$= \sqrt{36+9}$
$= \sqrt{45}$
$= 3\sqrt5$
Area $= \frac12\times3\sqrt5\times4\sqrt5$
$= 30$ units^2

b) i) $y = \frac{2x^2}{3x-1}$
$y' = \frac{(3x-1).4x-2x^2.3}{(3x-1)^2}$
$= \frac{12x^2-4x-6x^2}{(3x-1)^2}$
$= \frac{6x^2-4x}{(3x-1)^2}$

ii) $y = x^3\sin x$
$y' = x^3.\cos x - 3x^2\,\cancel{\sin x}$

QUESTION 4

a) $2\cancel{\cos}\,x = -\sqrt3$
$\cancel{\cos}\,x = -\frac{\sqrt3}{2}$



$x = \cancel{240^\circ}, \cancel{300^\circ}\ 150^\circ, 210^\circ$

b) i) $\alpha+\beta = -\frac ba \qquad a=2 \quad b=-6, \ c=1$
$= -\frac{-6}{2}$
$= 3$

ii) $\alpha\beta = \frac ca$
$= \frac12$

iii) $\alpha^2+\beta^2 = (\alpha+\beta)^2-2\alpha\beta$
$= (3)^2-2.\frac12$
$= 8$

iv) $\frac{1}{\alpha^2}+\frac{1}{\beta^2} = \frac{\beta^2+\alpha^2}{(\alpha\beta)^2}$
$= \frac{8}{1/4}$
$= 32$

c) $4+8+16+32+\cdots$ Geometric $r=2$
$S_n = \frac{a(r^n-1)}{r-1}$
$S_{15} = \frac{4(2^{15}-1)}{2-1}$
$= 131068$

d) i) $\int\frac{4x^3-x}{4}dx = \frac14\int4x^3-x\,dx$
$= \frac14\left(x^4-\frac{x^2}{2}\right)+c$
$= \frac{x^4}{4}-\frac{x^2}{8}+c$

ii) $\int(2-3x)^3dx = \frac{(2-3x)^4}{4\times-3}+c$
$= \frac{(2-3x)^4}{-12}+c$

-1 if no constant

QUESTION 5

a) i) $x^2+(k+2)x+4=0$

$\Delta = b^2-4ac$

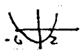
$=(k+2)^2-4.1.4$

$=k^2+4k+4-16$

$=k^2+4k-12$

ii) No real roots $\Delta<0$

$k^2+4k-12<0$

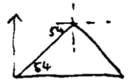
$(k+6)(k-2)<0$ 

$-6<k<2$

b) i) $\cos\angle BRM = \dfrac{70^2+89^2-75^2}{2\times70\times89}$

$\angle BRM = 54^\circ 43'$

$=54^\circ$ ~~to nearest degree~~.

ii) 

$270-54=216$

$\therefore\ 216^\circ$

c) 4, 4·5, 5, 5·5, ... Arithmetic

$a=4,\ d=0{\cdot}5$

i) $T_n=a+(n-1)d$

$T_{20}=4+(19)0{\cdot}5$

$=13{\cdot}5$ km

ii) $S_{20}=\frac{n}{2}(a+\ell)$

$=\frac{20}{2}(4+13{\cdot}5)$

$=175$ km

iii) $21=a+(n-1)d$

$21=4+(n-1)0{\cdot}5$

$21=3{\cdot}5+0{\cdot}5n$

$n=35 \therefore 35^{th}$ day

d) $y=\tan x$

at $x=\frac{\pi}{4}$, $y=\tan\frac{\pi}{4}$

$=1$ $\quad\therefore\left(\frac{\pi}{4},1\right)$

$y'=\sec^2x$

$=\sec^2\left(\frac{\pi}{4}\right)$

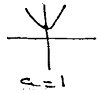
$=2$

$y-y_1=m(x-x_1)$

$y-1=2\left(x-\frac{\pi}{4}\right)$

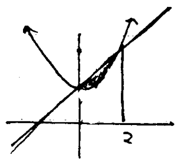
$y=2x-\frac{\pi}{2}+1$

QUESTION 6

a i) $x^2=4(y-1)$ 

vertex $(0,1)$

Focus $(0,2)$

ii) 

$x^2=4(y-1)$ $\quad 2y=x+2$

$x^2+4=4y$ $\quad y=\frac{1}{2}(x+2)$

$\frac{x^2}{4}+1=y$

$A=\int_0^2\left(\frac{1}{2}x+1\right)dx-\int_0^2\left(\frac{x^2}{4}+1\right)dx$

$=\left[\frac{x^2}{4}+x\right]_0^2-\left[\frac{x^3}{12}+x\right]_0^2$

$=(1+2-0)\ -\ \left(\frac{8}{12}+2-0\right)$

$=3\ -2\frac{2}{3}$

$=\frac{1}{3}$ unit^2

b) i) $x=a,\ x=c,\ x=e$ $\quad -\frac{1}{2}$ per mist.

ii) $x=b,\ x=d,\ x=e$

iii) $a<x<c$

iv) $0\le x<b,\ d<x<e$

c) i) $r=\frac{1}{\sqrt{3}}\doteqdot 0{\cdot}577$

For limiting sum $-1<r<1$

and r is between -1 and 1

ii) $S_\infty=\frac{a}{1-r}$

$=\dfrac{2}{1-\frac{1}{\sqrt3}}$

$=\dfrac{2}{\frac{\sqrt3-1}{\sqrt3}}$

$=2\times\dfrac{\sqrt3}{\sqrt3-1}$

$=\dfrac{2\sqrt3}{\sqrt3-1}\times\dfrac{\sqrt3+1}{\sqrt3+1}$

$=\dfrac{6+2\sqrt3}{3-1}$

$=\dfrac{6+2\sqrt3}{2}$

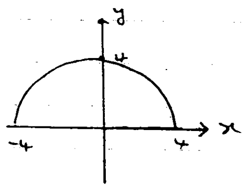
$=3+\sqrt3$.

QUESTION 7

a)

11	21	31	41	51✓	61✓
12	22	32	42✓	52	62✓
13	23	33✓	43	53	63✓
14	24✓	34	44	54	64✓
15✓	25	35	45	55	65✓
16✓	26✓	36✓	46✓	56✓	66✓

$\frac{16}{36}=\frac{4}{9}$

b) i) 

ii) D: $-4\le x\le 4$

R: $0\le y\le 4$.

iii) $A=\frac{2}{3}\left(4+0+4\sqrt{12}\right)$

x	0	2	4
y	4	$\sqrt{12}$	0

$=9{\cdot}2376$

$=9{\cdot}24\ u^2$

iv) $A=\frac{1}{4}\pi r^2$

$=\frac{1}{4}\pi.4^2$

$=4\pi\ u^2$

c) $\frac{dy}{dx}=3x^2-4x+c$

i) $\frac{dy}{dx}=0$ at $x=2$

$0=3(2)^2-4(2)+c$

$0=4+c$

$c=-4$

ii) $\frac{dy}{dx}=3x^2-4x-4$

$y=x^3-2x^2-4x+k$

$(2,-5)$ $\quad -5=(2)^3-2(2)^2-4(2)+k$

$3=k$

$\therefore y=x^3-2x^2-4x+3$

QUESTION 8

a) $1+\dfrac{\cos^2\theta}{\sin^2\theta}=1+\cot^2\theta$

$=\operatorname{cosec}^2\theta$

b) i) $\angle ADX=\angle CDY$ (given)

$\angle DAX=\angle DCY$ (opposite $\angle$s of parallelogram eq)

$\therefore \angle AXD=\angle CYD$ (3rd $\angle$ of Δ)

$\therefore \Delta AXD\ |||\ \Delta CYD$ (3 angles equal)

ii) $\frac{2}{CY}=\frac{5}{9}$

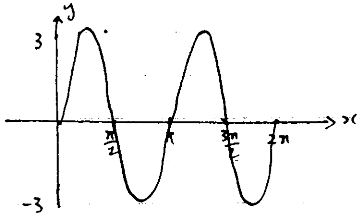
$CY=\frac{18}{5}=3{\cdot}6$

$\therefore BY=5-3{\cdot}6$

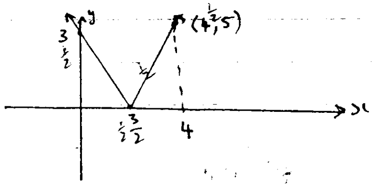
$=1{\cdot}4$ cm.

i) period $= \pi$.

ii)



d) i) $y = |2x-3|$



ii) $\int_0^4 |2x-3|\,dx = \left(\frac{1}{2}\times\frac{3}{2}\times 3\right) + \left(\frac{1}{2}\times 2\frac{1}{2}\times 5\right)$

$= 2\frac{1}{4} + 6\frac{1}{4}$

$= 8\frac{1}{2}$

QUESTION 9

a i) $2x^2 - x - 1 = 0$

$(2x+1)(x-1) = 0$

$x = -\frac{1}{2},\ 1$

ii) $2\cos^2 x - \cos x - 1 = 0$

$\cos x = -\frac{1}{2},\ 1$

$x = 120°, 240°, 0°, 360$

$= \frac{2\pi}{3}, \frac{4\pi}{3}, 0, 2\pi$

b) i) $\angle BCD = (180-90) \div 2$ (isosceles Δ)

$= 45°$

ii) Area ABD $= \left(\frac{1}{2}\times 4\times 4\right) - \left(\frac{1}{2}\cdot 4^2 \cdot \frac{\pi}{4}\right)$

$= 8 - 2\pi$

$= 2(4-\pi)\ \text{cm}^2$

$l = r\theta$

iii) $BD = 4 \cdot \frac{\pi}{4}$

$= \pi$

$AC^2 = 4^2 + 4^2$

$AC = \sqrt{32}$

$= 4\sqrt{2}$

$\therefore AD = 4\sqrt{2} - 4$

$\therefore$ Perimeter $= 4 + \pi + 4\sqrt{2} - 4$

$= \pi + 4\sqrt{2}$ m.

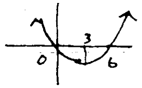
c) $V = 2\pi\int_0^3 36x^2 - 12x^3 + x^4\,dx$

$y^2 = 36x^2 - 12x^3 + x^4$

$y^2 = x^2(x^2 - 12x + 36)$

$y^2 = x^2(x-6)^2$

$y = x(x-6)$



$V = 2\pi\left[\frac{36x^3}{3} - \frac{12x^4}{4} + \frac{x^5}{5}\right]_0^3$

$= 2\pi\left(\frac{36(3)^3}{3} - \frac{12(3)^4}{4} + \frac{3^5}{5}\right) - 0$

$= 2\pi(129\cdot 6)$

$= 259\cdot 2\pi\ u^3$

QUESTION 10

a) $y = \frac{x}{\sqrt{1-x}}$ $y' = \frac{vu' - uv'}{v^2}$

$y' = \frac{\sqrt{1-x}\cdot 1 - x\cdot\frac{1}{2}(1-x)^{-1/2}\cdot -1}{(\sqrt{1-x})^2}$

$= \frac{\sqrt{1-x} + \frac{x}{2}(1-x)^{-1/2}}{1-x}$

at $x = -3$, $y' = \frac{5}{16}$

$\therefore$ gradient of normal $= -\frac{16}{5}$

b) i) $f(x) = (x^3 - x)^3$

$f(-x) = ((-x)^3 - (-x))^3$

$= (-x^3 + x)^3$

$= (-(x^3 - x))^3$

$= -(x^3 - x)^3$

$f(-x) = -f(x)$ $\therefore$ odd function

c) i) $A = 2\pi rh + 2\pi r^2$

$= 2\pi r\left(\frac{1024}{r^2}\right) + 2\pi r^2$ $\frac{1}{2}$

$\pi r^2 h = 1024\pi$ $\frac{1}{2}$

$h = \frac{1024\pi}{\pi r^2} = \frac{1024}{r^2}$

$= \frac{2048\pi}{r} + 2\pi r^2$ $\frac{1}{2}$ $= 2\pi r^2 + 2048\pi r^{-1}$

ii) $A' = 4\pi r - 1.2048\pi r^{-2}$ 1

$0 = 4\pi r - \frac{2048\pi}{r^2}$

$0 = 4\pi r^3 - 2048\pi$

$2048\pi = 4\pi r^3$

$512 = r^3$

$8 = r$ 1

$A'' = 4\pi + 2.2048\pi r^{-3}$ 1

at $r = 8$ $A'' > 0$ $\therefore$ min.

$\therefore A = 2\pi(8)^2 + \frac{2048\pi}{8}$

$= 128\pi + 256\pi$

$= 384\pi$ 1

$= 1206\cdot 3716$