The Scots College

2016

HIGHER SCHOOL CERTIFICATE
Pre-TRIAL EXAMINATION

Mathematics 2U

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- A reference sheet is provided at the back of this paper
- Write your Student Number on every Writing Booklet
- In Questions 11 - 16, show relevant mathematical reasoning and/or calculations

Total marks – 100

Section I

10 marks
- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II

90 marks
- Attempt Questions 11–16
- Allow about 2 hours and 45 minutes for this section

10 marks
Allow about 15 minutes for this section
Use the multiple-choice answer sheet for Questions 1–10.

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1 What is the value of $\sqrt{\frac{19}{2\pi}}$, correct to four significant figures?

(A) 3.0239

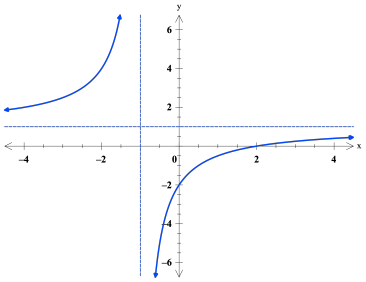
(B) 3.024

(C) 1.7389

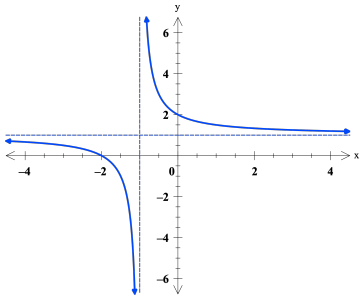
(D) 1.739

2 Which graph best represents $y = \frac{x+2}{x+1}$?

(A)



(B)



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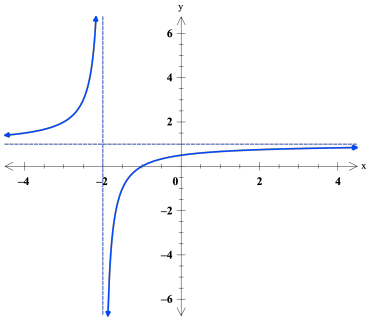

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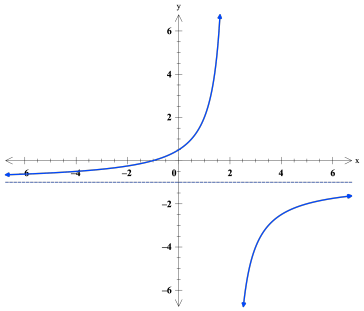


3 What is the solution to the equation $log_3(x + 1) = 7$

(A) 343 (B) 2187 (C) 2186 (D) 342

4 What is the derivative of $\frac{\sin x}{x}$?

(A) $\frac{\cos x - \sin x}{x^2}$

(B) $\frac{x \cos x - \sin x}{x^2}$

(C) $\frac{\cos x - x \sin x}{x^2}$

(D) $\frac{x \cos x + \sin x}{x^2}$

5 The gradient of the line which makes 120° with the positive direction of the x-axis is

(A) $\frac{1}{\sqrt{3}}$

(B) $-\sqrt{3}$

(C) $-\frac{1}{\sqrt{3}}$

(D) $\sqrt{3}$

6 For what values of c is the line $y = -3x + c$ a tangent to the parabola $y = x^2 + x - 1$?

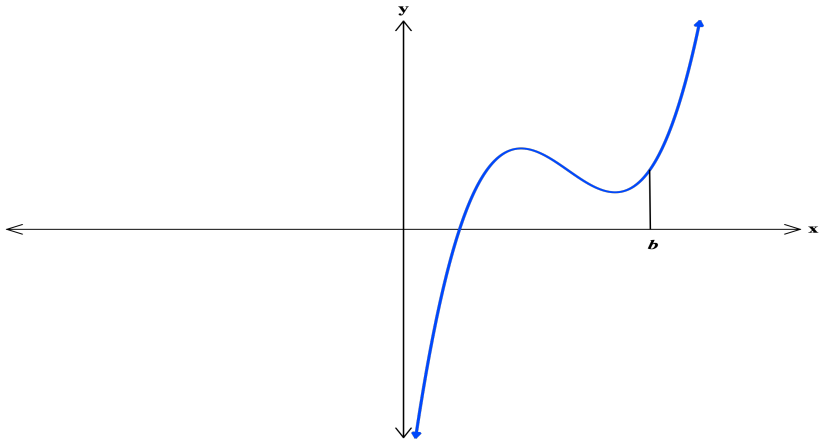
(A) -5

(B) -4

(C) 4

(D) 0

7 The diagram shows the graph of $y = f(x)$.



(A) $f'(b) < 0 \ and \ f''(b) < 0$

(B) $f'(b) < 0 \ and \ f''(b) > 0$

(C) $f'(b) > 0 \ and \ f''(b) < 0$

(D) $f'(b) > 0 \ and \ f''(b) > 0$

8 What is the value of $\int_1^2 \frac{1}{4x} \, dx$?

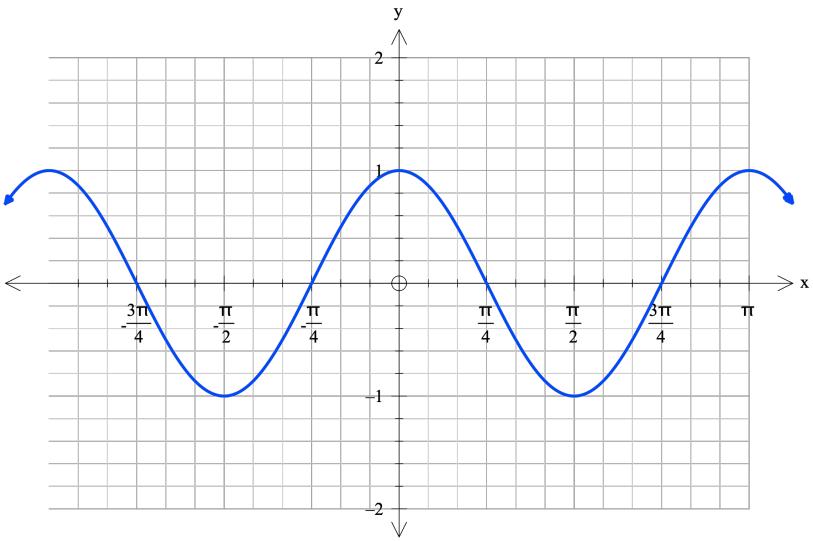
(A) $\frac{1}{2} \ln 4$

(B) $\ln 2$

(C) $\frac{1}{4} \ln 2$

(D) $\ln 4$

9 The graph below



is best represented by the function

(A) $y = \sin(2x + \pi)$

(B) $y = \sin\left(2x + \frac{\pi}{3}\right)$

(C) $y = \sin(x + \pi)$

(D) $y = \sin\left(2x + \frac{\pi}{2}\right)$

10 Let $y = e^x$ then $log_e(y^3)$ is equal to the expression

(A) e^{3x} (B) x^3 (C) e^{x^3} (D) $3x$

Section II (90 marks)

Attempt Questions 11–16

Allow about 2 hours and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11 - 16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE booklet.

a) Solve $x^2 - 5x + 6 \geq 0$ [2]

b) Solve $|3x - 2| = 4$. [2]

c) Find x and y if $x + y\sqrt{11} = \left(4 - 2\sqrt{11}\right)^2$. [2]

d) Evaluate $\log_5 9$, correct to 2 significant figures. [2]

e) If the line $kx + 5y - 7 = 0$ makes an angle of $45°$ with the positive side of the $x - axis$, find the value of k. [2]

f) Evaluate $\int_0^{\frac{\pi}{2}} \cos\frac{x}{2}\ dx$. [3]

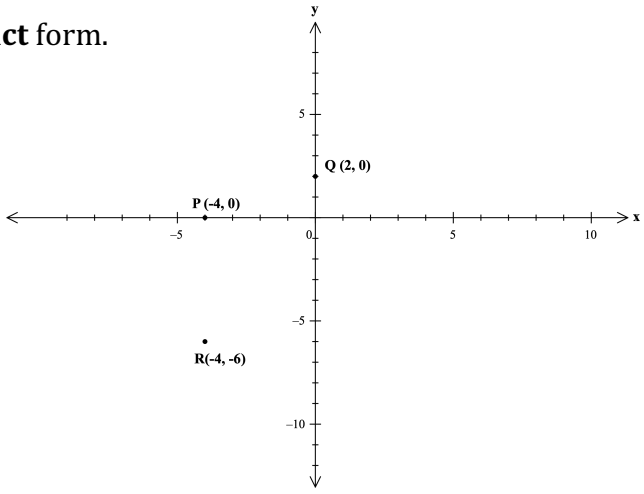
g) i) Differentiate $y = \cos x^2$ [2]

ii) Hence find $\int 2x \sin x^2\ dx$

Question 12 (15 marks) Use a SEPARATE booklet.

a) The coordinates of points P, Q and R are $(-4\ ,0), (0\ ,2)$ and $(-4\ ,-6)$ respectively.

Leave all your answers in **exact** form.



i) Find the gradient of the interval PQ. [1]

ii) Find the length of the interval PQ [1]

iii) Find the equation of the line l , parallel to PQ and passing through R. [2]

iv) The line l intersects the $x-axis$ at S. Find the coordinates of S. [2]

v) Find the area of the triangle ΔQRS to one decimal place. [3]

b) Sketch neatly, on half a side of a page, $y = 3\sin 2x$, [3]
for $0 \leq x \leq 2\pi$ stating its amplitude and period.

c) The sum of the first seven terms of an Arithmetic Progression is 56. If the sum of the first and third term is zero, find the first term and the common difference. [3]

Question 13 (15 marks) Use a SEPARATE booklet.

a) Differentiate the following with respect to x.

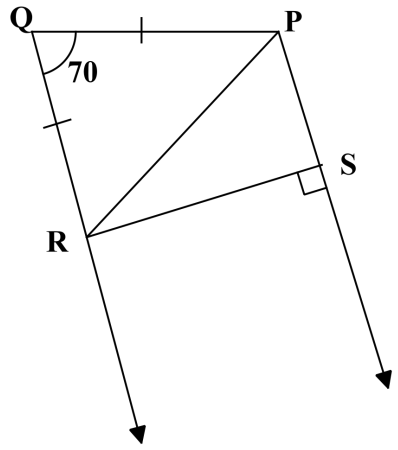
i) $y = 5x^4\left(3x^2 - \frac{2}{x^2}\right)$ [2]

ii) $y = x^2\, e^{5x}$ [2]

iii) $y = \frac{\log_e x}{3x^2}$ [3]

iv) $y = \frac{1}{x} + (\log_e x)^3$ [2]

b) In the figure, $PQ = QR$, $PS \parallel QR$, $\angle PQR = 70°$ and $\angle PSR = 90°$. Calculate the size of $\angle PRS$. Show all working and give reasons. [3]



c) Given $\sum_1^n (4k - 3)$ find S_{20}. [3]

Question 14 (15 marks) Use a SEPARATE booklet.

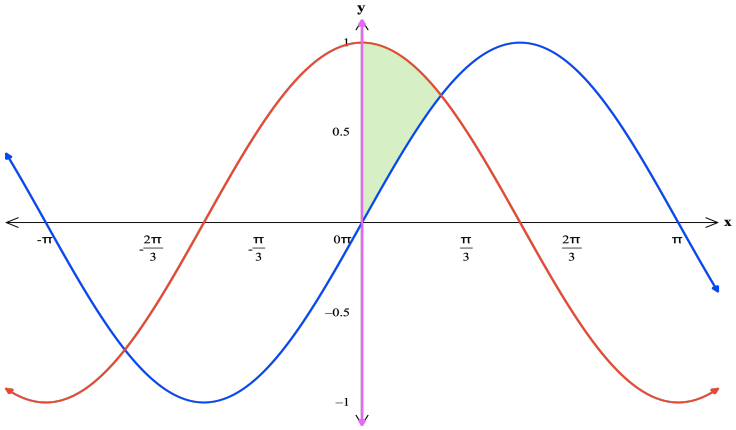
a) Use the trapezoidal rule with five function values to approximate the integral [4]

$$\int_1^2 x \log_{10} x \, dx$$

b) Use the Simpson's rule with five function values to approximate the integral [4]

$$\int_0^{\frac{\pi}{2}} \sin^2 x \; dx$$

c) Find the area of the region shaded in the diagram below. The region is bounded by $y = \sin x$, $y = \cos x$ and the $y - axis$. Hint : Find the point of intersection frist. [3]



d) If $y = e^{-x} \sin 2x$, simplify $2\frac{dy}{dx} + 5y$. [2]

e) Calculate the gradient of the function $y = \frac{1}{5}\tan^5 x + x$ at $x = \frac{\pi}{4}$. [2]

Question 15 (15 marks) Use a SEPARATE booklet.

a) Find the value of k if : [2]

$\int_1^k (k + 5x)\, dx = -\frac{5}{2}$

b) Find the following integrals:

i) $\int x^3(x^2 - 7)\, dx$ [1]

ii) $\int \left(\frac{x}{4} - 7\right)^3\, dx$ [2]

iii) $\int \frac{2}{1-2x}\, dx$ [2]

c) i) Solve simultaneously $y = x^3$ and $y = x^2$. [3]

ii) Sketch both the curves on the same graph for $0 \leq x \leq 2$ and shade the area between them.

iii) Find the area of the shaded region.

d) i) Show that $y = e^x - 1$ and $y = (e - 1)x$ intersect at $x = 0$ and $x = 1$ [1]
.

ii) Sketch both the curves on the same graph. [2]

iii) Find the area enclosed between the graphs from $x = 0$ *to* $x = 1$. [2]

Question 16 (15 marks) Use a SEPARATE booklet.

a) Consider the curve $y = x - e^x$. [5]

i) Find y' and y''.

ii) Find the coordinates of the stationary point.

iii) Explain why the curve is concave down for all x.

iv) Sketch the curve and write down its range.

b) i) Find the equation of the Normal to the curve $y = 5\log_e x + 7$ at $x = 1$. [3]

ii) Find the stationary point on the curve $y = x - \log_e x$ and determine it's nature. [2]

c) i) Sketch the curve $y = e^x - 1$ and shade the region bounded by $y = e^x - 1$, $y = 2$ and the $y - axis$. [1]

ii) Show that the volume of the solid generated by revolving the region in part i), about the $x - axis$ is [4]

$$3\pi \ln 3.$$

End Of Examination

Solution MC and Q11

Monday, 14 March 2016 11:59 AM

1	D	6	A
2	B	7	D
3	C	8	C
4	B	9	D
5	B	10	D

Q11.

(a) $x^2 - 5x + 6 \geq 0$

$(x-2)(x-3) \geq 0$ ✓

$x \leq 2$ or $x \geq 3$ ✓

(b) $|3x-2| = 4$

$3x - 2 = -4$ or $3x - 2 = 4$ ✓

$x = -\frac{2}{3}$ or $x = 2$ ✓

(c) $x + y\sqrt{11} = (4 - 2\sqrt{11})^2$

$= 16 - 16\sqrt{11} + 4 \times 11$ ✓

$= 60 - 16\sqrt{11}$

$\therefore x = 60$

$y = 16$ ✓

(d) $\log_5 9 = \frac{\log_{10} 9}{\log_{10} 5}$ ✓

$= 1.3652$ ✓

(e) $kx + 5y - 7 = 0$

$y = -\frac{k}{5}x + \frac{7}{5}$

$m = \tan 45 = 1$

$\Rightarrow -\frac{k}{5} = 1 \Rightarrow k = -5$ ✓ ✓

(f) $\int_0^{\pi/2} \cos \frac{x}{2}\, dx$

$= 2\left[\sin \frac{x}{2}\right]_0^{\pi/2}$ ✓

$= 2\left\{\sin \pi/4 - \sin 0\right\}$

$= 2 \times \frac{\sqrt{2}}{2}$ ✓

$= \sqrt{2}$ ✓

(g) $y = \cos x^2$

$\frac{dy}{dx} = 2x(-\sin x^2)$ ✓

$\int 2x \sin x^2\, dx$

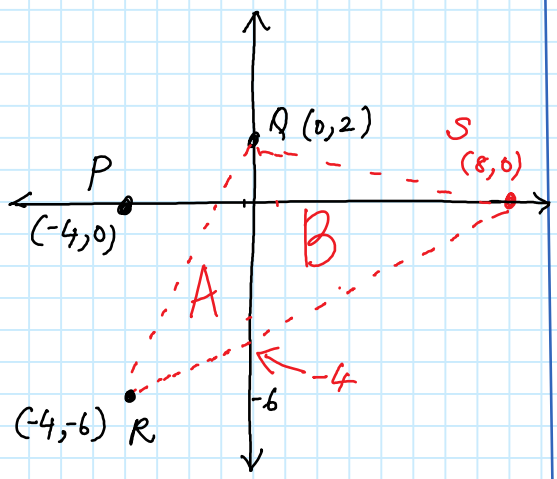
$= -\int -2x \sin x^2\, dx$

$= -\cos x^2$ ✓

Q 12

Monday, 14 March 2016 7:57 PM

(a)



(i) $m_{par} = \frac{2-0}{0--4} = \frac{1}{2}$ ✓

(ii) $d = \sqrt{(0--4)^2 + (2-0)^2} = \sqrt{16+4}$

$= \sqrt{20}$ ✓

(iii) $y - -6 = \frac{1}{2}(x--4)$ ✓

$2y + 12 = x + 4$

$l: \quad x - 2y - 8 = 0$ ✓

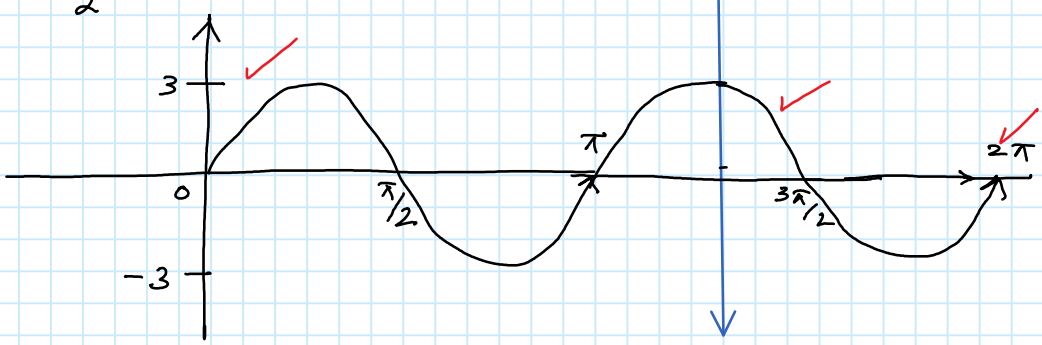
(iv) $y = 0 \Rightarrow x = 8 \quad S(8,0)$ ✓

(v) Area $\triangle QRS$

(b) $y = 3\sin 2x \qquad 0 \le x \le 2\pi$

Amplitude $= 3$

$T = \frac{2\pi}{2} = \pi$



(c) $S_7 = \frac{7}{2}\{2a + 6d\} =$

$56 = 7\{a + 3d\}$

$8 = a + 3d \quad -(1)$ ✓

$T_1 + T_3 = 0$

$a + a + 2d = 0$

$2a + 2d = 0$

$a + d = 0 \quad -(2)$ ✓

$(2) - (1) \Rightarrow 2d = 8$

$d = 4$ ✓

$\therefore \quad a = -4.$

Eqn of SR.

$y + 6 = \frac{6}{12}(x+4)$

$y = \frac{1}{2}x + 2 - 6$

$y = \frac{1}{2}x - 4$ ✓ y-intercept

Area of $\triangle A = \frac{1}{2} \times 6 \times 4 = 12$

Area of $\triangle B = \frac{1}{2} \times 6 \times 8 = 24$ ✓

Total Area $\triangle QRS = 36 \text{ u}^2$ ✓

Consider any other method too.

Q 13

Monday, 14 March 2016 10:04 PM

a (i) $y = 5x^4\left(3x^2 - \frac{2}{x^2}\right)$

$y = 15x^6 - 10x^2$ ✓

$y' = 90x^5 - 20x$ ✓

(ii) $y = x^2 e^{5x}$

$y' = e^{5x} 2x + x^2 5e^{5x}$

$= 2xe^{5x}$ ✓ $+ 5x^2 e^{5x}$ ✓

(iii) $y = \frac{\ln x}{3x^2}$

$y' = \frac{3x^2 \frac{1}{x} - \ln x (6x)}{9x^4}$ ✓

$= \frac{3x - 6x \ln x}{9x^4}$ ✓

$= \frac{1 - 2\ln x}{3x^3}$ ✓

(iv) $y = \frac{1}{x} + (\ln x)^3$

$y' = -\frac{1}{x^2}$ ✓ $+ 3 \ln x \times \frac{1}{x}$ ✓

$= -\frac{1}{x^2} + \frac{3}{x} \ln x$ ✓

(b) $\angle QRP = \angle QPR$ (isosceles $\triangle$)

$\angle QRP = \frac{180-70}{2} = 55°$ ✓

$\angle RPS = \angle QRP$ (alt. $\angle$s on parallel lines)

$= 55°$ ✓

$\therefore \angle PRS = 180° - 90° - 55°$ ($\angle$ sum in $\triangle$)

$= 35°$ ✓

(c) $\sum_{k=1}^{n} (4k-3)$

$= 4\times1 - 3 + 4\times2 - 3 + \ldots 4n - 3$

$= 1 + 5 + 9 + \ldots + 4n - 3$ ✓

$S_n = \frac{n}{2}\{2\times1 + (n-1)4\}$ ✓

$= n\{1 + (n-1)2\}$

$= n\{1 + 2n - 2\}$

$= n\{2n - 1\}$ ✓

Q 14

Tuesday, 15 March 2016 10:25 PM

(a)

x	1	1.25	1.5	1.75	2
$f(x)$	0	0.12113	0.26414	0.42532	0.60205

$$\int_1^2 x \log_{10} x \, dx \doteq \frac{0.25}{2}\{0 + 2\times 0.12113 + 2\times 0.26414 + 2\times 0.42532 + 0.60205\}$$

$$\doteq \frac{0.25}{2}\{2.22323\}$$

$$\doteq 0.2779$$

(b) Simpson's Rule

$$\int_0^{\pi/2} \sin^2 x \, dx$$

x	0	$\pi/8$	$\pi/4$	$\frac{3\pi}{8}$	$\pi/2$
$\sin^2 x$	0	0.1464	$\frac{1}{2}$	0.8536	1

$$\doteq \frac{\pi}{24}\{0 + 4\times 0.1464 + 2\times\tfrac{1}{2} + 4\times 0.8536 + 1\}$$

$$\doteq \frac{\pi}{24}\{6\}$$

$$\doteq \frac{\pi}{4}$$

(c) $$\text{Area} = \int_0^{\pi/4} \cos x - \sin x \, dx$$

$$= \sin x + \cos x \Big]_0^{\pi/4}$$

$$= (\sin \pi/4 + \cos \pi/4) - (\sin 0 + \cos 0)$$

$$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - (0 + 1)$$

$$= \frac{2\sqrt{2}}{2} - 1$$

$$= \sqrt{2} - 1$$

(c) $y = e^{-x}\sin 2x$

$$\frac{dy}{dx} = \sin 2x\,(-1)e^{-x} + e^{-x}\,2\cos 2x$$

$$= -e^{-x}\sin 2x + 2e^{-x}\cos 2x$$

$$2\frac{dy}{dx} = -2e^{-x}\sin 2x + 4e^{-x}\cos 2x \quad \text{— (1)}$$

$$5y = 5e^{-x}\sin 2x \quad \text{— (2)}$$

(1) + (2) $\Rightarrow$ $3e^{-x}\sin 2x + 4e^{-x}\cos 2x$

(d) $$y = \frac{1}{5}\tan^5 x + x$$

$$\frac{dy}{dx} = \frac{1}{5}\times 5\tan^4 x \sec^2 x + 1$$

$$= \tan^4 x(1 + \tan^2 x) + 1$$

at $x = \pi/4$

$$= 1(1 + 1) + 1$$

$$= 3$$

Q 15

Tuesday, 15 March 2016 11:16 PM

(a) $\int_1^k (k+5x)\,dx = -\frac{5}{2}$

$kx + \frac{5x^2}{2}\Big]_1^k = -\frac{5}{2}$ ✓

$k^2 + \frac{5k^2}{2} - \left(k + \frac{5}{2}\right) = -\frac{5}{2}$

$\frac{7k^2}{2} - k - \frac{5}{2} = -\frac{5}{2}$

$7k^2 - 2k = 0$

$k(7k-2) = 0$

$k = 0 \quad \text{or} \quad k = \frac{2}{7}$ ✓

(b) (i) $\int x^3(x^2-7)\,dx$

$= \int x^5 - 7x^3\,dx$

$\frac{x^6}{6} - \frac{7x^4}{4} + C$ ✓

(ii) $\int \left(\frac{x}{4} - 7\right)^3 dx$

$= \frac{\left(\frac{x}{4}-7\right)^4}{1} \times 4 + C$

$= \left(\frac{x}{4} - 7\right)^4 + C$ ✓

(iii) $\int \frac{2}{(1-2x)^4}\,dx$

$= 2\int (1-2x)^{-4}\,dx$

$= 2\,\frac{(1-2x)^{-3}}{-3} \times \left(\frac{1}{-2}\right) + 2C$

$= \frac{1}{3(1-2x)^3} + C$ ✓

(iv) $\int \frac{1}{\sqrt[3]{2x-1}}\,dx$

$= \int (2x-1)^{-1/3}\,dx$ ✓

$= \frac{(2x-1)^{2/3}}{2/3}\left(\frac{1}{2}\right) + C$

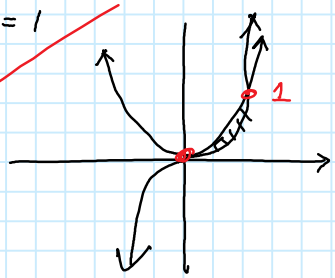
$= \frac{3}{4}(2x-1)^{2/3} + C$ ✓

(c) $y = x^3 \qquad y = x^2$

$x^3 - x^2 = 0$

$x^2(x-1) = 0$

$x = 0 \quad \text{or} \quad x = 1$ ✓



$A = \int_0^1 x^2 - x^3\,dx$

$= \frac{x^3}{3} - \frac{x^4}{4}\Big]_0^1$ ✓

$= \frac{1}{3} - \frac{1}{4} - 0 + 0$

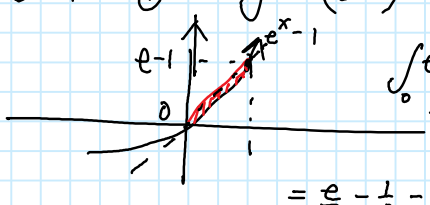
$= \frac{4-3}{12}$

$= \frac{1}{12}$ ✓

(d) $x = 0, \; y = e^0 - 1 = 0, \; y = e \times 0 - 0 = 0$

$x = 1, \; y = e^1 - 1 = e - 1 \qquad y = e \times 1 - 1 = e - 1$ ✓

$y = e^x - 1$ —(1) $\qquad y = (e-1)x$



$\int_0^1 ex - x - e^x + 1\,dx$

$\frac{ex^2}{2} - \frac{x^2}{2} - e^x + x\Big]_0^1$

$= \frac{e}{2} - \frac{1}{2} - e + 1 + 1$

$= -\frac{e}{2} + \frac{3}{2} = \frac{3-e}{2}$

Q 16

Monday, 21 March 2016 10:58 PM

(a) $y = x - e^x$

(i) $y' = 1 - e^x$

$y'' = -e^x$ ✓

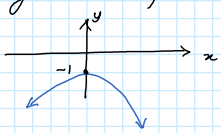
(ii) $y' = 0 \Rightarrow 1 - e^x = 0$

$e^x = 1$

$x = 0$ $\therefore y = 0 - e^0 = -1$

$(0, -1)$ ✓

(iii) Because $y'' = -e^x < 0$ for all x. ✓



✓

(iv) $R : y \leq -1$ ✓

(b)(i) $y = 5 \log_e x + 7$

$y' = 5 \times \frac{1}{x}$ ✓

at $x = 1$, $y' = 5$ = gradient

at $x = 1$, $y = 7$ point $(1, 7)$ } 1

Gradient of Normal $= -\frac{1}{5}$

equation : $y - 7 = -\frac{1}{5}(x - 1)$ } 1

$x + 5y - 36 = 0$

(ii) $y = x - \log_e x$

$y' = 1 - \frac{1}{x}$

$y' = 0 \Rightarrow 1 - \frac{1}{x} = 0$

$x = 1$

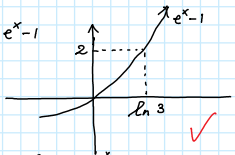
& $y = 1 - \ln 1 = 1$ $(1, 1)$ ✓

$y' = 1 - x^{-1}$

$y'' = \frac{1}{x^2} > 0$ $\therefore$ concave up for all x

$\therefore$ $(1, 1)$ is a minimum. ✓

(c) $y = e^x - 1$



✓

at $y = 2$, $e^x - 1 = 2$

$e^x = 3$

$x = \ln 3$

$V = \pi \int_0^{\ln 3} 2^2 - (e^x - 1)\, dx$ ✓

$= \pi \int_0^{\ln 3} 4 - e^{2x} + 2e^x - 1\, dx$

$= \pi \int_0^{\ln 3} 3 - e^{2x} + 2e^x\, dx$ ✓

$= \pi \left\{ 3x - \frac{e^{2x}}{2} + 2e^x \right\}_0^{\ln 3}$ ✓

$= \pi \left\{ \left(3\ln 3 - \frac{e^{2\ln 3}}{2} + 2e^{\ln 3} \right) - \left(0 - \frac{e^{2\times 0}}{2} + 2e^0 \right) \right\}$

$= \pi \left\{ 3\ln 3 - \frac{9}{2} + 6 + \frac{1}{2} - 2 \right\}$

$= \pi \{ 3\ln 3 + 0 \}$

$= 3\pi \ln 3$ ✓