Examination Number:

Set:

Shore

Year 12

Term II Examination

May 2016

Mathematics

General Instructions

- Reading time − 5 minutes
- Working time − 3 hours
- Write using black pen
- Board-approved calculators may be used
- A BOSTES Reference Sheet is provided
- Answer Questions 1–10 on the Multiple Choice Answer Sheet provided
- In Questions 11–16, show relevant mathematical reasoning and/or calculations
- Start each of Questions 11–16 in a new writing booklet
- Write your examination number on the front cover of each booklet
- If you do not attempt a question, submit a blank booklet marked with your examination number and "N/A" on the front cover

Total marks – 100

Section I Pages 3–6

10 marks

- Attempt Questions 1−10
- Allow about 15 minutes for this section

Section II Pages 7–14

90 marks

- Attempt Questions 11−16
- Allow about 2 hours 45 minutes for this section

DO NOT REMOVE THIS PAPER FROM THE EXAMINATION ROOM

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the Multiple Choice Answer Sheet for Questions 1–10.

1 What is the value of $\dfrac{4.56^3-\sqrt{78}}{\sqrt{6.8^2\times 9.3^6}}$ correct to 2 significant figures?

(A) 0.01

(B) 0.02

(C) 0.015

(D) 0.016

2 Which of the following is a simplification of $4m^{-2}\div\dfrac{1}{2}m^{-1}$?

(A) $\dfrac{8}{m^3}$

(B) $\dfrac{8}{m}$

(C) $\dfrac{2}{m^3}$

(D) $\dfrac{2}{m}$

3 Which of the following represents the solution to $|2x-3|\le 1$?

(A) $-2\le x\le 2$

(B) $x\le -2$ or $x\ge 2$

(C) $1\le x\le 2$

(D) $x\le 1$ or $x\ge 2$

4 What are the values of a and b if $\dfrac{3}{2+\sqrt{5}} = a + \sqrt{b}$?

(A) $a = 3 \quad b = -6$

(B) $a = -6 \quad b = 3$

(C) $a = -6 \quad b = 45$

(D) $a = 45 \quad b = -6$

5 Let $\log_a 2 = p$ and $\log_a 3 = q$.

Which of the following is the expression for $\log_a 24$?

(A) $3p + q$

(B) $p^3 + q$

(C) $3pq$

(D) $p^3 q$

6 The roots of $2x^2 - 4x + 7 = 0$ are α and β. What is the value of $\alpha^2 + \beta^2$?

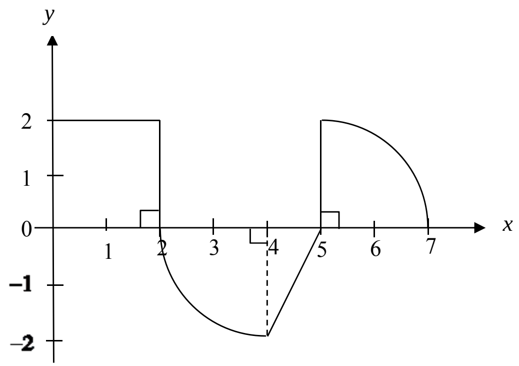
(A) –3

(B) 4

(C) 11

(D) $10\frac{1}{4}$

7 The graph of the function $y = f(x)$ is shown in the diagram.



Which of the following gives the exact value for $\int_0^7 f(x)\,dx$?

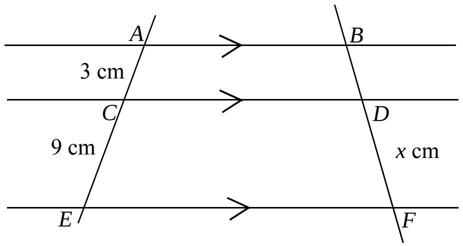
(A) π

(B) 3

(C) 5

(D) $\pi + 5$

8 In the diagram $AB \parallel CD \parallel EF$. $AC = 3$ cm, $CE = 9$ cm, $BF = 20$ cm and $DF = x$ cm.



NOT TO SCALE

What is the value of x?

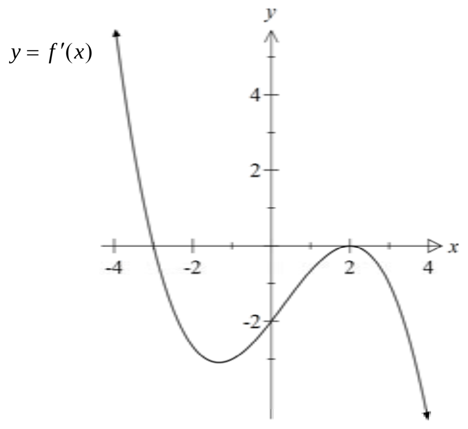
(A) 5

(B) $6\frac{2}{3}$

(C) 10

(D) 15

9 Consider the graph of the gradient function $y = f'(x)$ below.



Which one of the following statements is true for $y = f(x)$?

(A) There is a maximum turning point at $x = 2$.

(B) There is a minimum turning point at $x = -3$.

(C) There is a horizontal point of inflexion at $x = 2$.

(D) There is a point of inflexion at $x = 0$.

10 Let $c = e^x$. Which expression is equal to $\log_e(c^3)$?

(A) e^{3x}

(B) $3x$

(C) e^{x^3}

(D) x^3

End of Section I

Section II

90 marks
Attempt Questions 11–16
Allow about 2 hours 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11–16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet

(a) Simplify $3x - 2(x-1)$. **1**

(b) Evaluate $\log_3 5$ correct to three decimal places. **2**

(c) Evaluate $\lim_{x \to 2} \dfrac{x^3 - 8}{x - 2}$. **2**

(d) Find the primitive of $e^{5x} + 1$. **2**

(e) Given $f(x) = x^2 - 3x - 30$, find the value(s) of x if $f(x) = 10$. **2**

(f) Find the domain and range of $y = \dfrac{2}{x-3} + 1$. **2**

(g) Find the equation of the parabola with focus $(-3,1)$ and directrix $y = -1$. **2**

(h) Solve $\dfrac{2x-1}{4} + 1 = \dfrac{x}{3}$. **2**

Question 12 (15 marks) Use a SEPARATE writing booklet

(a) Solve $\sqrt{3}+\tan x=0$ for $0° \le x \le 360°$. **2**

(b) Differentiate the following with respect to x.

(i) $\log_e(3x^2-1)$. **2**

(ii) x^2e^x. **2**

(c) Find $\int xe^{5x^2}\,dx$. **2**

(d) Evaluate $\displaystyle\int_1^3 \frac{2x^3+x}{x^2}\,dx$. **3**

(e) Find the equation of the normal to the curve $y=3\log_e x$ at the point $(e,\,3)$. **3**

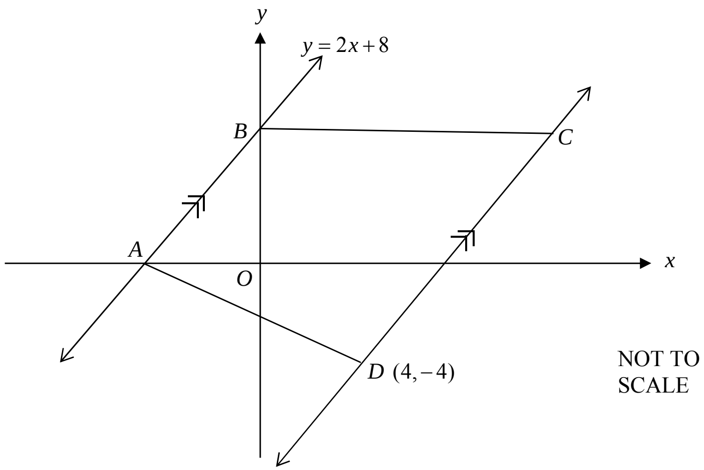
(f) Simplify $\sqrt{\sec^2 A-1}$. **1**

Question 13 (15 marks) Use a SEPARATE writing booklet

(a) In the diagram *ABCD* forms a trapezium.

Line *AB* has equation $y=2x+8$ and is parallel to line *CD*.

Line *BC* is parallel to the x-axis.



NOT TO SCALE

(i) Show that the equation of line *CD* is $2x-y-12=0$. **2**

(ii) Show that the perpendicular distance from *B* to the line *CD* is $4\sqrt{5}$ units. **2**

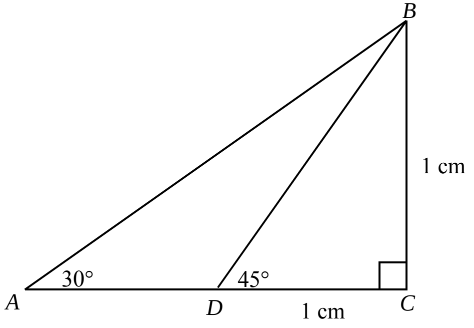
(iii) Find the co-ordinates of *C*. **1**

(iv) If $AB = 4\sqrt{5}$ units, find the area of the trapezium *ABCD*. **3**

(b) Find the values of k for which $x^2-2kx+1=0$ has real and different roots. **2**

Question 13 continues on the following page

(c) Consider the triangles *ABC*, *BAD* and *BDC* below.
$\angle ACB = 90°$, $\angle BDC = 45°$, $\angle BAD = 30°$ and *BC*= *DC* = 1 cm.



NOT TO SCALE

(i) Using triangle *ABC* show that the length of *AD* is $(\sqrt{3}-1)$ cm. **2**

(ii) Find the length of *BD*. **1**

(iii) Hence, or otherwise, show that $\sin 15° = \dfrac{\sqrt{3}-1}{2\sqrt{2}}$. **2**

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End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet

(a) Consider the series $4+7+10+...+301$.

(i) How many terms are in the series? **2**

(ii) Evaluate the sum of the series. **2**

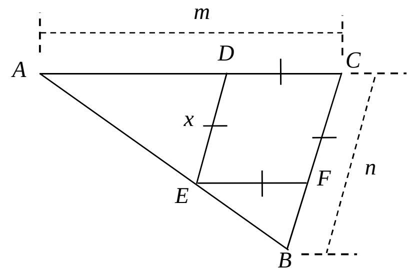
(b) Use Simpson's rule with 3 function values to find an approximation for **2**
$\displaystyle\int_4^8 \frac{1}{\log_e x}\,dx$. Give your answer correct to 3 significant figures.

(c) Consider the function $y = 2x^3$.

(i) Draw a neat sketch of this function. **1**

(ii) Find the area between the curve $y = 2x^3$, the *x*-axis and the lines $x = -1$ and $x = 3$. **3**

(d) The diagram shows a triangle *ABC* with sides $AC = m$ and $BC = n$. The points *D*, *E* and *F* lie on the sides *AC, AB* and *BC* respectively so that *CDEF* is a rhombus with sides of length *x*.



NOT TO SCALE

(i) Prove that ΔADE is similar to ΔEFB. **2**

(ii) Find an expression for *x* in terms of *m* and *n*. **3**

Question 15 (15 marks) Use a SEPARATE writing booklet

(a) Evaluate $\int_0^3 \frac{x}{x^2+3}\,dx$. Express your answer in simplest form. **3**

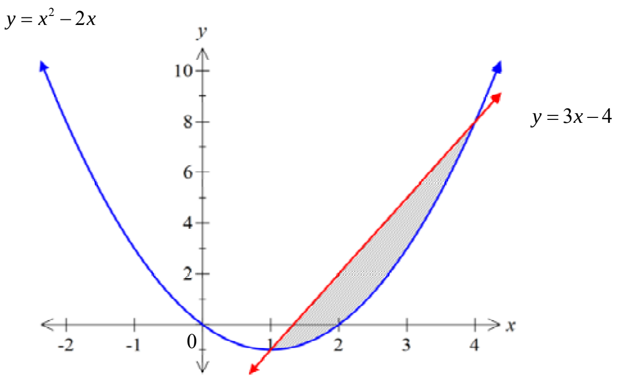
(b) Consider the curve given by $y = 1 + 3x - x^3$.

(i) Find the coordinates of the stationary point(s) and determine their nature. **2**

(ii) Find any points of inflexion. **2**

(iii) Hence, sketch the curve showing features found in parts (i) and (ii). **2**
(Do not find the x intercepts).

(c) The shaded region in the diagram is bounded by the curve $y = x^2 - 2x$ and the line $y = 3x - 4$.



(i) State the points of intersection of the line and the parabola. **1**

(ii) Find the area of the shaded region. **2**

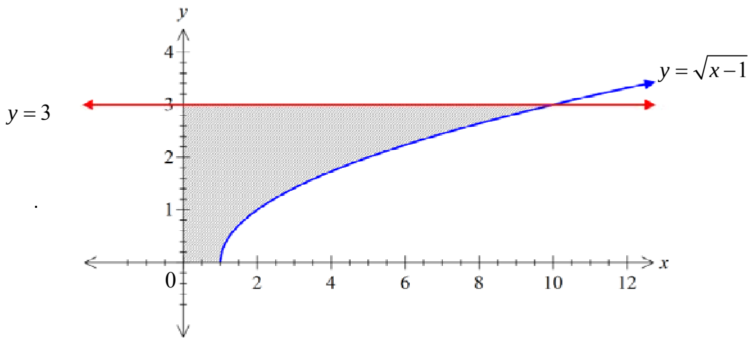
(d) Consider the series $3 + 6p + 12p^2 + 24p^3 + \ldots$

(i) For what values of p does this series have a limiting sum? **1**

(ii) If the limiting sum of this series is $4\frac{1}{2}$, find the value of p. **2**

Question 16 (15 marks) Use a SEPARATE writing booklet

(a) The graph of $y = \sqrt{x-1}$ is shown below. **4**



The shaded region in the diagram is bounded by the curve $y = \sqrt{x-1}$, the y-axis and the lines $y = 0$ and $y = 3$.

Find the volume of the solid of revolution formed when the shaded region is rotated about the y axis.

(b) Mr Smith borrowed \$180 000 to buy a unit. The interest rate was 18% per annum, compounded monthly. He agreed to repay the loan in 20 years with equal monthly repayments. Let the monthly repayments be \$$M$. The amount owing, \$$A_n$, on the loan after the nth monthly repayment is given by the formula

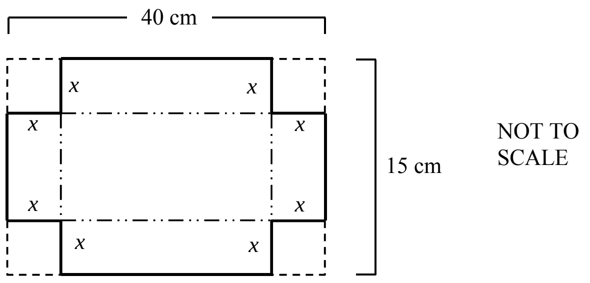
$$A_n = 180\,000(1.015)^n - M(1 + 1.015 + 1.015^2 + \ldots + 1.015^{n-1}) \text{ (Do not prove this)}$$

(i) If the loan is repaid in full in 20 years show that \$$M$, the monthly repayment is \$2777.96. **2**

(ii) Mr Smith decides to round this monthly repayment to the nearest \$1000. How long will it take him to repay the loan, to the nearest month? **3**

Question 16 continues on the following page

(c) An open rectangular box is to be formed by cutting squares of side length x cm from each corner of a rectangular sheet of metal that has length 40 cm and width 15 cm and folding up the sides.



(i) Find expressions for the length and breadth of the box in terms of x. **1**

(ii) Show that the volume of the box is given by $V = 600x - 110x^2 + 4x^3$. **2**

(iii) Find the value of x that gives the box its greatest volume. **3**

END OF PAPER

YEAR 12 TERM II EXAMINATION

MAY 2016.

Section 1.

1. $0.0157\ldots \doteq \underline{0.016}$ (D)

2. $4m^{-2} \div \frac{1}{2}m^{-1} = 8m^{-2--1}$
$= 8m^{-1}$
$= \dfrac{8}{m}$ (B)

3. $-1 \leq 2x - 3 \leq 1$
$2 \leq 2x \leq 4$
$\underline{1 \leq x \leq 2}$ (C)

or critical values

$2x - 3 = 1$ or $2x - 3 = -1$

$2x = 4$ or $2x = 2$

$x = 2$ or $x = 1$

Test $x = 0$ $\quad |2(0) - 3| \leq 1$

$3 \leq 1$ False

$\therefore \underline{1 \leq x \leq 2}$.

4. $\dfrac{3}{2+\sqrt{5}} = a + \sqrt{b}$

$\dfrac{3}{2+\sqrt{5}} \times \dfrac{2-\sqrt{5}}{2-\sqrt{5}} = \dfrac{6-3\sqrt{5}}{2^2 - (\sqrt{5})^2}$

$= \dfrac{6-3\sqrt{5}}{4-5}$

$= \dfrac{6-3\sqrt{5}}{-1}$

$= -6 + 3\sqrt{5}$

$a + \sqrt{b} = -6 + \sqrt{45}$

$\therefore \underline{a = -6 \quad b = 45}$ (C)

5. $\log_a 24 = \log_a(3 \times 8)$
$= \log_a 3 + \log_a 8$
$= \log_a 3 + \log_a 2^3$
$= \log_a 3 + 3\log_a 2$
$= \underline{q + 3p}$ (A)

6. $2x^2 - 4x + 7 = 0$

$\alpha + \beta = \dfrac{-b}{a}$
$= \dfrac{4}{2}$
$= 2$

$\alpha\beta = \dfrac{c}{a}$
$= \dfrac{7}{2}$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$
$= 2^2 - 2 \times \frac{7}{2}$
$= 4 - 7$
$= \underline{-3}$ (A)

7. $\displaystyle\int_0^7 f(x)\,dx = A_{square} + Area_{\triangle}$

$= 4 + -\frac{1}{2} \times 1 \times 2$
$= 4 - 1$
$= \underline{3}$ (B)

8. $\dfrac{x}{20} = \dfrac{9}{12}$

$x = \dfrac{9}{12} \times 20$
$= \underline{15}$ (D)

9. C

10. $\log_e(c^3) = \log_e(e^x)^3$
$= \log_e e^{3x}$
$= 3x \log_e e$
$= \underline{3x}$ (B)

1	D	6	A
2	B	7	B
3	C	8	D
4	C	9	C
5	A	10	B

Question 11.

(a) $3x - 2(x-1)$ $\boxed{1}$
$= 3x - 2x + 2$
$= \underline{x + 2}$

(b) $\log_3 5 = \dfrac{\log_e 5}{\log_e 3}$ $\boxed{2}$
$= 1.4649\ldots$
$\doteq \underline{1.465}$ (3 dp)

(c) $\displaystyle\lim_{x \to 2} \frac{x^3 - 8}{x - 2}$ $\boxed{2}$

$= \displaystyle\lim_{x \to 2} \frac{(x-2)(x^2+2x+4)}{(x-2)}$

$= 2^2 + 2(2) + 4$

$= \underline{12}$

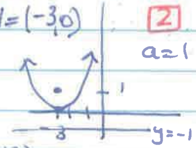
(d) $\dfrac{dy}{dx} = e^{5x} + 1$ $\boxed{2}$

$\underline{y = \dfrac{e^{5x}}{5} + x + c}$

(e) $f(x) = 10$ $\boxed{2}$
$x^2 - 3x - 30 = 10$
$x^2 - 3x - 40 = 0$
$(x-8)(x+5) = 0$
$\underline{x = 8 \text{ or } -5}$

(f) Domain: all real x, $x \neq 3$ $\boxed{2}$
Range: all real y, $y \neq 1$

(g) $\underline{(x+3)^2 = 4y}$ $\quad V = (-3, 0)$ $\boxed{2}$
$a = 1$



(h) $\dfrac{2x-1}{4} + 1 = \dfrac{x}{3}$ $(\times 12)$ $\boxed{2}$

$3(2x-1) + 12 = 4x$
$6x - 3 + 12 = 4x$
$2x = -9$
$\underline{x = -4\frac{1}{2}}$

Question 12.

(a) $\sqrt{3} + \tan x = 0$
$\tan x = -\sqrt{3}$

related $\angle = 60°$

$x = 180 - 60°, \ 360° - 60°$

$\underline{x = 120° \text{ or } 300°}$ $\boxed{2}$

(b)(i) $y = \log(3x^2-1)$ $\boxed{2}$

$\frac{dy}{dx} = \frac{1}{3x^2-1} \times 6x$

$= \frac{6x}{3x^2-1}.$

(ii) $y = x^2 e^x$

$\left[\begin{array}{ll} u = x^2 & v = e^x \\ u' = 2x & v' = e^x \end{array}\right.$

$y' = vu' + uv'$

$= e^x \times 2x + x^2 \times e^x$ $\boxed{2}$

$= 2xe^x + x^2e^x$

$= xe^x(2+x)$

(c) $\int x e^{5x^2} dx = \frac{1}{10} e^{5x^2} + c$ $\boxed{2}$

(d) $\int_1^3 \left(\frac{2x^3}{x^2} + \frac{1}{x}\right) dx$

$= \int_1^3 \left(2x + \frac{1}{x}\right) dx$

$= \left[\frac{2x^2}{2} + \ln x\right]_1^3$ $\boxed{3}$

$= \left[x^2 + \ln x\right]_1^3$

$= [(3^2 + \ln 3) - (1^2 + \ln 1)]$

$= 9 + \ln 3 - 1$

$= 8 + \ln 3.$

(e) $y = 3\log_e x$ $\boxed{3}$

$\frac{dy}{dx} = 3 \times \frac{1}{x}$

$= \frac{3}{x}$

When $x = e$

$\frac{dy}{dx} = \frac{3}{e} = m_T$

$\therefore m_N = -\frac{e}{3}$

Eqn of normal. $(e, 3)$

$y - 3 = -\frac{e}{3}(x - e)$

$3y - 9 = -ex + e^2$

$ex + 3y - 9 - e^2 = 0$

(f) $\sqrt{\sec^2 A - 1} = \sqrt{\tan^2 A}$ $\boxed{1}$

$= \tan A.$

$[1 + \tan^2 A = \sec^2 A$

$\therefore \sec^2 A - 1 = \tan^2 A]$

Question 13.

(i) $m_{AB} = 2$ $\therefore m_{CD} = 2$ $(AB \parallel CD)$

Eqn CD. $\boxed{2}$

$y + 4 = 2(x - 4)$

$y + 4 = 2x - 8$

$0 = 2x - y - 12.$

(ii) $2x - y - 12 = 0$

$a = 2 \quad b = -1 \quad c = -12$

$B(0, 8)$

$pd = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$ $\boxed{2}$

$= \frac{|2 \times 0 + -1 \times 8 - 12|}{\sqrt{4+1}}$

$= \frac{|-20|}{\sqrt{5}}$

$= \frac{20}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$

$= \frac{20\sqrt{5}}{5}$

$= 4\sqrt{5}$ units.

(iii) $A + C$, $y = 8$ $\boxed{1}$

Sub into $2x - y - 12 = 0$

$2x - 8 - 12 = 0$

$2x = 20$

$x = 10$

$\therefore C = (10, 8)$

(iv) $d_{DC} = \sqrt{(10-4)^2 + (8+4)^2}$ $\boxed{3}$

$= \sqrt{36 + 144}$

$= \sqrt{180}$

$= 3\sqrt{20}$

$= 3 \times 2\sqrt{5}$

$= 6\sqrt{5}$

$\therefore A_{trap} = \frac{1}{2}h(a+b)$

$= \frac{1}{2} \times 4\sqrt{5}(4\sqrt{5} + 6\sqrt{5})$

$= 2\sqrt{5} \times 10\sqrt{5}$

$= 100$ units^2.

(b) real + diff roots when $\Delta > 0$

$\Delta = b^2 - 4ac$ $\boxed{2}$

$= (-2k)^2 - 4(1)$

$= 4k^2 - 4$

$\therefore 4k^2 - 4 > 0$

$k^2 - 1 > 0$

$(k-1)(k+1) > 0$

$k = \pm 1$

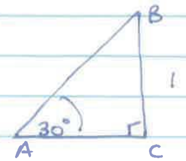
test $x = 0$ $\quad 0 - 1 > 0$

$-1 > 0$ False

$\therefore k < -1$ or $k > 1$

(c) In ΔABC

(i)



$\tan 30° = \frac{1}{AC}$

$AC = \frac{1}{\tan 30°}$ $\boxed{2}$

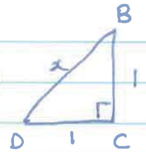
$= \frac{1}{\frac{1}{\sqrt{3}}}$

$= \sqrt{3}$

$\therefore AD = AC - DC$

$= (\sqrt{3} - 1)$ cm

(ii)



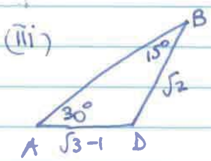
$x^2 = 1^2 + 1^2$

$= 2$

$x = \sqrt{2}$

$BD = \sqrt{2}$ $\boxed{1}$

(iii)



sine rule : $\boxed{2}$

$\therefore \dfrac{\sin 15^\circ}{\sqrt{3}-1} = \dfrac{\sin 30^\circ}{\sqrt{2}}$

$\sin 15^\circ = \dfrac{\frac{1}{2}}{\sqrt{2}}(\sqrt{3}-1)$

$\sin 15^\circ = \dfrac{\sqrt{3}-1}{2\sqrt{2}}$

Question 14.

(i) $a = 4 \quad d = 3 \quad T_n = 301$

$T_n = a + (n-1)d$ $\boxed{2}$

$301 = 4 + (n-1)3$

$301 = 4 + 3n - 3$

$300 = 3n$

$100 = n$

there are 100 terms

(ii) $S_{100} = ?$ $\boxed{2}$

$S_n = \frac{n}{2}[a + l]$

$S_{100} = \frac{100}{2}[4 + 301]$

$= 15\,250$

(b) $y = \dfrac{1}{\log x}$ $\boxed{2}$

x	4	6	8
y	$\frac{1}{\ln 4}$	$\frac{1}{\ln 6}$	$\frac{1}{\ln 8}$

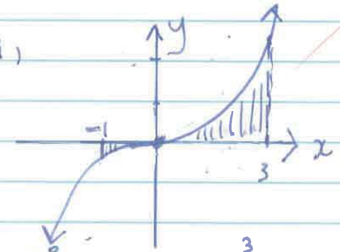
$\int_a^b f(x)\,dx \doteqdot \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$

$\int_4^8 \frac{1}{\log x}\,dx \doteqdot \frac{8-4}{6}\left[\frac{1}{\ln 4} + 4\times\frac{1}{\ln 6} + \frac{1}{\ln 8}\right]$

$\doteqdot 2\cdot289\ldots$

$\doteqdot 2\cdot29$ (3 sf)

(c) (i) $\boxed{1}$



(ii) $A = \left|\int_{-1}^{0} 2x^3\,dx\right| + \int_0^3 2x^3\,dx$ $\boxed{3}$

$= \left|\left[\frac{2x^4}{4}\right]_{-1}^{0}\right| + \left[\frac{2x^4}{4}\right]_0^3$

$= \left|\left[\frac{x^4}{2}\right]_{-1}^{0}\right| + \left[\frac{x^4}{2}\right]_0^3$

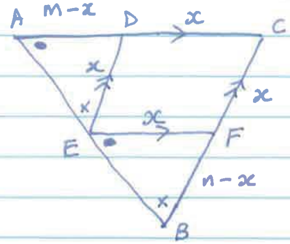
$= \left|\frac{0}{2} - \frac{(-1)^4}{2}\right| + \left[\frac{3^4}{2} - 0\right]$

$= \left|-\frac{1}{2}\right| + \frac{81}{2}$

$= \frac{82}{2}$

$= 41$ units^2

(d)



$\boxed{2}$

In $\triangle ADE$ and $\triangle EFB$

$\angle DAE = \angle FEB$ (corresponding $\angle$s, $AC \parallel EF$)

$\angle DEA = \angle FBE$ (corresponding $\angle$s, $ED \parallel BC$)

$\therefore \triangle ADE \,|||\, \triangle EFB$ (equiangular)

(ii) $\therefore \dfrac{x}{m-x} = \dfrac{n-x}{x}$ (matching sides of similar △s in proportion)

$x^2 = (m-x)(n-x)$

$\cancel{x^2} = mn - mx - nx + \cancel{x^2}$

$0 = mn - mx - nx$ $\boxed{3}$

$= mn - x(m+n)$

$x(m-n) = mn$

$x = \dfrac{mn}{m+n}$

Question 15. $\boxed{3}$

(a) $\int_0^3 \dfrac{x}{x^2+3}\,dx = \left[\frac{1}{2}\ln(x^2+3)\right]_0^3$

$= \frac{1}{2}\ln(3^2+3) - \frac{1}{2}\ln(0+3)$

$= \frac{1}{2}\ln 12 - \frac{1}{2}\ln 3$

$= \frac{1}{2}\left[\ln\frac{12}{3}\right]$

$= \frac{1}{2}\ln 4$

$= \frac{1}{2}\times 2\ln 2$

$= \ln 2$

(b) $y = 1 + 3x - x^3$

(i) $\dfrac{dy}{dx} = 3 - 3x^2$ $\boxed{2}$

$\dfrac{d^2y}{dx^2} = -6x$

Stat. pts when $\dfrac{dy}{dx} = 0$

$3 - 3x^2 = 0$

$3 = 3x^2$

$x^2 = 1$

$x = \pm 1$

Stat pts $(-1, -1)$ and $(1, 3)$

$x = -1 \quad y = 1 + (3\times -1) - (-1)^3$

$= 1 - 3 + 1$

$= -1$

$x = 1 \quad y = 1 + 3 - 1$

$= 3.$

When $x = -1$ $\dfrac{d^2y}{dx^2} = -6\times -1$

$= 6 > 0$

$\therefore$ Min. turning pt at $(-1, -1)$

when $x = 1$ $\dfrac{d^2y}{dx^2} = -6\times 1$

$= -6 < 0$

$\therefore$ Max. turning pt $(1, 3)$.

(ii) pts of inflection when $\frac{d^2y}{dx^2} = 0$ and change of concavity

$\frac{d^2y}{dx^2} = -6x$

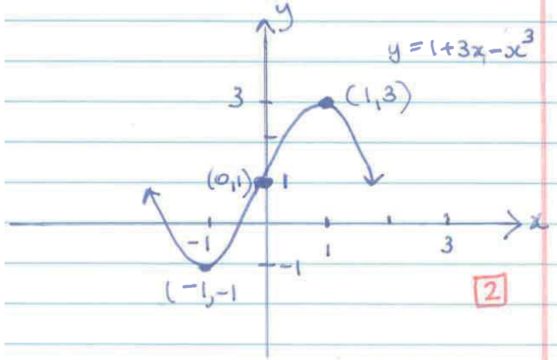
$-6x = 0$

$\therefore x = 0$

x	$-\frac{1}{2}$	0	$\frac{1}{2}$
$\frac{d^2y}{dx^2}$	3	0	-3

$\boxed{2}$

$\therefore$ pt of inflexion at $(0,1)$



$\boxed{2}$

(c) (i) Pts of intersection $(1,-1)$ and $(4,8)$ $\boxed{1}$

(ii) $A = \int_1^4 [f(x) - g(x)]\,dx$

$= \int_1^4 [(3x-4) - (x^2-2x)]\,dx$

$= \int_1^4 (3x-4-x^2+2x)\,dx$

$= \int_1^4 (5x-4-x^2)\,dx$ $\boxed{2}$

$= \left[\frac{5x^2}{2} - 4x - \frac{x^3}{3}\right]_1^4$

$= \left[\left(\frac{5(4)^2}{2} - 4(4) - \frac{4^3}{3}\right) - \left(\frac{5}{2} - 4 - \frac{1}{3}\right)\right]$

$= 4.5$ units^2

(d) $3 + 6p + 12p^2 + \ldots$

$r = 2p$

For a limiting sum

$-1 \leq r \leq 1$

$-1 \leq 2p \leq 1$

$-\frac{1}{2} \leq p \leq \frac{1}{2}$ $\boxed{1}$

(ii) $S_\infty = \frac{a}{1-r}$

$4\frac{1}{2} = \frac{3}{1-2p}$ $\boxed{2}$

$1-2p = \frac{3}{4\frac{1}{2}}$

$-2p = \frac{3}{4\frac{1}{2}} - 1$

$p = \frac{1}{6}$

Question 16.

$V = \pi \int x^2\,dy$

$= \pi \int_0^3 (y^2+1)^2\,dy$

$= \pi \int_0^3 (y^4 + 2y^2 + 1)\,dy$ $\boxed{4}$

$= \pi \left[\frac{y^5}{5} + \frac{2y^3}{3} + y\right]_0^3$

$= \pi \left[\left(\frac{3^5}{5} + \frac{2(3)^3}{3} + 3\right) - \left(\frac{0}{5} + 2(0) + 0\right)\right]$

$= \frac{348\pi}{5}$ units^3

$y = \sqrt{x-1}$

$y^2 = x-1$

$y^2 + 1 = x$

$\therefore x^2 = (y^2+1)^2$

$= y^4 + 2y^2 + 1$

(b) $A_{240} = 180\,000(1.015)^{240} - M(1 + 1.015 + \ldots + 1.015^{239})$

S_n, $n = 240$ $a = 1$ $r = 1.015$

$S_n = \frac{a(r^n - 1)}{r-1}$ $\boxed{2}$

$S_{240} = \frac{1.015^{240} - 1}{0.015}$

$A_{240} = 0$ after 20yrs.

$\therefore 0 = 180\,000(1.015)^{240} - M\left(\frac{1.015^{240} - 1}{0.015}\right)$

$M\left(\frac{1.015^{240} - 1}{0.015}\right) = 180\,000(1.015)^{240}$

$M = 180\,000(1.015)^{240} \div \frac{1.015^{240} - 1}{0.015}$

$= \$2\,777.96$

$M = \$3000$

(ii) From (i) $A_n = 180\,000(1.015)^n - 3000\left(\frac{1.015^n - 1}{0.015}\right)$

$A_n = 0$ $\quad 0 = 180\,000(1.015)^n - 200\,000(1.015^n - 1)$

$0 = 180\,000(1.015)^n - 200\,000(1.015)^n + 200\,000$

$0 = -20\,000(1.015)^n + 200\,000$

$20\,000(1.015)^n = 200\,000 \qquad (\div 20\,000)$

$1.015^n = 10$

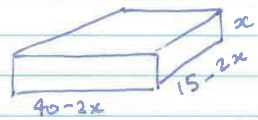
$\log_e(1.015)^n = \log_e 10$

$n \log_e(1.015) = \log_e 10$ $\boxed{3}$

$n = \frac{\log_e 10}{\log_e(1.015)}$

$= 154.65$

$= 155$ mths (nearest mth)

(c) 

(i) length $= 40 - 2x$ $\boxed{1}$

breadth $= 15 - 2x$

(ii) $V = lbh$

$= x(40-2x)(15-2x)$

$= x[600 - 80x - 30x + 4x^2]$

$= 600x - 110x^2 + 4x^3$ $\boxed{2}$

(iii) $\frac{dV}{dx} = 600 - 220x + 12x^2$

$\frac{d^2V}{dx^2} = -200 + 24x$ $\boxed{3}$

Stat pts when $\frac{dV}{dx} = 0$ $\quad 600 - 220x + 12x^2 = 0 \qquad (\div 4)$

$3x^2 - 55x + 150 = 0$

$x = \frac{55 \pm \sqrt{55^2 - 4(3)150}}{6}$

$= \frac{55 \pm 35}{6}$

$x = \frac{90}{6}$ or $x = \frac{20}{6}$

$x = 15$ or $x = 3\frac{1}{3}$

When $x = 15$ $\quad \frac{d^2V}{dx^2} = -200 + 24 \times 15$

$= 160 > 0$

$\therefore$ Min

When $x = 3\frac{1}{3}$ $\quad \frac{d^2V}{dx^2} = -200 + 24 \times 3\frac{1}{3}$

$= -120 < 0 \quad \therefore$ Max

$\therefore$ Max volume when $x = 3\frac{1}{3}$ cm.