SECTION 1

QUESTION 1

(a) Differentiate :

(i) $8x^6 - 5$

(ii) $\sin 3x$

(iii) $3\cos x + \sqrt{x}$

(iv) $x \tan x$ **6**

(b) Evaluate: $\sum_{n=1}^{3} 2^{1-n}$ **1**

(c) Evaluate:

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cos \frac{1}{2} x \, dx$$

Answer in exact, simplified form. **3**

QUESTION 2

(a) For the function $f(x) = 3 \cos \frac{1}{2} x$

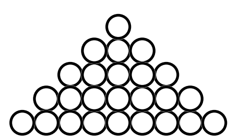
(i) State the amplitude and period of the function

(ii) Sketch the function for $0 \le x \le 2\pi$ 4

(b) The first term of a geometric series is $\frac{1}{2}$ and the limiting sum, $S_\infty = \frac{3}{4}$.

Find the common ratio of the series. **2**

(c) Cans of fruit are stacked in a supermarket display as shown in the following diagram.



The top row has one can and each row has two more than the row above.

(i) If the top row is row one, how many cans are in the 15th row ?

(ii) Which is the first row to have more than 17 cans ?

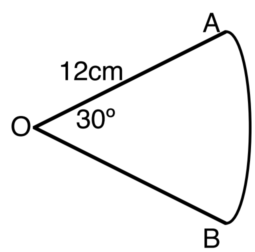
(iii) Show that the number of pins in n rows will be n^2. **4**

SECTION 2
Start a new booklet

QUESTION 3

(a) The probability that Tan's train will be late on any given day is $\frac{1}{9}$. What is the probability that it will be late at least once in three consecutive days ? **1**

(b) Below is a diagram of a sector of a circle.



Find the length of the arc AB in terms of π. **1**

(c) The region bounded by the curve $y = \tan x$, the x-axis and the line $x = \frac{\pi}{4}$ is rotated about the x-axis. Find, in terms of π, the volume of the solid of revolution. **3**

(d) Find a primitive of $x^3 + \sqrt{x}$. **2**

(e) (i) On the same set of axes, sketch the functions $y = \sin x$ and $y = x - \frac{\pi}{2}$.

(ii) Use your sketch to approximate a solution to the equation

$$\sin x = x - \frac{\pi}{2}.$$ **3**

QUESTION 4

(a) (i) Express 0.323232........ as a geometric series.

(ii) Use the limiting sum of the series to express 0.323232...as a fraction. **2**

(b) A class consists of 10 girls and 15 boys. Two of the girls and five of the boys are left handed. Give each answer as a simplified fraction.

(i) If a boy and a girl are chosen at random, what is the probability that they are both left handed ?

If two students are chosen at random find the probability that:

(ii) they are both left handed?

(iii) One is left handed and the other is not. **5**

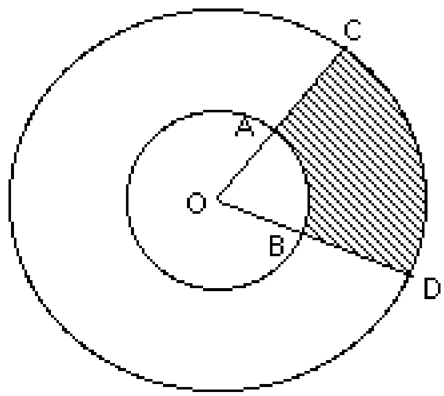
(c) Use Simpson's Rule with three function values to estimate to 3 decimal places:

$$\int_0^1 \sin\ (1+x^2)\ dx$$ **3**

QUESTION 5

(a) Express $\frac{5\pi}{9}$ radians in degrees. **1**

(b) The diagram below shows two concentric circles with centre O. The larger circle has radius 20 cm and the small circle has radius 15 cm. Sectors COD and AOB subtend an angle of 60° at O. The diagram is not to scale.



(i) Calculate the exact area of the sector COD.

(II) Calculate the exact perimeter of the region ACDB. **3**

(c) The second derivative of the curve $y = f(x)$ is given by

$f''(x) = 3x - 6$ and $f'(0) = f(0) = 0$.

(i) Find the equation of the first derivative of the curve.

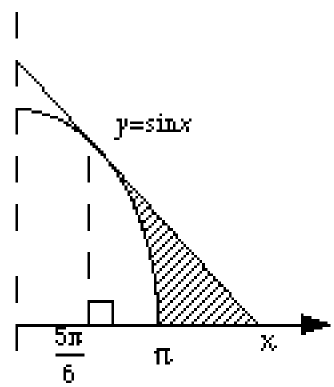
(ii) Sketch the curve $y = f(x)$, showing any turning point(s) and point(s) of inflexion. **6**

QUESTION 6

(a) Differentiate $\dfrac{x}{\cos x + 1}$ with respect to x. **2**

(b) \$30 000 is borrowed to buy a car. the interest rate is 12% per annum compounded monthly. The loan is to be repaid in equal monthly instalments over a four year period. If M is the monthly payment, write an expression containing M for the amount owing after 3 months. **2**

(c) The diagram below (not to scale) shows part of the curve $y = \sin x$ meeting the tangent at the point where $x = \dfrac{5\pi}{6}$.



(i) Find the equation of the tangent to the curve $y = \sin x$ at the point where $x = \dfrac{5\pi}{6}$.

(ii) Show that the tangent meets the x-axis at the point where

$$x = \frac{5\pi + 2\sqrt{3}}{6}$$

(iii) Find, in simplified form, the exact area of the shaded region. **6**

END OF PAPER

SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2004

HIGHER SCHOOL CERTIFICATE
ASSESSMENT TASK # 2

Mathematics

Sample Solutions

Section ①

①

(a) (i) $\frac{d}{dx}\, 8x^6 - 5 = 48x^5$

(ii) $\frac{d}{dx}\, \sin 3x = 3\cos 3x$

(iii) $\frac{d}{dx}\, 3\cos x + \sqrt{x} = -3\sin x + \frac{1}{2\sqrt{x}}$

(iv) $\frac{d}{dx}\, x\tan x = \tan x + x\sec^2 x$

(b) $\sum_{n=1}^{3} 2^{1-n} = 2^0 + 2^{-1} + 2^{-2}$

$= 1 + \frac{1}{2} + \frac{1}{4}$

$= 1\frac{3}{4}$

(c) $\int_{\pi/3}^{\pi/2} \cos\frac{x}{2}\,dx = \left[2\sin\frac{x}{2}\right]_{\pi/3}^{\pi/2}$

$= \sqrt{2} - 1$

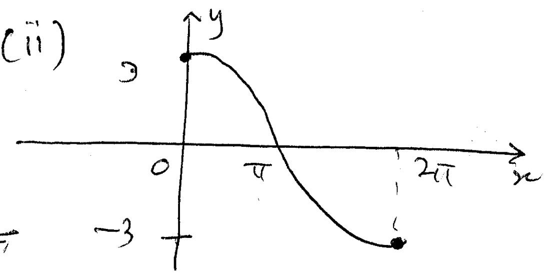
②

$f(x) = 3\cos\frac{x}{2}$

(a) amplitude = 3

(i) $T = 2\pi/\tfrac{1}{2} = 4\pi$

(ii)



(b) $a = \frac{1}{2}$, $S = \frac{3}{4}$,

$S = \frac{a}{1-r}$, $\frac{3}{4} = \frac{1/2}{1-r}$

$3 - 3r = 2$, $3r = 1 \Rightarrow r = \frac{1}{3}$

(c) 1, 3, 5, 7, ...

(i) $a = 1$, $d = 2$, $T_{15} = 1 + 14\times 2$
$= 29$.

(ii) $1 + (n-1)2 > 17$, $n > 9 \Rightarrow n = 10$

(iii) $1 + 3 + \cdots + (2n-1)$

$= \frac{n}{2}[1 + (2n-1)]$

$= n^2$.

Question 3

a) P(late at least once) = 1 − P(not lat at all)

$$= 1 - \frac{8}{9} \times \frac{8}{9} \times \frac{8}{9}$$

$$= 1 - \frac{512}{729}$$

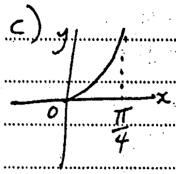
$$= \frac{217}{729}$$

b) $\ell = r\theta$

$$= 12 \times \frac{\pi}{6}$$

$$= 2\pi \text{ cm}$$

c)



$$V = \pi \int_0^{\frac{\pi}{4}} y^2 \, dx$$

$$= \pi \int_0^{\frac{\pi}{4}} \tan^2 x \, dx$$

$$= \pi \int_0^{\frac{\pi}{4}} (\sec^2 x - 1) \, dx$$

$$= \pi \left[\tan x - x \right]_0^{\frac{\pi}{4}}$$

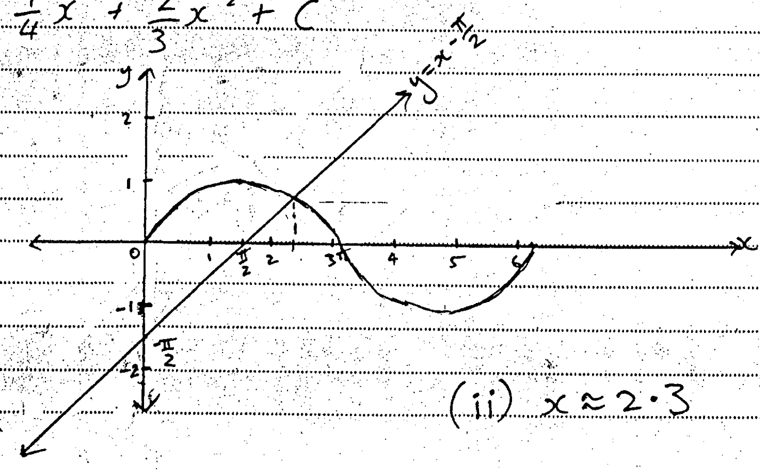
$$= \pi \left[\tan\frac{\pi}{4} - \frac{\pi}{4} - (\tan 0 - 0) \right]$$

$$= \pi \left(1 - \frac{\pi}{4} \right) \quad \text{units}^3$$

d) $\int (x^3 + x^{\frac{1}{2}}) \, dx$

$$= \frac{1}{4} x^4 + \frac{2}{3} x^{\frac{3}{2}} + C$$

e) i)



(ii) $x \approx 2 \cdot 3$

Question 4

a) i) $0.32323232\ldots = \frac{32}{100} + \frac{32}{10000} + \frac{32}{1000000} + \ldots$

or $0.32 + 0.0032 + 0.000032 + \ldots$

ii) $a = \frac{32}{100}$ $\qquad S_\infty = \frac{a}{1-r}$

$r = \frac{1}{100}$

$$= \frac{\left(\frac{32}{100}\right)}{1 - \frac{1}{100}}$$

$$= \frac{\frac{32}{100}}{\frac{99}{100}}$$

$$= \frac{32}{99}$$

b) i) $P = \frac{2}{10} \times \frac{5}{15}$

$$= \frac{1}{15}$$

ii) $P(LL) = \frac{7}{25} \times \frac{6}{24}$

$$= \frac{42}{600}$$

$$= \frac{7}{100}$$

iii) $P = P(LR) + P(RL)$

$$= \frac{7}{25} \times \frac{18}{24} + \frac{18}{25} \times \frac{7}{24}$$

$$= 2 \times \frac{7}{25} \times \frac{18}{24}$$

$$= \frac{252}{600}$$

$$= \frac{21}{50}$$

c) $\int_0^1 \sin(1+x^2)\,dx \approx \frac{b-a}{6}\left[f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right]$

$f(0) = \sin(1)$

$\approx 0.841470984\ldots$

$f(0.5) = \sin(1+0.5^2)$

$= \sin(1.25)$

$= 0.948984619\ldots$

$f(1) = \sin(1+1)$

$= \sin(2)$

$= 0.909297426$

$\int_0^1 \sin(1+x^2)\,dx \approx \frac{\left(\frac{1}{2}\right)}{6}\left[0.84147 + 4\times 0.94898 + 0.9093\right]$

$\approx 0.924451147\ldots$

≈ 0.924 to 3 decimal places

Question 5

(a) $\frac{5\pi}{9}^c = \frac{5\pi}{9} \times \frac{180^\circ}{\pi}$

$= 100^\circ$ [1]

(b) (i) Sector Area

$A = \frac{1}{2} r^2 \theta$

$= \frac{1}{2}(20)^2 \frac{\pi}{3}$

$= \frac{200\pi}{3}$ [1]

(ii) Perimeter $= 10 + \frac{15\pi}{3} + \frac{20\pi}{3}$

$= 10 + \frac{35\pi}{3}$ [2]

(c) $f''(x) = 3x - 6$, $f'(0) = f(0) = 0$

(i) $f'(x) = \frac{3x^2}{2} - 6x$ [1]

(ii) $f'(x) = 0$ when $\left(\frac{3}{2}x - 6\right)x = 0$

ie $x = 0, 4$

At $x = 0$, $f''(x) = -6$

$\therefore$ Rel Max $(0,0)$

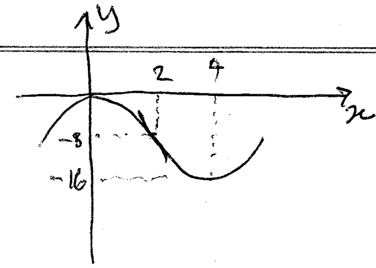
At $x = 4$, $f''(x) = 6$

$\therefore$ Rel Min $(4, -16)$

$f''(x) = 0$ at $x = 2$

$f''(2^-) < 0$, $f''(2^+) > 0$

$\therefore$ PoI at $(2, -8)$



[5]

Question 6

(a) $\frac{d}{dx} \frac{x}{\cos x + 1} = \frac{(\cos x + 1).1 - x.(-\sin x)}{(\cos x + 1)^2}$

$= \frac{1 + \cos x + x \sin x}{(\cos x + 1)^2}$

(b) Monthly rate $= 1\%$

Let $P = 30000$, $R = 1.01$

A_n = Amount owing after n periods

$\therefore A_1 = PR - M$

$A_2 = (PR - M)R - M$

$= PR^2 - MR - M$

$\therefore A_3 = (PR^2 - MR - M)R - M$

$= PR^3 - M(R^2 + R + 1)$

$\therefore A_3 = 30000(1.01)^3 - M(1.01^2 + 1.01 + 1)$

[2]

(c) $y = \sin x$, $y' = \cos x$

(i) $m = y'\left(\frac{5\pi}{6}\right) = \cos\left(\frac{5\pi}{6}\right)$

$= -\frac{\sqrt{3}}{2}$

$y\left(\frac{5\pi}{6}\right) = \sin\left(\frac{5\pi}{6}\right)$

$= \frac{1}{2}$

$\therefore$ Eq'n is $y - \frac{1}{2} = -\frac{\sqrt{3}}{2}\left(x - \frac{5\pi}{6}\right)$

$6\sqrt{3}x + 12y - 6 - 5\sqrt{3}\pi = 0$

[2]

(ii) When $y = 0$

$$6\sqrt{3}x = 5\sqrt{3}\pi + 6$$

$$x = \frac{5\sqrt{3}\pi + 6}{6\sqrt{3}}$$

$$= \frac{5\pi + 2\sqrt{3}}{6}$$

[2]

(iii) Shaded Area

$$A = A(\text{triangle}) - A(\text{under curve})$$

$$= \frac{1}{2} \times \frac{1}{2}\left(\frac{5\pi + 2\sqrt{3}}{6} - \frac{5\pi}{6}\right) - \int_{5\pi/6}^{\pi} \sin x \, dx$$

$$= \frac{1}{4}\left(\frac{\sqrt{3}}{3}\right) + \Big[\cos x\Big]_{5\pi/6}^{\pi}$$

$$= \frac{\sqrt{3}}{12} + (-1) + \frac{\sqrt{3}}{2}$$

$$= \frac{7\sqrt{3}}{12} - 1 \quad \text{unit}^2$$

[2]