SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2008
HIGHER SCHOOL CERTIFICATE
ASSESSMENT TASK #2

Mathematics

General Instructions

- Reading Time – 5 Minutes
- Working time – 90 Minutes
- Write using black or blue pen. Pencil may be used for diagrams.
- Board approved calculators may be used.
- Each **section** is to be returned in a separate bundle.
- All necessary working should be shown in every question.

Total Marks – 100

- Attempt all questions.
- All questions are of equal value.

Examiner: *F.Nesbitt*

SECTION A

QUESTION 1(16 marks)

(a) Find $\frac{d}{dx}\cos 3x$ 1

(b) Convert 36° to radians. Give your answer in terms of π. 1

(c) For the series

$2 + 9 + 16$ .... find: 3

(i) the 12th term

(ii) the sum ot the first 15 terms

(d) State the Domain and Range of $y = \sqrt{5-2x}$ 2

(e) A sector of a circle with radius 5 cm subtends an angle of 50° at the centre.

Find the area of the sector in terms of π. 2

(f) Differentiate the following with respect to x:

(i) $\frac{1}{2}\sin x - \cos 2x$ 1

(ii) $\sin^2 2x$ 2

(iii) $e^{3x-1} - \frac{1}{2}e^{-2x-5}$ 2

(iv) $\frac{2x}{\cos 2x}$ 2

QUESTION 2 (17 marks)

(a) Evaluate: $\sum_{n=1}^{20}(3n-4)$ 2

(b) Express $0.\dot{4}\dot{3}$ as a simplified fraction. 2

(c) Find the value of x if $3x$, $2x+4$ and $6x-2$ are consecutive terms of an arithmetic series. 2

(d) Find a primitive function of $\cos 3x - \sin 2x$ 1

(e) If $f'(x) = 4(3x-4)^3$ and $f(2) = 3$, Find $f(x)$ 2

(f) For the function $y = x^3 - 3x - 4$

(i) Find the co-ordinates of any stationary points and determine their nature. 4

(ii) Find any point(s) of inflexion 1

(iii) Sketch the curve showing all important features 2

(iv) For which values of x is the curve concave up? 1

SECTION B (start a new booklet)

QUESTION 3 (18 marks)

Find
(a)

(i) $\int (2x-3)(x-1)dx$ 2

(ii) $\int (4x-1)^3 \, dx$ 2

(iii) $\int_0^1 \frac{x^3+2x^2-x}{x} dx$ 2

(iv) $\int_1^2 \frac{1+x}{\sqrt{x}} dx$ 2

(b) In a certain series $S_n = n^2 - 2n$

(i) Find Term 1 1

(ii) Find the sum of the first two terms 1

(iii) Show that this is an arithmetic series. 2

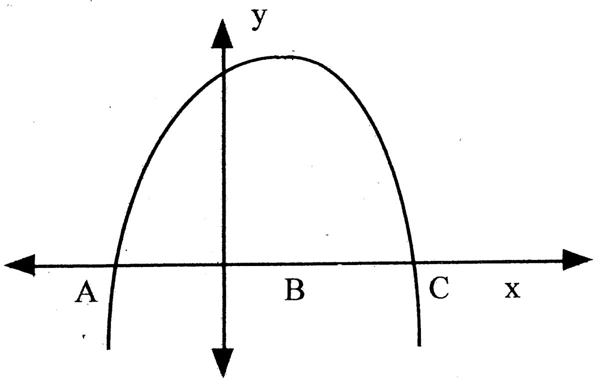
(iv) Write an expression for T_n. 1

(c) (i) On the same set of axes, sketch the curve $y = x^2$ and the line $y = x$ 1

(ii) Find any point(s) of intersection. 1

(iii) Find the volume of the solid formed when the area enclosed between the curve and the line is rotated about the x axis. 3

QUESTION 4 (16 marks)



(a) The diagram above shows the gradient function $y = f'(x)$ of a curve $y = f(x)$

(i) Describe the shape of the curve $y = f(x)$ where x = A and x = B 2

(ii) For what values of x is the curve $y = f(x)$ decreasing? 1

(iii) Sketch a possible curve $y = f(x)$, showing where x =A,B and C. 2

(b) For the function $y = 3\cos 2x$:

(i) State the amplitude and the period. 2

(ii) Sketch $y = 3\cos 2x$ in the domain $-\pi \le x \le \pi$ showing all important features, 3

(c) A loan of \$50 000 is to be repaid in equal monthly instalments over 10 years. The interest rate is 12%p.a. reducible and each monthly repayment is \$P.

(i) If A_n is the amount owing at the end of the nth month, show that

$A_1 = 50000 \times 1.01 - P$ 1

(ii) Find an expression for A_3. 2

(iii) Find the amount of each monthly repayment. 3

SECTION C (start a new booklet)

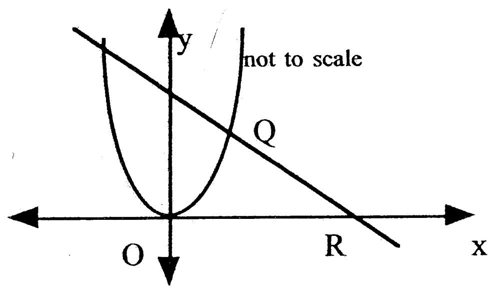
QUESTION 5 (15 marks)

(a) (i) On a number plane plot the points A(1,2), B(5,4) and C(7,8) 1

(ii) Show that the triangle ABC is isosceles. 2

(iii) The point D has coordinates (3,6). Show that ABCD is a rhombus. 2

(b)



(i) Copy the diagram above of $y = x^2$ and $y = 6 - x$ into your booklet. 1

(ii) In the first quadrant, the line $y = 6 - x$ meets the x axis at R and the curve intersects the line at Q. Show that the point Q has coordinates (2, 4) 2

(iii) Find the coordinates of R. 1

(iv) Find the area enclosed by QR, the curve OQ and the x axis. 3

(c) Use Simpson's Rule with 5 function values to find an approximation for

$\int_0^4 e^x dx$ to 2 dec. pl. 3

QUESTION 6 (18 marks)

(a) Evaluate

(i) $\int (\cos x - \sin x)dx$ 1

(ii) $\int 3\sec^2 2x \, dx$ 1

(iii) $\int_0^{\frac{\pi}{2}} 5\sin\frac{x}{2}dx$ 3

(b) (i) Sketch the curve $y = e^x$ 1

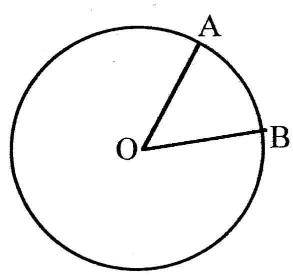
(ii) Find the coordinates of the point P where $x = a$ 1

(iii) Find the equation of the tangent at P. 2

(iv) Find the coordinates of the point T where the tangent cuts the x axis. 1

(iii) A perpendicular from P to the x axis meets the axis at N. Show that the distance TN is constant 2

(c)



The circle above has radius 10 cm. The sector AOB has angle 45° at the centre.

(i) Find the length of the arc AB in terms of π. 1

(ii) The radii AO and OB are joined so that the sector AOB makes a cone.

Find the radius of the cone. 2

(iii) Find the volume of the cone correct to 2 dec. pl. 3

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1;\ x \neq 0, \text{if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x,\ x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\ a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\ \ a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\ \ a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\ \ a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax,\ \ a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\ \ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a},\ \ a > 0,\ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right),\ x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x,\ \ x > 0$

2 unit Assess Task 2 2008 /16.

(1) (a) $-\sin 3x \times 3 = -3\sin 3x$ (1)

(b) $180^\circ = \pi$

$1^\circ = \frac{\pi}{180}$

So $36^\circ = 36 \times \frac{\pi}{180} = \frac{\pi}{5}$ (1)

(c)(i) $a = 2 \quad d = 7$

$U_n = a + (n-1)d$

$U_{12} = 2 + 11 \times 7 = 79$ (1)

(ii) $S_n = \frac{n}{2}(2a + (n-1)d)$

$S_{15} = \frac{15}{2}(4 + 14 \times 7)$

$= 765$ (2)

(d) domain $5 - 2x \geq 0$

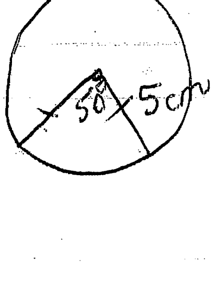
$-2x \geq -5$

$x \leq \frac{-5}{-2}$

$x \leq 2\frac{1}{2}$ (1)

Range $y \geq 0$ (1)

(e)



$A = \frac{1}{2}r^2\theta$

$= \frac{1}{2} \times 5^2 \times \frac{50\pi}{180}$

$= \frac{25 \times 50\pi}{2 \times 180}$

$= 3\frac{17}{36}\pi\ u^2 = \frac{125\pi}{36}\ u^2$ (2)

(f) (i) $\frac{1}{2}\cos x - {}^{-}\sin 2x \times 2 = \frac{1}{2}\cos x + 2\sin 2x$ (1)

(ii) $2\sin 2x \times \cos 2x \times 2 = 4\sin 2x \cos 2x$ (2)

(iii) $e^{3x-1} \times 3 - \frac{1}{2}e^{-2x-5} \times -2 = 3e^{3x-1} + e^{-2x-5}$ (2)

(f)(iv)

$\frac{\cos 2x \times 2 - 2x \times {}^{-}\sin 2x \times 2}{(\cos 2x)^2}$

$\frac{2\cos 2x + 4x\sin 2x}{(\cos 2x)^2}$ (2)

or/ as $2x(\cos 2x)^{-1}$

$\Rightarrow 2x \times -1(\cos 2x)^{-2} \times -2\sin 2x + 2(\cos 2x)^{-1}$

$= \frac{+4x\sin 2x}{(\cos 2x)^2} + \frac{2}{(\cos 2x)^1}$

$= \frac{2\cos 2x + 4x\sin 2x}{(\cos 2x)^2}$

(2) (a) $n=1, \quad 3-4=-1$
$n=2, \quad 6-4=2$
$n=3, \quad 9-4=5$
$\vdots$
$n=20 \quad 60-4=56$

$a=-1$
$d=3$
$L=56$
$n=20$

$S_n = \frac{n}{2}(a+L)$

$S_{20} = \frac{20}{2}(-1+56)$

$= 10 \times 55 = 550$ (2)

(b) $0.\dot{4}\dot{3} = \frac{43}{99}$ (2)

(c) $T_1 = 3x \quad T_2 = 2x+4 \quad T_3 = 6x-2$

So $T_2 - T_1 = T_3 - T_2$
$2x+4-3x = 6x-2-(2x+4)$
$-x+4 = 4x-6$
$10 = 5x$
$x = 2 \qquad \{6, 8, 10\}$ (2)

(d) $\int \cos 3x - \sin 2x \; dx$

$= \frac{1}{3}\int 3\cos 3x \, dx + \frac{1}{2}\int 2\sin 2x \, dx = \frac{1}{3}\sin 3x + \frac{1}{2}\cos 2x + c$ (1)

(e) $\int 4(3x-4)^3 dx$

$= 4\int (3x-4)^3 dx = \frac{4(3x-4)^4}{4 \times 3} + c$

$\Rightarrow f(x) = \frac{(3x-4)^4}{3} + c$

$3 = \frac{(6-4)^4}{3} + c$

$c = 3 - \frac{16}{3} = -2\frac{1}{3}$

So $f(x) = \frac{(3x-4)^4}{3} - 2\frac{1}{3}$

(2)

(2) (f) $y = x^3 - 3x - 4$

$y' = 3x^2 - 3$

$y'' = 6x$

(i) Stat pts exist when $y' = 0$, $3x^2 - 3 = 0$

$3x^2 = 3$

$x^2 = 1$ $x = \pm 1$

When $x = 1$, $y = 1 - 3 - 4 = -6$ $(1, -6)$ (1)

When $x = -1$, $y = -1 + 3 - 4 = -2$ $(-1, -2)$. (1)

When $x = 1$, $y'' = 6 > 0$ min (1)

$x = -1$, $y'' = -6 < 0$ max (1)

(ii) Inf occur when $y'' = 0$ and there is a sign change

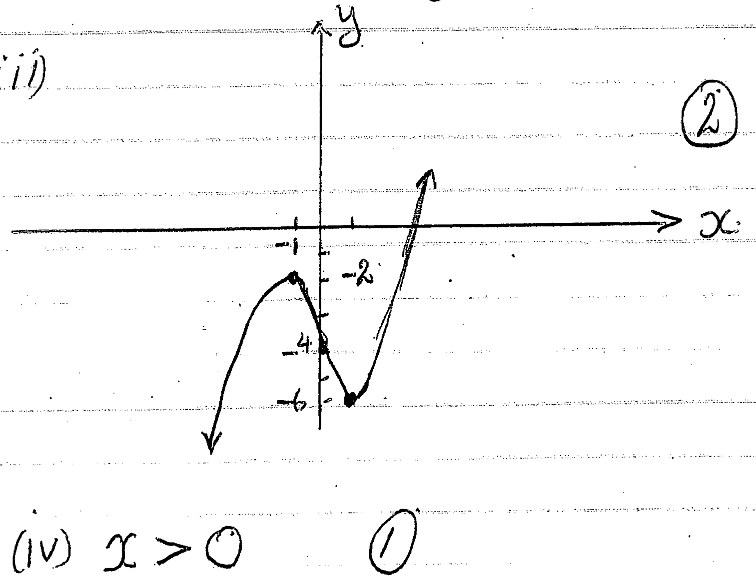
$y'' = 6x = 0$, $x = 0$

when $x = 0$, $y = 0 - 0 - 4$ $(0, -4)$

At $x = 0 - \varepsilon$ $y'' < 0$ } sign change Yes, inflexion (1)

$x = 0 + \varepsilon$ $y'' > 0$ }

(iii)



(2)

(iv) $x > 0$ (1)

2unit - Mathematics Section B Solutions.

Question 3

a) i) $\int (2x-3)(x-1)\,dx$

$= \int (2x^2 - 2x - 3x + 3)\,dx$

$= \int (2x^2 - 5x + 3)\,dx$ (1)

$= \frac{2x^3}{3} - \frac{5x^2}{2} + 3x + C$ (1)

$-\frac{1}{2}$ for 1st inc of no $+C$.

) $\int (4x-1)^3\,dx$

$= \frac{(4x-1)^4}{4 \times 4} + C$

$= \frac{(4x-1)^4}{16} + C$

i) $\int_0^1 \frac{x^3 + 2x^2 - x}{x}\,dx$

$\int_0^1 \left(\frac{x^3}{x} + \frac{2x^2}{x} - \frac{x}{x}\right)dx$

$\int_0^1 (x^2 + 2x - 1)\,dx$ (1)

$\left[\frac{x^3}{3} + \frac{2x^2}{2} - x\right]_0^1$

$= \left[\frac{1}{3} + 1 - 1\right] - \left[0 + 0 - 0\right]$

$= \frac{1}{3}$ (1)

iv) $\int_1^2 \frac{1+x}{\sqrt{x}}\,dx$

$= \int_1^2 \left(\frac{1}{\sqrt{x}} + \frac{x}{\sqrt{x}}\right)dx$

$= \int_1^2 \left(x^{-1/2} + x^{1/2}\right)dx$

$= \left[\frac{x^{1/2}}{1/2} + \frac{x^{3/2}}{3/2}\right]_1^2$ (1)

$= \left[2x^{1/2} + \frac{2x^{3/2}}{3}\right]_1^2$

$= \left[2 \times 2^{1/2} + \frac{2}{3} \times 2^{3/2}\right] - \left[2 \times 1 + \frac{2}{3} \times 1\right]$

$= \left[2\sqrt{2} + \frac{2}{3}\sqrt{8}\right] - \left[2 + \frac{2}{3}\right]$

$= \left[2\sqrt{2} + \frac{4}{3}\sqrt{2}\right] - \frac{8}{3}$

$= \frac{10\sqrt{2}}{3} - \frac{8}{3}$ (1)

$= \frac{2}{3}\left(5\sqrt{2} - 4\right)$

)) $S_n = n^2 - 2n$

') $S_1 = 1^2 - 2 \times 1$
$= 1 - 2$
$= -1$ (1)

i) $S_2 = 2^2 - 2 \times 2$
$= 4 - 4$
$= 0$ (1)

i) Formula for sum of an Arithmetic series

$S_n = \frac{n}{2}[2a + (n-1)d]$

$n = 2$, $a = -1$, $S_n = 0$

$0 = \frac{2}{2}[2 \times -1 + (2-1)d]$

$0 = 1[-2 + d]$

$\therefore$ $\boxed{d = 2}$

$S_3 = 3^2 - 2 \times 3$
$= 3$ (2)

$x + x + d + x + 2d = 3$
$3x + 3d = 3$
$-3 + 3d = 3$
$3d = 6$
$\therefore$ $\boxed{d = 2}$

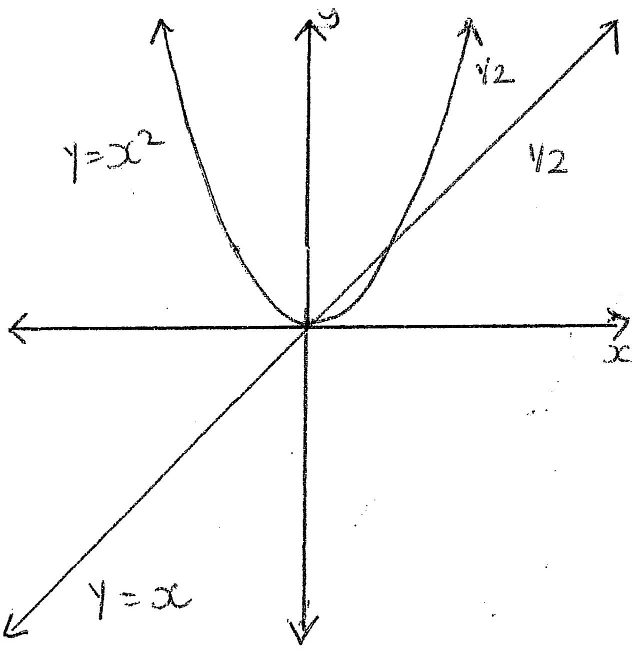
Since we have proven that this series has a common difference & not a common ratio

$S_n = n^2 - 2n$ is an arithmetic series.

iv) $T_n = a + (n-1)d$

$\therefore$ $T_n = -1 + (n-1) \times 2$ (1)

c) i



ii) $x = x^2$
$x^2 - x = 0$
$x(x - 1) = 0$
$\therefore$ $x = 0$, $x = 1$ (1)
$y = 0$ $y = 1$

or can read straight from an accurate graph.

) iii)

$V = \pi \int_0^1 x^2 dx - \pi \int_0^1 (x^2)^2 dx$ ①

$= \pi \int_0^1 (x^2 - x^4)\, dx$

$= \pi \left[\frac{x^3}{3} - \frac{x^5}{5} \right]_0^1$ ①

$= \pi \left[\left(\frac{1}{3} - \frac{1}{5} \right) - (0) \right]$

$= \frac{2\pi}{15} u^3$ ①

1½ for $\pi/30$. 1 for $\pi/6$

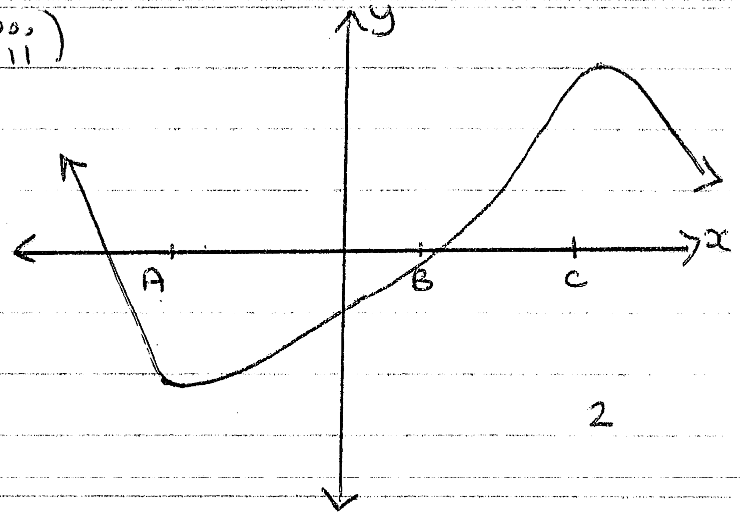
Question 4.

i) at $x = A$ as $f'(A) = 0$
$f(x)$ has a zero gradient, ①
∴ a stationary point.

at $x = B$ ①
pt of inflexion

) $f(x)$ is decreasing when
$f'(x) < 0$
∴ $x < A$ & $x > C$
½ ½

iii)



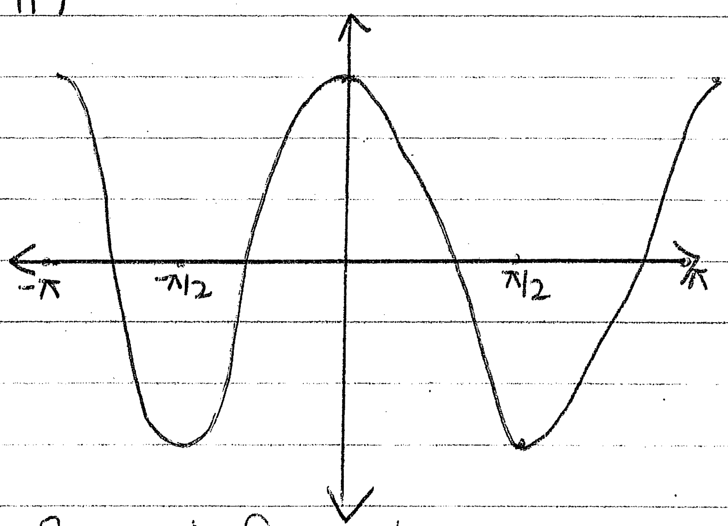
2

b) $y = 3\cos 2x$

i) amplitude $= 3$ ①

period $\frac{2\pi}{2} = \pi$ ①

ii)



3 − 1 for 1st error
½ for extra

c) \$50,000 loan
monthly instalments 10 years ∴ 120 payments.
I R = 12% pa ∴ 1% per month.
repayment = \$P.

i) $S_n = \frac{1.01(1.01^1 - 1)}{1.01 - 1}$ Amount Owing = balance * interest − Payment. ①

$= 1.01.$

$A_1 = 50000 \times 1.01 - P.$

~~ii) $A_3 = 50000\left[\frac{1.01(1.01^3 - P)}{1.01 - 1}\right] - P$~~

$A_2 =$ ~~$[50500 - P] \times 1.01 - P$~~

$= [50000 \times 1.01 - P]\, 1.01 - P$

$= 50000 \times (1.01)^2 - P(1.01 + 1)$

∴ $A_3 = A_2 \times (1.01) - P$

$= [50000 \times 1.02^2 - P(1.01 + 1)](1.01) - P$

$= 50000 \times 1.01^3 - P(1.01^2 + 1.01 + 1)$ ②

iii) $A_{120} = \frac{50000 \times 1.01^{120} * 0.01}{1.01^{120} - 1}$

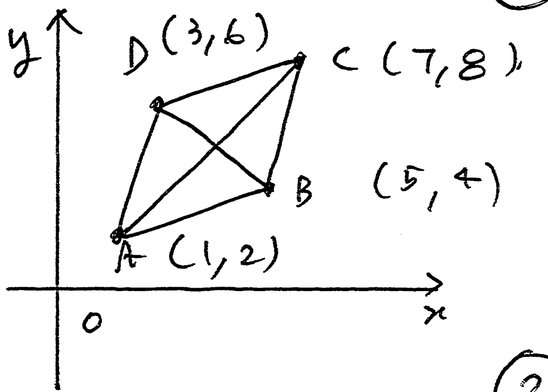
$= \$717.35$ per month. ③

15 (marks)

Question 5 (a)

(1)

(i)



(2)

(ii) $AB = \sqrt{(-4)^2 + 4} = \underline{2\sqrt{5}}$

$BC = \sqrt{(-2)^2 + (-4)^2} = \underline{2\sqrt{5}}$

$\because |AB| = |BC| = 2\sqrt{5}$

i.e $\triangle ABC$ is isosceles

(2)

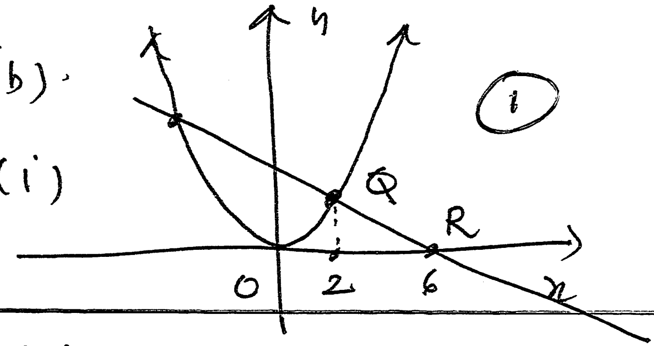
(iii) $|CD| = |AD| = 2\sqrt{5}$.

i.e $|AB| = |BC| = |CD| = |DA|$

i.e ABCD is rhombus.

(b). (1)

(i)



(ii) $x^2 = 6 - x$

$\therefore x^2 + x - 6 = 0$.

$\therefore (x+3)(x-2) = 0$

$x = -3$ $x = 2$

$y = 9$ $y = 4$.

$\therefore Q(2,4)$. (2)

(iii) $y = 6 - x$, $0 = 6 - x$

$\Rightarrow R(6,0)$. (1)

(iv)

$A = \int_0^2 x^2 dx + \int_2^6 (6-x) dx$ 1

$= \left[x^3/3\right]_0^2 + \left[6x - x^2/2\right]_2^6$. 1

$= 8/3 + [18 - 10]$

$= 10\,2/3$ $(32/3)$. (3)

(c)

$\int_0^4 e^x dx$

$\doteqdot \frac{1}{3}\left[(y_1 + y_5) + 2y_3 + 4(y_2 + y_4)\right]$

x	y's	$f(x) = e^x$
0	y_1	1
1	y_2	2.7183
2	y_3	7.3891
3	y_4	20.0855
4	y_5	54.5982.

$\doteqdot \frac{1}{3}[55.5982 + 14.7782 + 91.2152]$

$\doteqdot \underline{53.8639}$. (3)

18 marks

Question 6 (a)

(i) ① $\int(\cos x - \sin x)\,dx$

$= \sin x + \cos x + c$

(ii) ① $3\int \sec^2 2x\,dx$

$= \frac{3}{2}\tan 2x + c.$

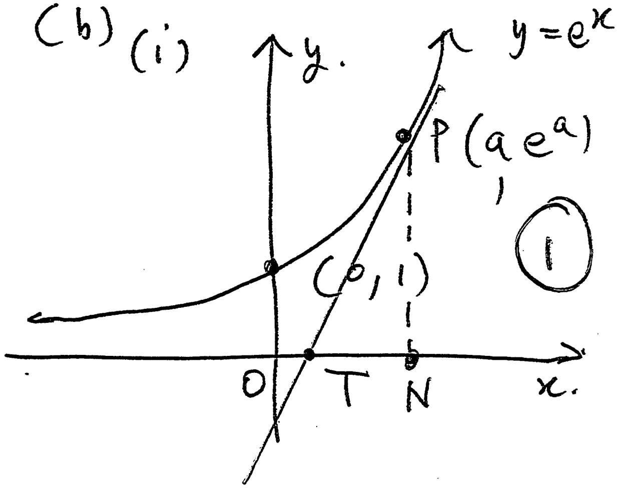
(iii) ③ $\int_0^{\pi/2} 5\sin\frac{x}{2}\,dx$

$= \left[-10\cos\frac{x}{2}\right]_0^{\pi/2}$

$= 10\left(\frac{\sqrt{2}-1}{\sqrt{2}}\right)$ 1

$= 5(2-\sqrt{2}) \doteqdot (2.93)$

(b) (i)



①

(ii) $(a, e^a) = P.$ ①

(iii) $\left.\frac{dy}{dx}\right|_{x=a} = e^a.$

$\therefore\ y - e^a = e^a(x-a)$

$y = (e^a)x - e^a(a-1)$ ②

(iv) $y = 0.$

$e^a(a-1) = e^a x$

$\therefore\ x = (a-1)$

$T\,(a-1, 0).$ ①

$N\,(a, 0).$

$\therefore\ |TN| = 1$ (which is constant). ②

(c)

(i) arc $AB = 10 \times \frac{\pi}{4}$

$= 5\pi/2$ cm. ①

(ii) $C = 2\pi r$

$C = 10 \times \left(2\pi - \frac{\pi}{4}\right)$

i.e. $\frac{35\pi}{2} = 2\pi r$ ②

$r = 35/4 = 8.75$ cm.

(iii) $h^2 + 8.75^2 = 100$

$\therefore\ h = 4.84$ 1

$V = \frac{1}{3}\pi \times (8.75)^2 \times 4.84$

$= 388.15\ \text{cm}^3.$ ③