SYDNEY TECHNICAL HIGH SCHOOL
MATHEMATICS DEPARTMENT

Year 12 2 Unit HSC Task No 2
March 2002

Name: ______________________

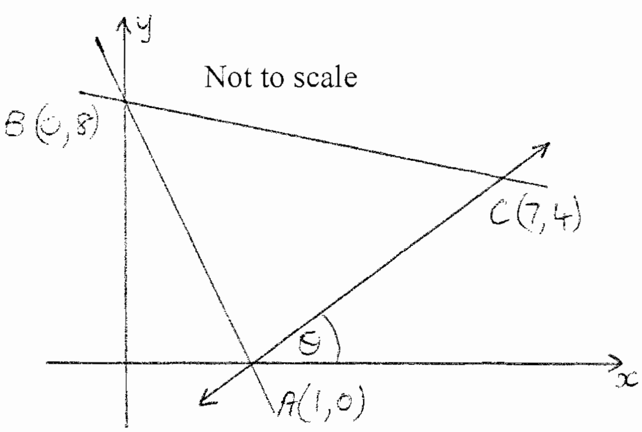
Teacher: ______________________

Instructions:

- Show all necessary working
- Full marks may not be awarded for incomplete working or poor setting out.
- Approximate marks are indicated
- Hand in this question paper on top of your answer pages.
- TIME ALLOWED : 70 MINUTES

Question 1	Question 2	Question 3	Question 4	Question 5	Total
/10	/10	/10	/10	/10	/50

Question 1



The points A,B,C have coordinates (1,0), (0,8) and (7,4) as shown. The angle between line AC and the x-axis is θ.

Copy this diagram onto your Answer page.

a)	Find the gradient of the line AC	1
b)	Calculate the size of θ to the nearest degree.	1
c)	Find the equation of the line AC	2
d)	Show that Δ ABC is isosceles	2
e)	Find the perpendicular distance from B to AC	2
f)	Find the area of Δ ABC	1
g)	Write down the coordinates of a point E such that ABCE is a rhombus	1

Question 2

Give the curve $y = x^3 - 6x^2 + 9x - 5$

a) Find the stationary points and determine their nature. **4**

b) Sketch the curve for $x \geq -1$ and find the absolute minimum value. **2**

c) For what values of x is: **2**

(i) $\frac{dy}{dx} < 0$ (ii) the curve concave up?

d) Find the equation of the tangent to the curve when $x = 0$ **2**

Question 3

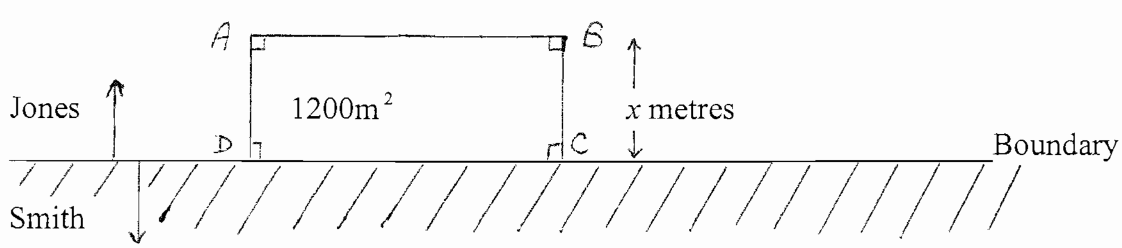
a) The government announces that " **while unemployment is currently rising its increase will slow.**" **2**

Given a mathematical model y = f(*x*) for unemployment, what does this statement imply for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$?

b) If $\frac{d^2y}{dx^2} = 6x - 4$ and that $\frac{dy}{dx} = 7$ at (1,12), find y as a function of x. **2**

c) Find $\int (4x-6)^9 \, dx$. **1**

d)



Farmer Jones wishes to fence off a rectangular yard ABCD of area 1200m^2, as in the figure above, with the side CD against the property of Farmer Smith. Fencing costs \$3 per metre and Smith has agreed to pay for half the cost of side CD. Let \$C be the cost to Jones of fencing the yard and x metres be the length of BC.

(i) Show that $C = 6x + \frac{5400}{x}$ **1**

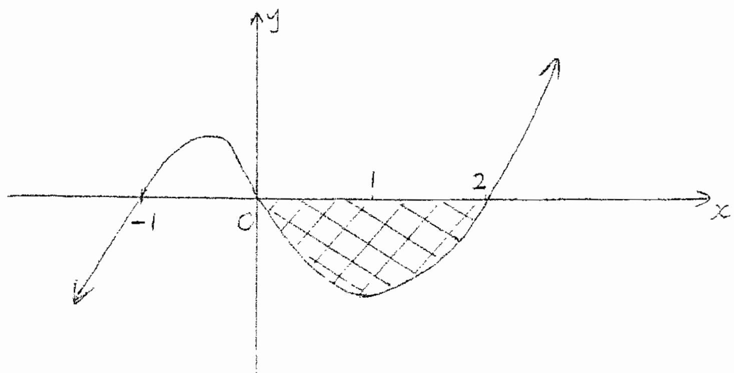
(ii) Prove that the minimum cost to Jones for fencing the yard is \$360. **4**

Question 4

a) Evaluate $\int_0^3 \sqrt{x+1}\, dx$ **2**

b) The graph of $y = x(x+1)(x-2)$ is shown. **3**

Find the shaded area



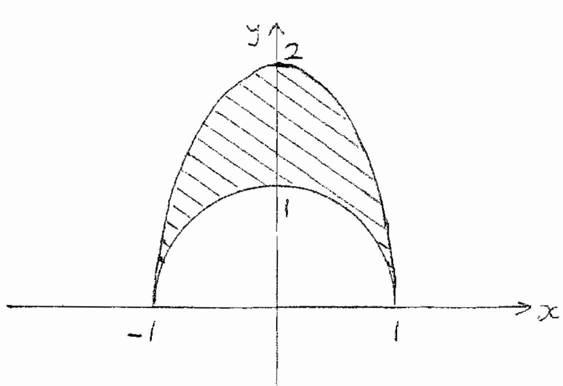
c) Find the area enclosed between the graphs of $y = x^3$ and $y = 4x$ **5**

Question 5

a) Find $\int \frac{3x^3 + 2}{x^2}\, dx$ **2**

b) A cone is generated by rotating about the y – axis the area bounded by the line y = 2x, the y– axis and the line $y = 2$. Use integral calculus to find the volume of this cone. **3**

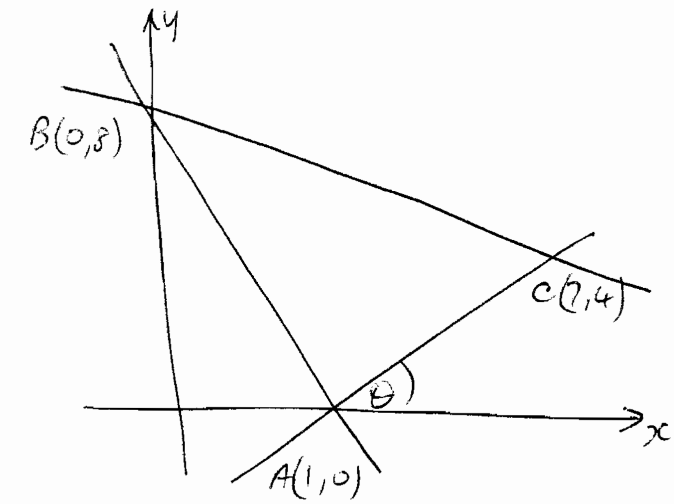
c) The graphs of $y = \sqrt{1-x^2}$ and $y = 2\sqrt{1-x^2}$ are shown below. **5**



Find the volume of the resultant solid when the enclosed region shown above is rotated about the x – axis.

Solutions · 2u HSC Ass #2 2002

Question 1



a) $M_{AC} = \frac{4-0}{7-1}$

$= \frac{2}{3}$ ①

b) $\tan\theta = \frac{2}{3}$

$\therefore \theta \doteqdot 34°$ ①

c) $y - 0 = \frac{2}{3}(x-1)$ ①

$\therefore y = \frac{2}{3}x - \frac{2}{3}$

or $2x - 3y - 2 = 0$ ①

d) $AB = \sqrt{8^2 + 1^2}$

$= \sqrt{65}$ ①

$BC = \sqrt{7^2 + 4^2}$

$= \sqrt{65}$ ①

$\therefore \triangle ABC$ is isosceles (2 equal sides)

e) $2x - 3y - 2 = 0$ and $(0,8)$

$\therefore$ p.d. $= \left| \frac{2\times 0 - 3\times 8 - 2}{\sqrt{2^2+3^2}} \right|$ ①

f) area $= \frac{1}{2} \times \sqrt{6^2+4^2} \times \frac{26}{\sqrt{13}}$

$= \frac{1}{2} \times \sqrt{52}^{\sqrt{4}} \times \frac{26}{\cancel{\sqrt{13}}}$

$= 26u^2$ ①

g) $E(8,-4)$ ①

Question 2

a) S.P.'s when $\frac{dy}{dx} = 0$

$\therefore 3x^2 - 12x + 9 = 0$ ①

$3(x^2 - 4x + 3) = 0$

$3(x-3)(x-1) = 0$

$\therefore x = 3$ or 1 ①

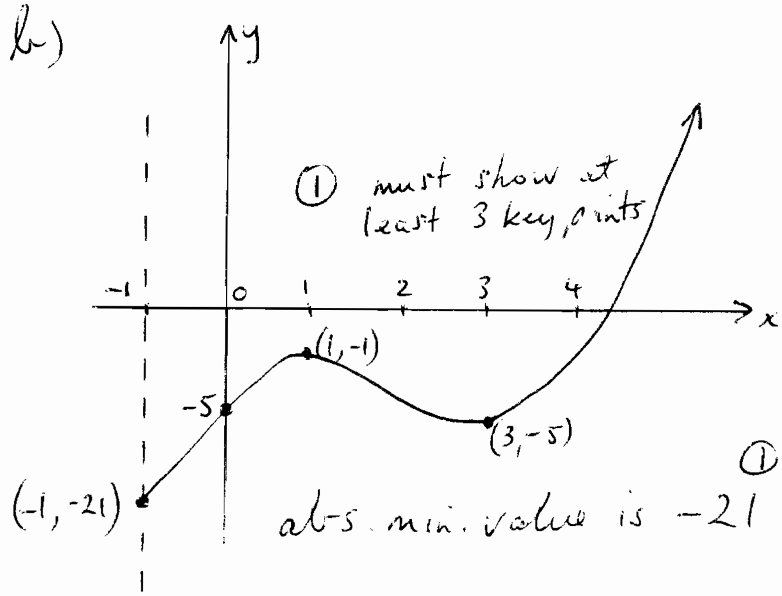
x	2·9	3	3·1
$\frac{dy}{dx}$	−	0	+

x	0·9	1	1·1
$\frac{dy}{dx}$	+	0	−

① ①

$\therefore$ min. T.P. at $(3,-5)$ max T.P. at $(1,-1)$

b)



①

abs. min. value is -21

c) (i) $1 < x < 3$ ①

(ii) $\frac{d^2y}{dx^2} = 0$, $6x - 12 = 0$

d) $\frac{dy}{dx} = 3x^2 - 12x + 9$

when $x = 0$, $\frac{dy}{dx} = m_T = 9$ (1)

$\therefore$ eqn of tangent at $(0, -5)$

is $y + 5 = 9(x - 0)$

$\therefore y = 9x - 5$ (1)

Question 3

a) $\frac{dy}{dx} > 0$ (1) and $\frac{d^2y}{dx^2} < 0$ (1)

b) $\frac{dy}{dx} = 3x^2 - 4x + c$

and $7 = 3 - 4 + c$ ($\therefore c = 8$) (1)

$\therefore \frac{dy}{dx} = 3x^2 - 4x + 8$

$\therefore y = x^3 - 2x^2 + 8x + k$

and $12 = 1 - 2 + 8 + k$ ($\therefore k = 5$)

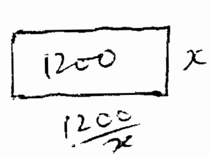
$\therefore y = x^3 - 2x^2 + 8x + 5$ (1)

c) $\frac{(4x-6)^{10}}{10 \times 4} + c = \frac{(4x-6)^{10}}{40} + c$ (1)

d) (i) metres of fencing

$= x + x + \frac{1200}{x} + \frac{600}{x}$

$= 2x + \frac{1800}{x}$ (1)



$\therefore$ cost to Jones

$= \left(2x + \frac{1800}{x}\right) \times \$3/m$

(ii) min. cost when $\frac{dC}{dx} = 0$

$\frac{dC}{dx} = 6 - 5400x^{-2} = 0$ (1)

$\therefore 6 - \frac{5400}{x^2} = 0$

$\therefore 6 = \frac{5400}{x^2}$

$\therefore x^2 = \frac{5400}{6} = 900$

$\therefore x = 30$ (1) (+ve only)

x	29	30	31
$\frac{dC}{dx}$	$-$	0	$+$

$\therefore$ minimum

(1) cost is proved when $x = 30$m,

and cost to Jones is

$6 \times 30 - \frac{5400}{30} = 180 + 180$ (1)

$= \$360$ as reqd.

Question 4

a) $\int_0^3 (x+1)^{1/2}\, dx = \left[\frac{2}{3}(x+1)^{3/2}\right]_0^3$ (1)

$= \frac{2}{3} \cdot 4^{3/2} - \frac{2}{3} \cdot 1^{\frac{3}{2}}$

$= \frac{2}{3} \times 8 - \frac{2}{3}$

$= 4\frac{2}{3}$ (1)

b) $y = x(x^2 - x - 2) = x^3 - x^2 - 2x$

$\therefore S.A. = \left|\int_0^2 (x^3 - x^2 - 2x)\, dx\right|$ (1)

$= \left|\left[\frac{x^4}{4} - \frac{x^3}{3} - x^2\right]_0^2\right|$ (1)

$= \left|\left(4 - 2\frac{2}{3} - 4\right) - (0 - 0 - 0)\right|$

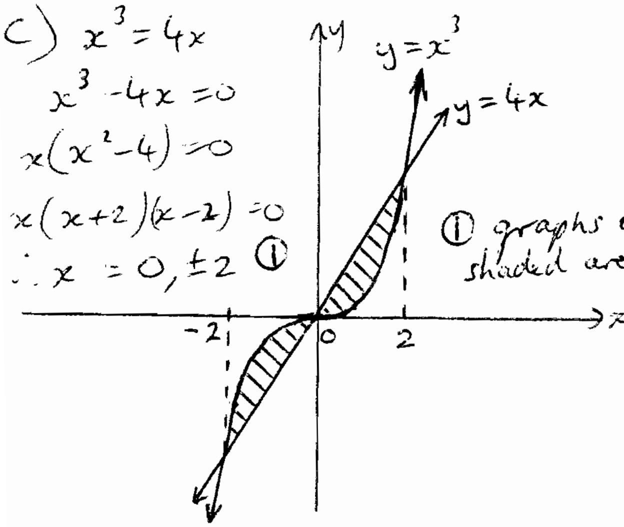
c) $x^3 = 4x$

$x^3 - 4x = 0$

$x(x^2-4) = 0$

$x(x+2)(x-2) = 0$

$\therefore x = 0, \pm 2$ ①



① graphs and shaded areas

$$\text{Area} = 2 \times \int_0^2 (4x - x^3)\,dx \quad ①$$

$$= 2 \times \left[2x^2 - \frac{x^4}{4}\right]_0^2 \quad ①$$

$$= 2 \times \left[(8-4) - (0-0)\right]$$

$$= 8\ u^2 \quad ①$$

Question 5

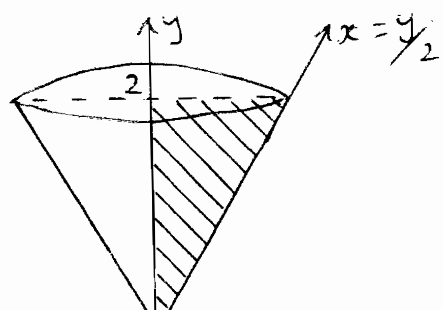
a) $$\int \left(\frac{3x^3}{x^2} + \frac{2}{x^2}\right) dx$$

$$= \int (3x + 2x^{-2})\,dx \quad ①$$

$$= \frac{3x^2}{2} + \frac{2x^{-1}}{-1} + c$$

$$= \frac{3x^2}{2} - \frac{2}{x} + c \quad ①$$

b)



$$\text{Vol} = \pi \int_0^2 \left(\frac{y}{2}\right)^2 dy \quad ①$$

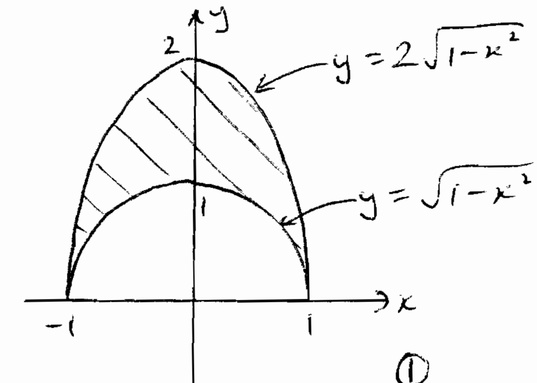
$$= \pi \int_0^2 \frac{y^2}{4}\,dy$$

$$= \pi \times \left[\frac{y^3}{12}\right]_0^2 \quad ①$$

$$= \pi \times \left(\frac{8}{12} - \frac{0}{12}\right)$$

$$= \frac{2\pi}{3}\ u^3 \quad ①$$

c)



$$\text{Vol} = 2 \times \pi \times \left[\int_0^1 \left(2\sqrt{1-x^2}\right)^2 - \int_0^1 \left(\sqrt{1-x^2}\right)^2\right] \quad ①$$

$$= 2\pi \times \left[\int_0^1 4(1-x^2) - \int_0^1 (1-x^2)\right] \quad ①$$

$$= 2\pi \times \int_0^1 (4 - 4x^2 - 1 + x^2)\,dx$$

$$= 2\pi \times \int_0^1 (3 - 3x^2)\,dx \quad ①$$

$$= 2\pi \times \left[3x - x^3\right]_0^1 \quad ①$$

$$= 2\pi \times \left[(3-1) - (0-0)\right]$$

$$= 2\pi \times 2$$

$$= 4\pi\ u^3 \quad ①$$