Sydney Technical High School

Year 12
2 Unit Mathematics

HSC Assessment Task 2 - March 2003

Name: ______________________________ **Class:** ______________

Time Allowed: 70 minutes

Instructions:

1. Answer questions on paper provided
2. Begin each question on a fresh page.
3. Marks may be deducted for careless or untidy work
4. Show all working
5. Marks for each question are indicated next to the question. These marks are a guide and may be adjusted slightly if necessary.
6. Approved calculators may be used.

Question	**1**	**2**	**3**	**4**	**5**
Mark					
Total					

Question 1 (12 marks)

5 (a) Find: i) $\int (4x^2 + 6)\, dx$

ii) $\int \frac{1}{x^2}\, dx$

iii) $\int \sqrt{2x+1}\, dx$

iv) $\int_1^4 (x^2 + 4)\, dx$

4 (b) Find the equation of a curve for which $y'' = 6x - 4$ and when $x = 1, y = 12$ and $y' = 7$.

3 (c) If $f(x) = \sqrt{2x^2 + 4}$, find $f''(x)$.

Question 2 (12 marks) (Begin a new page)

2 (a) Use calculus to find the values of x for which the curve $y = 4 + x - x^2$ is decreasing.

4 (b) Evaluate $\sum_{n=1}^{40} 3n - 1$

3 (c) For a certain function, $f'(x) = \dfrac{(x-2)(x-4)^2}{\sqrt{x(x+2)^3}}$.

i) Give a reason why the function has turning points when $x = 2$ and $x = 4$.
ii) Determine the nature of the turning point at $x = 2$.

3 (d) Julie is building a huge deck using 151 timber planks which decrease uniformly in length from 2500 mm to 400 mm so that the lengths of the planks form an arithmetic sequence. Find

i) the difference in length between adjacent planks.
ii) the total length (in metres) of planks needed.

Question 3 (10 marks) (Begin a new page)

4 (a) Find the values of x for which the curve $y = 4x^3 - 12x^2 + 2$ is

i) concave up

ii) concave down

4 (b) The ground floor of a twenty story office block will cost \$200 000 to construct. The next floor will cost \$230 000, and the next, \$264 500. The cost of the remaining 17 floors will follow the same pattern. Find the total cost of building the twenty floors.

2 (c) The point (1, 6) lies on the curve $y = f(x)$. If $f''(x) = 12(x-1)^2$, determine whether or not (1, 6) is a point of inflexion.

Question 4 (11 marks) (Begin a new page)

For the curve $y = x^3 - 6x^2 + 9x,\ -1 \le x \le 4.$

2 (a) Find y' and y''.

4 (b) Find the coordinates of any stationary points and determine their nature.

2 (c) Find where the curve touches the x axis.

2 (d) Sketch the curve in the given domain showing all features determined above.

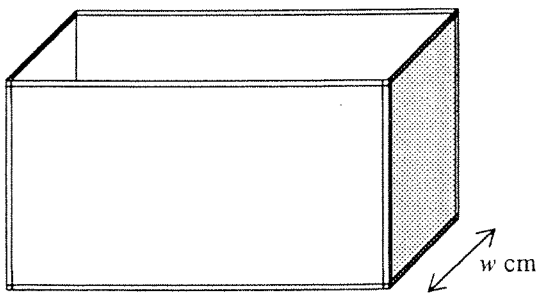
1 (e) Find the minimum value of the curve in the given domain.

Question 5 (10 marks) (Begin a new page)

4 (a) On a half page number plane diagram, sketch a possible curve for $y = f(x)$ which satisfies the conditions given in the following table:

x	0	1	2	3
$f(x)$		0	-1	
$f'(x)$	-1	0	-1	0
$f''(x)$		0	0	1

6 (b) An open cardboard box is twice as long as it is wide. The volume is 24cm^3 and all edges of the box are to be taped.



i) Show that the length of tape needed is $12w + \frac{48}{w^2}$ where w is the width (in cm) of the box.

ii) Find the dimensions of the box which will give a minimum amount of tape.

End of Test

2003 2U HSC ASSESSMENT #2 SOLUTIONS.

Q1

a) i) $\int (4x^2+6)\,dx$

$= \frac{4x^3}{3} + 6x + c$

ii) $\int \frac{1}{x^2}\,dx = \int x^{-2}\,dx$

$= -x^{-1} + c$

or

$\frac{-1}{x} + c.$

iii) $\int \sqrt{2x+1}\,dx = \int (2x+1)^{\frac{1}{2}}\,dx$

$= \frac{(2x+1)^{3/2}}{\frac{3}{2}\times 2} + c$

$= \frac{(2x+1)^{3/2}}{3} + c$

iv) $\int_1^4 (x^2+4)\,dx = \left[\frac{x^3}{3} + 4x\right]_1^4$

$= \left(\frac{64}{3}+16\right) - \left(\frac{1}{3}+4\right)$

$= 33$

b) $y'' = 6x - 4$

$\therefore y' = 3x^2 - 4x + c$

but $y' = 7$, $x = 1$

$\therefore 7 = 3 - 4 + c \Rightarrow c = 8$

$\therefore \quad y' = 3x^2 - 4x + 8$

$y = x^3 - 2x^2 + 8x + k$

but $y = 12$, $x = 1$

$\therefore 12 = 1 - 2 + 8 + k \Rightarrow k = 5$

$\therefore y = x^3 - 2x^2 + 8x + 5$ is equation of the curve.

c) $y = (2x^2+4)^{\frac{1}{2}}$

$y' = \frac{1}{2}(2x^2+4)^{-\frac{1}{2}} \times 4x$

$= 2x(2x^2+4)^{-\frac{1}{2}}$

$y'' = -\frac{1}{2} \times 2x(2x^2+4)^{-3/2} \times 4x + 2(2x^2+4)^{-\frac{1}{2}}$

$= 2(2x^2+4)^{-\frac{1}{2}} - 4x^2(2x^2+4)^{-3/2}$

Q2. (a) $y = 4 + x - x^2$

$y' = 1 - 2x$

decreasing when $1 - 2x < 0$

ie $2x > 1$

$x > \frac{1}{2}$

(b) $\sum_{1}^{40} 3n - 1$

$= 2 + 5 + 8 + \cdots + 119$

$= \frac{40}{2}(2 + 119)$

$= 2420$

(c) i) $f'(2) = f'(4) = 0$

ii) when $x < 2$, $f'(x) < 0$

" $x > 2$, $f'(x) > 0$

$\therefore$ local minimum when $x = 2$.

(d) i) $a = 400$, $T_{151} = 2500$

$\therefore 400 + 150d = 2500$

$\therefore d = 14$

ii) $S_{151} = \frac{151}{2}(400 + 2500)$

$= 218950$ mm

$= 218.95$ m.

Q3 (a) $y = 4x^3 - 12x^2 + 2$

$y' = 12x^2 - 24x$

$y'' = 24x - 24$

i) Concave up when $24x - 24 > 0$

$\Rightarrow x > 1$

ii) Concave down when $x < 1$

(b) G.P. where $a = 200\,000$

$r = 1.15$, $n = 20$

$S_{20} = 200000\frac{(1.15^{20} - 1)}{0.15}$

$= \$20,488,717$

Q3(c) $f''(x) = 12(x-1)^2$

(1,6) is NOT a pt of inflexion since $f''(x)$ is always positive (does not change sign).

Q4. $y = x^3 - 6x^2 + 9x$, $-1 \le x \le 4$.

a) $y' = 3x^2 - 12x + 9$

$y'' = 6x - 12$

b) St. points occur when $y' = 0$

ie $3x^2 - 12x + 9 = 0$

$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

$\therefore x = 1, 3.$

When $x = 1$, $y = 4$ and $y'' < 0$

$\therefore (1,4)$ is a local max.

When $x = 3$, $y = 0$ and $y'' > 0$

$\therefore (3,0)$ is a local min.

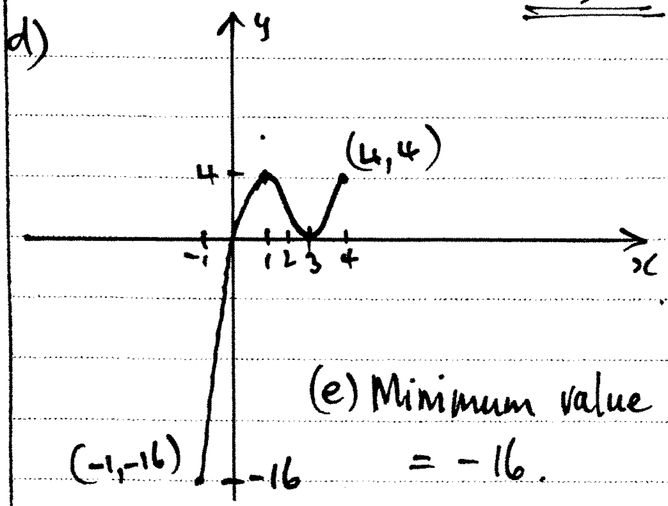
c) $y = 0 \Rightarrow x^3 - 6x^2 + 9x = 0$

$x(x^2 - 6x + 9) = 0$

$x(x-3)^2 = 0$

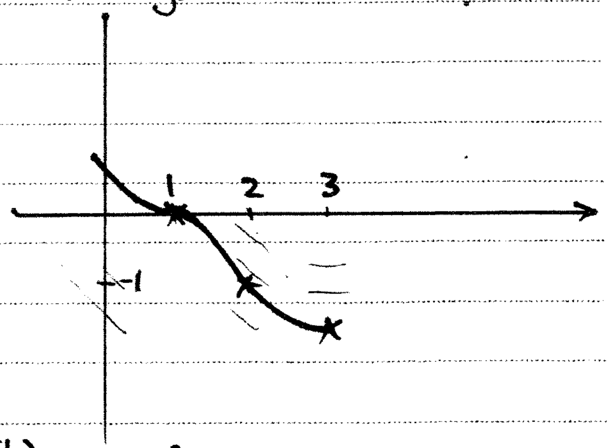
$\therefore x = 0, 3.$

d)

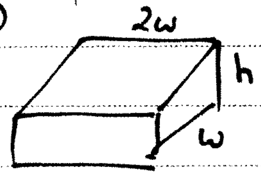


(e) Minimum value $= -16$.

Q5. (a) Rough in with pencil first then go over with ink.



(b)



$V = 2w^2h = 24$

$\therefore h = \frac{12}{w^2}$ ✓

i) $L = 4 \times 2w + 4 \times w + 4 \times h$

$= 12w + 4h$

$= 12w + \frac{48}{w^2}$ ✓

ii) Min L occurs when $L' = 0$ and $L'' > 0$.

$L' = 12 - \frac{96}{w^3}$ $\quad -96w^{-3}$

$L'' = \frac{288}{w^4}$ which is always +ve. $\quad 288w^{-4}$

When $L' = 0$, $\frac{96}{w^3} = 12$

ie $12w^3 = 96$

$w^3 = 8$

$\therefore w = 2.$

$\therefore$ Dimensions are $2 \times 4 \times 3$ cm.