SYDNEY TECHNICAL HIGH SCHOOL

YEAR 12 ASSESSMENT TASK 2

MARCH 2004

MATHEMATICS 2 UNIT

Time allowed: 70 minutes

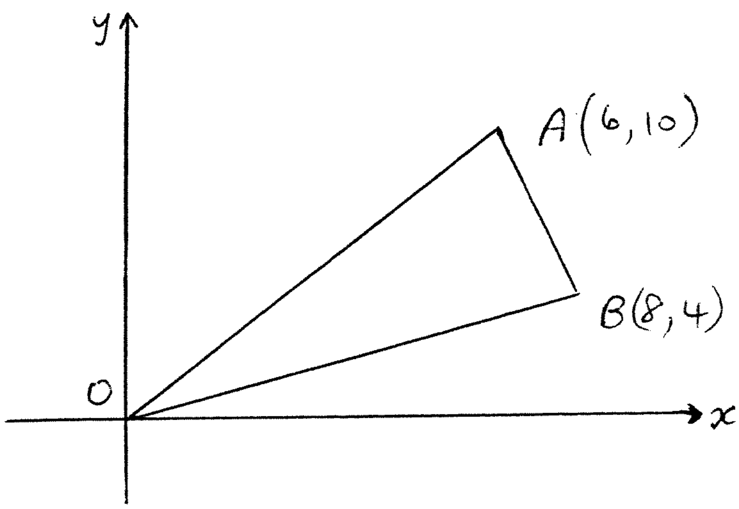
Instructions:

- Start each question on a new page
- Full marks may not be awarded if working is incomplete or illegible.

Name:____________________ **Teacher:**____________________

Q1	Q2	Q3	Q4	Q5	Total
/10	/10	/10	/10	/10	/50

Question 1



Not to scale

a) Copy this diagram above onto your answer page

b) Find the length of OA in surd form. 1

c) Find the gradient of OA 1

d) Find the equation of OA in general form **2**

e) K is on OA such that BK $\perp$ OA. Use your answer in part (c) to find the gradient of BK and hence the equation of BK. **2**

f) Find the perpendicular distance BK and hence the area of Δ OAB. **3**

g) Find the coordinates of a point C such that OABC is a parallelogram **1**

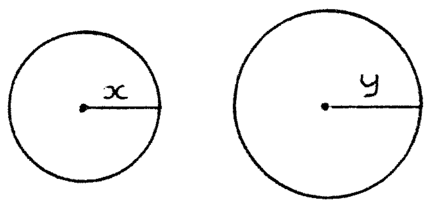
Question 2

a) Given $\frac{dy}{dt} = 12t^2 + t$ and that $y = 20$ when $t = 2$, find y in terms of t **2**

b) (i) Find $\int (x^2 + \frac{3}{x^2})\, dx$ **2**

(ii) Evaluate $\int_1^3 \frac{1}{\sqrt{x}}\, dx$ **2**

c) Two circles are such that the sum of their radii is constant at 10 cm.



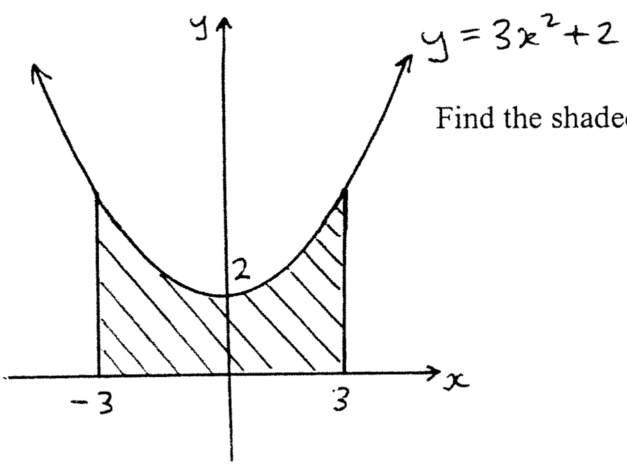
Let the radii of the two circles be x cm and y cm.

(i) Show that the sum of their areas is given by $A = 2\pi x^2 - 20\pi x + 100\pi$ **1**

(ii) Show that the sum of their areas will be a minimum when the radii are equal **3**

Question 3

a)



Find the shaded area **3**

b) (i) Find the x values of the points of intersection of the graphs of $y = x^2 - 6x$ and $y = 2x$. **1**

(ii) Find the area between the two graphs above. **3**

c) Find the area between the curve $y = x^3$, the $y-$axis and the lines $y = 1$ and $y = 8$ **3**

Question 4

a) Find a primitive of $(3x+2)^4$ 1

b) For the curve $y = 6x^2 - x^3 + 9$:

(i) Find the stationary points and determine their nature 4

(ii) Determine any point (s) of inflexion 2

(iii) Sketch the curve over the domain $-3 \le x \le 5$ 2

(iv) In this domain, determine the maximum value of the function. 1

Question 5

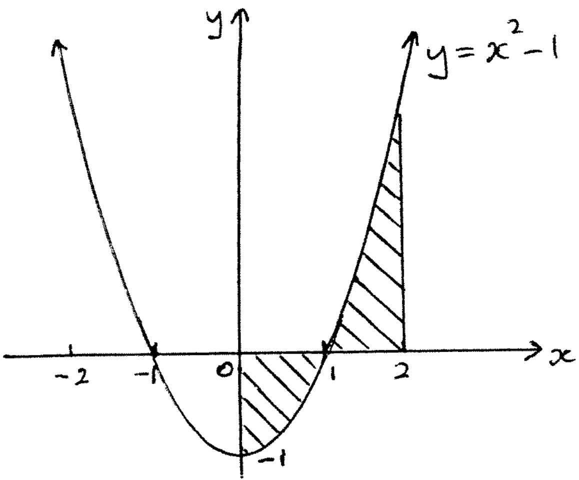
a) Sketch a curve on a number plane that satisfies all conditions below: 3

$f(0) = 2$

When $x < 0, f'(x) < 0$ and $f''(x) > 0$

When $x > 0, f'(x) < 0$ and $f''(x) < 0$.

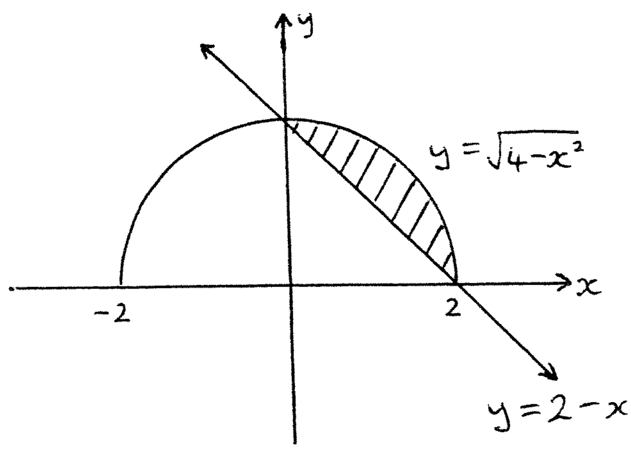
b)



Find the shaded area 3

c) The graphs of $y = 2 - x$ and $y = \sqrt{4 - x^2}$ are shown. 4

Find the volume generated when the shaded area shown is rotated about the $x - axis$.



Solutions.

(1) b) $OA = \sqrt{6^2+10^2}$

$= \sqrt{136}$ (1)

c) $m_{AB} = \frac{10}{6}$

$= \frac{5}{3}$ (1)

d) $y = \frac{5}{3}x$, $3y = 5x$

(1)

$\therefore 5x - 3y = 0$ (1)

e) $m_{BK} = -\frac{3}{5}$ (1)

$\therefore$ eqn. of BK is

$y - 4 = -\frac{3}{5}(x-8)$

$5y - 20 = -3x + 24$

$\therefore 3x + 5y - 44 = 0$ (1)

f) line $5x - 3y = 0$
point $(8,4)$

$\therefore$ p.d. $= \dfrac{5 \times 8 + (-3) \times 4 + 0}{\sqrt{5^2+3^2}}$

$= \dfrac{28}{\sqrt{34}}$ (1) correct idea (1) correct applic.

$\therefore$ area $\triangle OAB = \frac{1}{2} \times \sqrt{136} \times \frac{28}{\sqrt{34}}$

$= 28u^2$ (1)

g) $C(2, -6)$ (1)

(2) a) $y = 4t^3 + \frac{t^2}{2} + c$ (1)

$y = 20, t = 2$:

$\therefore 20 = 32 + 2 + c$ $(c = -14)$

$\therefore y = 4t^3 + \frac{t^2}{2} - 14$ (1)

b) (i) $\int (x^2 + 3x^{-2})\,dx$

$= \frac{x^3}{3} + \frac{3x^{-1}}{-1} + c$ (1)

$= \frac{x^3}{3} - \frac{3}{x} + c$ (1)

(ii) $\int_1^3 x^{-1/2}\,dx = \left[\frac{x^{1/2}}{1/2}\right]_1^3$ (1)

$= \left[2\sqrt{x}\right]_1^3$

$= 2\sqrt{3} - 2$ (1)

c) (i) $A = \pi x^2 + \pi y^2$
(and $x + y = 10$
$\therefore y = 10 - x$)

$\therefore A = \pi x^2 + \pi(10-x)^2$

$= \pi x^2 + \pi(100 - 20x + x^2)$ (1)

$= \pi x^2 + 100\pi - 20\pi x + \pi x^2$

$= 2\pi x^2 - 20\pi x + 100\pi$

(ii) min. area when $\frac{dA}{dx} = 0$ ①

$\frac{dA}{dx} = 4\pi x - 20\pi = 0$

$4\pi(x-5) = 0$

$\therefore x = 5$ ①

x	5^-	5	5^+
$\frac{dA}{dx}$	$-$	0	$+$

① $\diagdown\!\_\!\diagup$

$\therefore$ area must be a minimum when $x = 5$ and $y = 5$, ie. equal radii.

③ a) $A = 2 \times \int_0^3 (3x^2+2)\,dx$ ①

$= 2\left[x^3 + 2x\right]_0^3$ ①

$= 2\left[27 + 6 - (0+0)\right]$

$= 66\,u^2$ ①

b) (i) At intersection,

$x^2 - 6x = 2x$

$x^2 - 8x = 0$

$x(x-8) = 0$

$\therefore x = 0$ or 8 ①

(ii) $A = \left|\int_0^8 (x^2 - 6x - 2x)\,dx\right|$ ①

$= \left|\int_0^8 (x^2 - 8x)\,dx\right|$

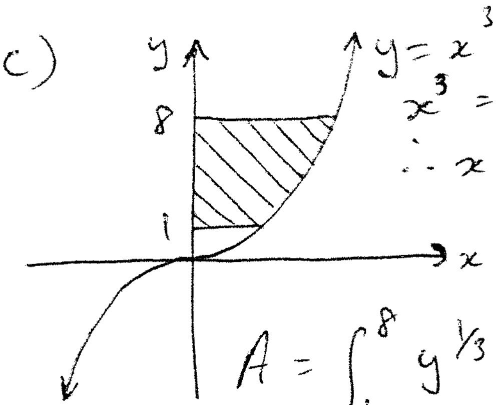
$= \left|\left[\frac{x^3}{3} - 4x^2\right]_0^8\right|$ ①

$= \left|\left(\frac{512}{3} - 256\right) - (0 - 0)\right|$

$= \left|170\tfrac{2}{3} - 256\right|$

$= \left|-85\tfrac{1}{3}\right|$

$= 85\tfrac{1}{3}\,u^2$ ①

c) 

$x^3 = y$

$\therefore x = y^{1/3}$

$A = \int_1^8 y^{1/3}\,dy$ ①

$= \left[\frac{3}{4} y^{4/3}\right]_1^8$ ①

$= \frac{3}{4}\left(8^{4/3} - 1^{4/3}\right)$

$= \frac{3}{4}(16 - 1)$

$= \frac{3}{4} \times 15$

$= 11\tfrac{1}{4}\,u^2$ ①

④ a) $\frac{(3x+2)^5}{5 \times 3} + c$

$= \frac{(3x+2)^5}{15} + c$

①

b) (i) S.P.'s when $\frac{dy}{dx} = 0$ ①

$$\frac{dy}{dx} = 12x - 3x^2 = 0$$

$$3x(4-x) = 0$$

$$\therefore x = 0, 4 \quad ①$$

x	0^-	0	0^+
$\frac{dy}{dx}$	$-$	0	$+$

x	4^-	4	4^+
$\frac{dy}{dx}$	$+$	0	$-$

$\therefore$ min. T.P. at $(0, 9)$ ①

$\therefore$ max. T.P. at $(4, 41)$ ①

ii) P. of I. when $\frac{d^2y}{dx^2} = 0$ and concavity changes

$$\frac{d^2y}{dx^2} = 12 - 6x = 0$$

$$\therefore x = 2 \quad ①$$

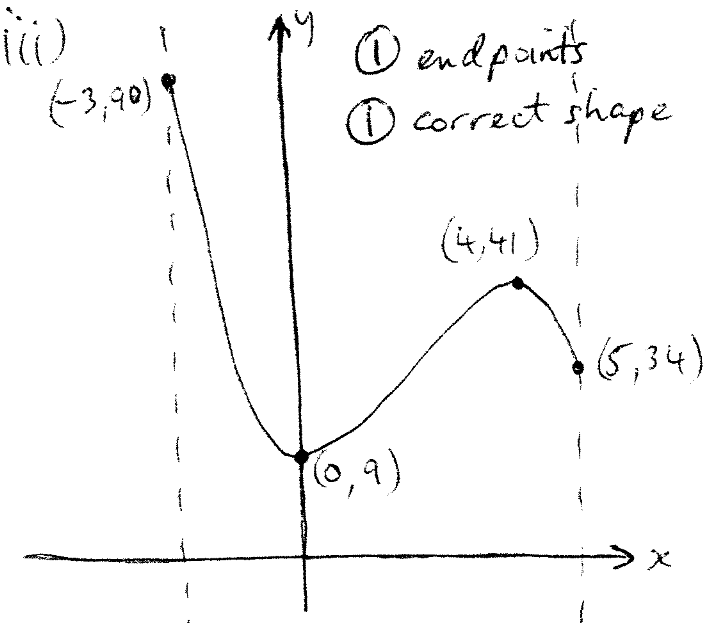
x	2^-	2	2^+
$\frac{d^2y}{dx^2}$	$+$	0	$-$

①

$\therefore$ concavity changes

$\therefore$ P. of I. at $(2, 25)$

iii)



① endpoints

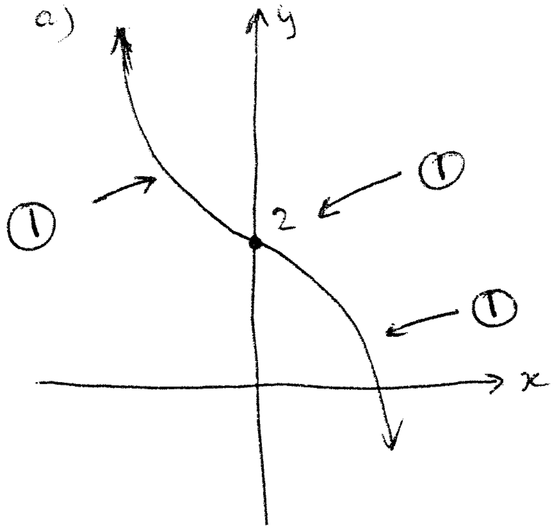
① correct shape

(iv) maximum value is 90

or $y = 90$ ①

5

a)



b) $A = \left|\int_0^1 (x^2-1)\,dx\right| + \int_1^2 (x^2-1)\,dx$ ①

$$= \left|\left[\frac{x^3}{3} - x\right]_0^1\right| + \left[\frac{x^3}{3} - x\right]_1^2$$

$$= \left|\left(\tfrac{1}{3} - 1\right) - (0-0)\right| + \left(\tfrac{8}{3} - 2\right) - \left(\tfrac{1}{3} - 1\right) \quad ①$$

$$= \left|-\tfrac{2}{3}\right| + \tfrac{2}{3} - \left(-\tfrac{2}{3}\right)$$

$$= \tfrac{2}{3} + \tfrac{2}{3} + \tfrac{2}{3}$$

$$= 2u^2 \quad ①$$

c) $V = \pi \int_0^2 \left(\sqrt{4-x^2}\right)^2 - (2-x)^2\,dx$ ①

$$= \pi \int_0^2 4 - x^2 - (4 - 4x + x^2)\,dx$$

$$= \pi \int_0^2 (4 - x^2 - 4 + 4x - x^2)\,dx$$

$$= \pi \int_0^2 (4x - 2x^2)\,dx \quad ①$$

$$= \pi\left[2x^2 - \frac{2x^3}{3}\right]_0^2 \quad (1)$$

$$= \pi\left[\left(8 - \frac{16}{3}\right) - (0 - 0)\right]$$

$$= \pi \times 2\tfrac{2}{3}$$

$$= \frac{8\pi}{3}\ u^3 \quad (1)$$