SYDNEY TECHNICAL HIGH SCHOOL

YEAR 12 MATHEMATICS

H.S.C. Assessment Task 2

March 2006

Name: ________________________________ **Class:** ______________

Time Allowed: 70 minutes

Instructions:

1. Begin each question on a new page.
2. Marks may be deducted for careless or untidy work.
3. Show all necessary working.
4. Marks indicated should be taken as a guide and may change slightly during the marking process.

Question	1	2	3	4	5	Total
Mark						

Question 1

a) Differentiate with respect to x:

i) $\frac{2}{x^4}$ (1)

ii) $\sqrt{2-x^2}$ (1)

iii) $x^2(x+3)^2$ (2)

b) The first two terms in an arithmetic sequence are 100 and 95.

i) Find the 20th term (2)

ii) Find the sum of the first 20 terms (1)

iii) What positive number of terms must be taken for their sum to be zero? (2)

c) Evaluate $\sum_{n=11}^{20} 4-4n$ (3)

Question 2 (Begin on a new page)

a) Find the value of a if 2, a, 100 are in geometric progression. (2)

b) In a geometric sequence, the first term is 6 and the tenth term is 3072. Find the second term. (3)

c) Find the sum of the first 12 terms of the series 128 – 64 +32 . . .
(Answer correct to 2 dec. places) (3)

d) Find i) the primitive of x^4 (1)

ii) $\int 4x^{-3}dx$ (1)

iii) $\int x(x+3)^2 dx$ (2)

Question 4 (Begin on a new page)

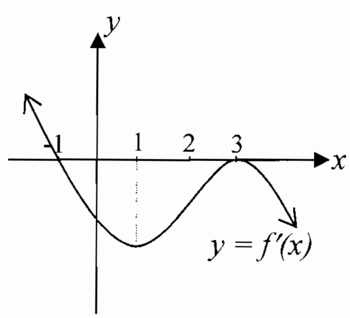
a) Find the area between the curve $y = 4 - x^2$ and the x axis between $x = 0$ and $x = 2$. (3)

b) P (x, y) is a point on the line $y = 2x + 4$. A is the point $(1, 1)$.

i) Show that the length (l) of the interval PA is given by $l = \sqrt{5x^2 + 10x + 10}$. (2)

ii) Use calculus to find the coordinates of P which will make l^2 a minimum. (3)

c) By inspecting the graph of the derivative (gradient function), $y = f'(x)$ shown below



we may conclude that the curve $y = f(x)$ (not shown) must have stationary points when $x = -1$ and $x = 3$.

i) Give a reason for this conclusion. (1)

ii) Describe the type of stationary point on the curve $y = f(x)$ at $x = -1$. (1)

iii) Describe the type of stationary point on the curve $y = f(x)$ at $x = 3$. (1)

iv) Describe the type of point there must be on the curve $y = f(x)$ when $x = 1$. (1)

Question 3 (Begin on a new page)

a) The population (P) of a country town is increasing due to new arrivals but the rate at which people are arriving is decreasing. Describe the effect this situation has on $\frac{dP}{dt}$ and $\frac{d^2P}{dt^2}$. (2)

b) Find the equation of the curve $y = f(x)$ if $f'(x) = 12x$ and the curve passes through the point (1, 5). (2)

c) A water tank 8.4 metres high is full when it springs a leak. The water level drops 10 cm on the first day, a further 18 cm on the second day and a further 26 cm on the third day. If the water level continues to fall in this manner, on which day will the tank be emptied? (4)

d) i) Find 2 values of c for which $\int_0^c (4-2x)\, dx = 3$ (3)

ii) Suggest a reason involving areas which explains why there are two values of c which make the integral in i) equal to 3. (1)

Question 5 (Begin on a new page)

a) A function is defined as $f(x) = (x-1)(x+2)^2$.

i) Show that $f'(x) = 3x(x+2)$ (1)

ii) Find the coordinates of any stationary points on the curve $y = f(x)$ and determine their nature. (2)

iii) Sketch the graph of $y = f(x)$ showing clearly the turning points and where the curve meets the x axis. (3)

iv) For what values of x is the curve concave up? (1)

b) i) At the beginning of the year Jack borrowed \$60 000. Interest is charged at 4% p.a. (compounded yearly). The loan plus interest is to be repaid in a single payment at the end of 10yrs.
Calculate the amount (to the nearest dollar) that Jack will repay. (2)

ii) At the same time that Jack took out the loan, his wife, Jill, deposited \$M in an investment fund at 5% p.a. (compounded yearly). She deposits \$M at the beginning of each year thereafter. Show that at the end of 10 years, her investment has grown to \$21M(1.05^{10} – 1). (2)

iii) If Jill's investment is to be used to repay Jack's loan, calculate the value of M (to the nearest integer). (1)

End of paper

2U HSC Ass. #2 MARCH 2006
Marking Scheme & Rubric.

Q1(a)

(i) $y' = -8x^{-5}$ or equivalent (1)

(ii) $y' = -x(2-x^2)^{-\frac{1}{2}}$

or $\dfrac{-x}{\sqrt{2-x^2}}$ (1)

(iii) $y' = 2x(x+3)^2 + x^2.2(x+3)$ (2) Give 1 mark for each correct half of the product rule.

or $2x(x+3)(2x+3)$ ✓ (2)

or $4x^3 + 18x^2 + 18x$ ✓ (2) Give 1 for correct differentiation of an incorrect expansion.

(b) $100 + 95 + \ldots$

(i) $d = -5$ (1) If nothing else correct

$T_{20} = 100 + 19 \times -5$

$= 5$ (1) Total (2)

(ii) $S_{20} = \frac{20}{2}(100+5)$

$= 1050$ (1)

(iii) $\frac{n}{2}(200 + (n-1) \times -5) = 0$ (1) for this if rest incorrect

$\therefore 200 - 5(n-1) = 0$

$n - 1 = 40$

$\therefore n = 41$ (2) if correct (Total (2))

(c) $-40 + -44 + -48 \cdots + -76$ 1. to here

$S_{10} = \frac{10}{2}(-40 + -76)$ 2 to here

$= -580$ 3 if correct

Q2. (a) $2, a, 100$

$a = \sqrt{200}$ ✓

$= 10\sqrt{2}$ ✓ Total 2

(b) $a = 6$

$ar^9 = 3072$

$\therefore r^9 = 512$ ✓

$\therefore r = 2$ ✓

$\therefore T_2 = 12$ ✓ Total 3

(c) $a = 128$

$r = -\frac{1}{2}$ ✓

$$S_{12} = \frac{128\left(\left(-\frac{1}{2}\right)^{12} - 1\right)}{-1\frac{1}{2}}$$ ✓

$= 85.31$ ✓ Total 3

(d) i) $\frac{x^5}{5} + c$ 1 zero if no constant.

ii) $\int 4x^{-3}\,dx = 4 \cdot \frac{x^{-2}}{-2} + c$

$= -2x^{-2} + c$ ✓ Allow if no constant.

or $\frac{-2}{x^2} + c$

(iii) $\int x(x+3)^2\,dx$

$= \int (x^3 + 6x^2 + 9x)\,dx$ ✓ Give 1 for correct primitive of an incorrect expansion.

$= \frac{x^4}{4} + 2x^3 + \frac{9x^2}{2} + c$ ✓ Allow if no Constant

No marks for an attempt to use reverse fn of fn rule.

Q3. (a) $\dfrac{dP}{dt} > 0$ (1)

$\dfrac{d^2P}{dt^2} < 0$ (1)

(b) $f'(x) = 12x$

$f(x) = 6x^2 + c$ (1) for this

but $5 = 6 + c \Rightarrow c = -1$ (1) for this

$\therefore y = 6x^2 - 1$ Allow $f(x) = 6x^2 - 1$.

(c) $10 + 18 + 26 + \cdots$

$S_n = \frac{n}{2}(20 + (n-1).8)$

$= 4n^2 + 6n$ (1) for correct $\exp^n$ for S_n.

When empty $4n^2 + 6n \geq 840$

Solving $4n^2 + 6n - 840 = 0$

ie $2n^2 + 3n - 420 = 0$

$\Big\}$ + (1) for a correct eqn

$n = \dfrac{-3 \pm \sqrt{9 + 3360}}{4}$ +

$= \dfrac{-3 \pm 58.04}{4}$ (1) for the correct value of n

ie $n \doteqdot 13.8$ (taking $n > 0$)

$\therefore$ Tank empties on 14^{th} day Correct answer (4) marks.

(d) (i) $\displaystyle\int_0^c (4 - 2x)\,dx = 3$

ie $\left[4x - x^2\right]_0^c = 3$ (1)

ie $4c - c^2 = 3$

ie $c^2 - 4c + 3 = 0$ (1)

$(c-1)(c-3) = 0$

$c = 1, 3.$ (3) for correct answer

(ii) "$\displaystyle\int_0^3 (4-2x)\,dx = \int_0^1 4 - 2x\,dx + \int_1^3 4 - 2x\,dx$. So $\displaystyle\int_1^3 4 - 2x\,dx$" (1)

Q4 (a) $A = \int_0^2 (4-x^2)\,dx$

$= \left[4x - \frac{x^3}{3} \right]_0^2$ ✓

$= (8-\frac{8}{3}) - (0-0)$ ✓

$= \frac{16}{3}\ u^2$ ✓ (3) for correct answer.

(b)(i) $l = \sqrt{(x-1)^2 + (y-1)^2}$ ✓ (1) Correct distance formula.

$= \sqrt{(x-1)^2 + (2x+4-1)^2}$ ✓ (1) for correct substitution for y (even if into an incorrect formula)

Total (2)

(ii) $\frac{dl^2}{dx} = 10x + 10$

$= 0$ when $x = -1$ (1) Correct value for x

$\frac{d^2l^2}{dx^2} = 10$ (which > 0) $\Rightarrow$ minimum (1) for a minimum test

$\therefore P$ is $(-1, 2)$ (1) for coordinates.

(c) (i) "Because $f'(x) = 0$ at $x = -1$ and 3." or equivalent (1)

(ii) "Local maximum" or Maximum etc (1)

(iii) "Point of Inflexion" - no need to say horizontal (1)

(iv) "Inflexion point" or equivalent (1)

Q5 (a) (i) $f'(x) = 3x^2 + 6x$ ✓ ① for correct differentiation from product rule or expansion

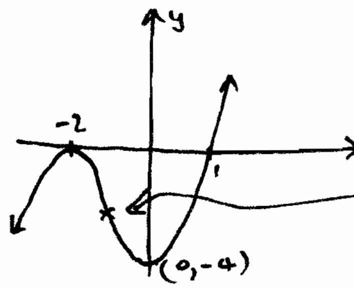
(ii)

$(0,-4)$ is local minimum ✓ Total ②. Allow ①

$(-2,0)$ is local maximum. ✓ if both x values correct, or for equivalent merit.

Full marks if y values omitted here but shown correctly on graph.

(iii)



③ 1 mark for each correct feature.

(iv) $x > -1$ ①

B) i) $A = 60000(1\cdot04)^{10}$ ✓

$= \$88\,815$ ✓ Total ②

ii) $A = M.1\cdot05 + M.1\cdot05^2 + \cdots + M.1\cdot05^{10}$ ✓

$= M(1\cdot05 + 1\cdot05^2 + \cdots + 1\cdot05^{10})$

$= M \dfrac{1\cdot05(1\cdot05^{10} - 1)}{1\cdot05 - 1}$ ✓✓ G.P. with correct a, r and n.

$= M\,21(1\cdot05^{10} - 1)$ Total ②*

(iii) $21M(1\cdot05^{10} - 1) = 88\,815$

$\therefore M = \dfrac{88\,815}{21(1\cdot05^{10} - 1)}$

$= \$6725$ ①