

NAME:.....

CLASS:

SYDNEY TECHNICAL HIGH SCHOOL

YEAR 12

HSC ASSESSMENT TASK 2

MARCH 2007

MATHEMATICS

Time Allowed : 70 minutes

Instructions:

- Write your name and class at the top of each page
- All necessary working must be shown. Marks may be deducted for careless or badly arranged work.
- Marks indicated are a guide only and may be varied if necessary.
- Start each question on a new page.

Question 1	Question 2	Question 3	Question 4	Question 5	Total
/12	/12	/12	/12	/10	/58

QUESTION 1**Marks****a)** Differentiate the following:

i) $y = x^3 + 4x^2 + 2$

1

ii) $y = \frac{3x}{x+2}$

2

iii) $y = (2x+1)^4$

2**b)** Find the gradient of the tangent to the curve $y = 4x^3 + x$ at the point (1, 5)**2****c)** Find:

i) $\int x^4 + 3x^2 dx$

1

ii) $\int (x-5)(x+4) dx$

2

iii) $\int \frac{x^3 - 3x^4}{x^2} dx$

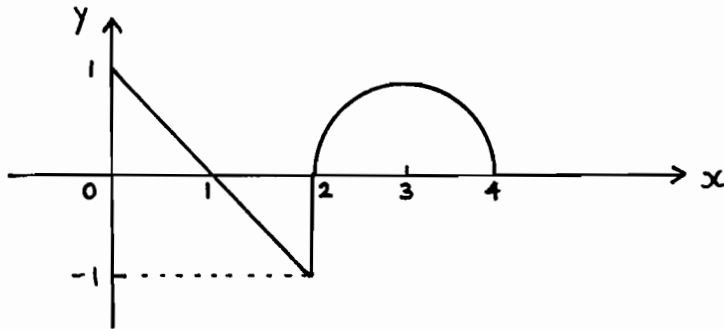
2

QUESTION 2

Marks

- a) Find the exact value of $\int_0^4 f(x) dx$ given

2

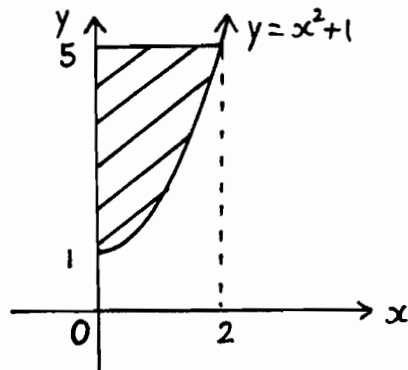


- b) Find $\int (2x-1)^5 dx$

2

- c) The sketch shows an arc of the curve $y = x^2 + 1$.

3



Calculate the shaded area.

- d) The gradient of a curve is given by

$$\frac{dy}{dx} = 3x^2 - 12$$

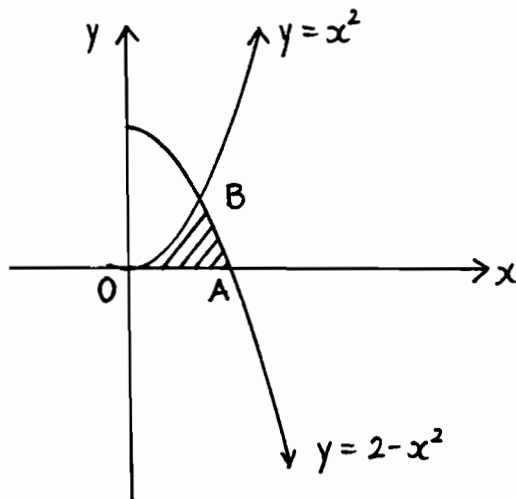
- i) Find $\frac{d^2y}{dx^2}$ 1
- ii) Find the values of x for which the curve both increases **and** is concave downwards. 2
- iii) If the curve passes through $(1, -2)$ find the equation of the curve. 2

QUESTION 3

Marks

- a) Find the primitive function of \sqrt{x} 2
- b) Melanie joined a Superannuation Fund, investing \$P at the beginning of every year at 8% p.a. compound interest (compounded yearly).
- i) Write an expression for the amount of her investment at the end of the first year. 1
 - ii) Write an expression for the amount of her investment at the end of the second year. 1
 - iii) Write an expression for the amount of her investment at the end of twenty five years 1
 - iv) If at the end of twenty five years, she wishes to collect \$500,000 calculate the value of \$P to the nearest dollar. 2

c)

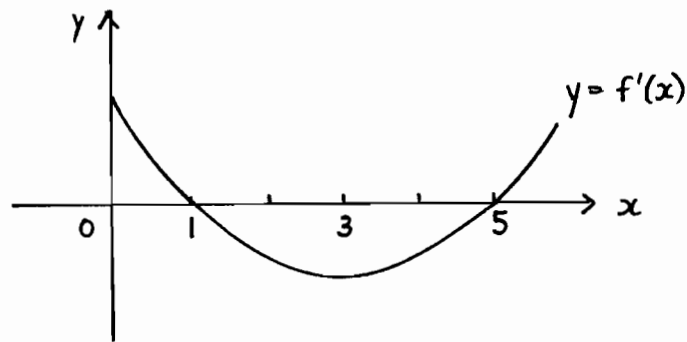


The shaded region OAB is bounded by the parabolas $y = x^2$ and $y = 2 - x^2$ and the x axis from $x = 0$ to $x = \sqrt{2}$.

- i) B is the point of intersection of the two parabolas in the first quadrant. 2
Find the co-ordinates of B.
- ii) Calculate the area of the shaded region OAB (2 dp). 3

QUESTION 4**Marks**

- a) A function is defined by $y=3x^2 - 2x^3$
- i) Find the co-ordinates of any turning points and determine their nature **3**
 - ii) Given that there is a point of inflexion, find its co-ordinates. **1**
 - iii) Sketch the function from $x=-1$ to $x=2$ **2**
- Note** Your sketch must be neat
Use a ruler to draw the axes
Label all important points
- iv) Find the area bounded by the curve $y=3x^2 - 2x^3$ and the x axis from $x=0$ to $x=2$ **4**
- b) **2**



The diagram shows the graph of the gradient function of the curve $y=f(x)$.

For what value of x does $f(x)$ have a local minimum?

QUESTION 5**Marks**

- a)** A cylindrical container closed at both ends is made from a sheet of thin plastic.
The surface area of the cylinder is 600π centimetres².

- i) Show that the height h of the cylinder is given by the expression: **2**

$$h = \frac{300}{r} - r, \text{ where } r \text{ is the radius.}$$

- ii) Find an expression for the volume V in terms of r . **1**

- iii) Find the height of the container if the volume is to be a maximum. **3**

- b)** i) Differentiate $y = x^3(1+x)^3$ **2**

- ii) Hence, solve $\frac{dy}{dx} = 0$ **2**

End of Test

HSC ASSESSMENT TASK 2 - MARCH 2007

QUESTION 1

a) i. $y = x^3 + 4x^2 + 2$
 $\frac{dy}{dx} = \underline{\underline{3x^2 + 8x}}$

ii. $y = \frac{3x}{x+2}$

$u = 3x \quad v = x+2$
 $u' = 3 \quad v' = 1$

$\frac{dy}{dx} = \frac{3(x+2) - 3x(1)}{(x+2)^2}$
 $= \frac{3x+6-3x}{(x+2)^2}$
 $= \underline{\underline{\frac{6}{(x+2)^2}}}$

iii. $y = (2x+1)^4$
 $\frac{dy}{dx} = 4(2x+1)^3 \times 2$
 $= \underline{\underline{8(2x+1)^3}}$

b) $y = 4x^3 + x$
 $\frac{dy}{dx} = 12x^2 + 1$

when $x=1$, $m_{\text{tangent}} = 12 \times 1^2 + 1$
 $= \underline{\underline{13}}$

c) i. $\int x^4 + 3x^2 dx$
 $= \underline{\underline{\frac{x^5}{5} + x^3 + c}}$

ii. $\int (x-5)(x+4) dx$
 $= \int x^2 - x - 20 dx$
 $= \underline{\underline{\frac{x^3}{3} - \frac{x^2}{2} - 20x + c}}$

iii. $\int \frac{x^3 - 3x^4}{x^2} dx$
 $= \int x - 3x^2 dx$
 $= \underline{\underline{\frac{x^2}{2} - x^3 + c}}$

QUESTION 2

a) $\int_0^4 f(x) dx = \frac{1}{2} \times \pi \times 1^2$
 $= \underline{\underline{\frac{\pi}{2}}}$

b) $\int (2x-1)^5 dx = \frac{(2x-1)^6}{6 \times 2} + c$
 $= \underline{\underline{\frac{(2x-1)^6}{12} + c}}$

c) $A = 5 \times 2 - \int_0^2 x^2 + 1 dx$
 $= 10 - \left[\frac{x^3}{3} + x \right]_0^2$
 $= 10 - \left[\frac{2^3}{3} + 2 - 0 \right]$
 $= 10 - 4\frac{2}{3}$
 $= \underline{\underline{5\frac{1}{3} u^2}}$

d) $\frac{dy}{dx} = 3x^2 - 12$

i. $\frac{d^2y}{dx^2} = \underline{\underline{6x}}$

ii. Increasing: $\frac{dy}{dx} > 0$
 $3x^2 - 12 > 0$
 $3(x+2)(x-2) > 0$
 $x < -2, x > 2$

Concave down: $\frac{d^2y}{dx^2} < 0$
 $6x < 0$
 $x < 0$

\therefore both increasing and concave down $\underline{\underline{x < -2}}$

iii. $y = \int 3x^2 - 12 dx$

$y = x^3 - 12x + c$
 when $x=1, y=-2$
 $-2 = 1^3 - 12 \cdot 1 + c$
 $c = 9$

$\therefore y = x^3 - 12x + 9$

QUESTION 3

$$a) \sqrt{x} = x^{\frac{1}{2}}$$

$$\text{primitive} = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + C$$

$$= \frac{2}{3} x^{\frac{3}{2}} + C$$

$$b) \text{ i. } A_1 = P(1.08)$$

$$\text{ ii. } A_2 = P(1.08)^2 + P(1.08)$$

$$= P(1.08^2 + 1.08)$$

$$\text{ iii. } A_{25} = P(1.08^{25} + 1.08^{24} + \dots + 1.08)$$

$$\text{ iv. } A_{25} = \frac{P \times 1.08(1.08^{25} - 1)}{1.08 - 1}$$

$$500\,000 = \frac{P \times 1.08(1.08^{25} - 1)}{0.08}$$

$$P = \underline{\underline{\$6333}} \text{ (nearest dollar)}$$

$$c) \text{ i. } x^2 = 2 - x^2$$

$$2x^2 = 2$$

$$x^2 = 1$$

$$x = 1 \text{ (1st quadrant)}$$

$$y = 1^2$$

$$\therefore \underline{\underline{B(1,1)}}$$

$$\text{ ii. } A = \int_0^1 x^2 dx + \int_1^{\sqrt{2}} 2 - x^2 dx$$

$$= \left[\frac{x^3}{3} \right]_0^1 + \left[2x - \frac{x^3}{3} \right]_1^{\sqrt{2}}$$

$$= \left[\frac{1^3}{3} - 0 \right] + \left[(2\sqrt{2} - \frac{\sqrt{2}^3}{3}) - (2 \times 1 - \frac{1^3}{3}) \right]$$

$$= \frac{1}{3} + \left[2\sqrt{2} - \frac{2\sqrt{2}}{3} - (2 - \frac{1}{3}) \right]$$

$$\therefore A = \underline{\underline{0.55}} \text{ (2 dp)}$$

QUESTION 4

$$a) y = 3x^2 - 2x^3$$

$$\frac{dy}{dx} = 6x - 6x^2$$

$$\frac{d^2y}{dx^2} = 6 - 12x$$

$$\text{ i. Stat pts: } \frac{dy}{dx} = 0$$

$$6x - 6x^2 = 0$$

$$6x(1-x) = 0$$

$$x = 0, 1$$

$$\text{ When } x=0, y=0$$

$$\frac{d^2y}{dx^2} > 0$$

$$\therefore \underline{\underline{\text{min at } (0,0)}}$$

$$\text{ When } x=1, y=1$$

$$\frac{d^2y}{dx^2} < 0$$

$$\therefore \underline{\underline{\text{max at } (1,1)}}$$

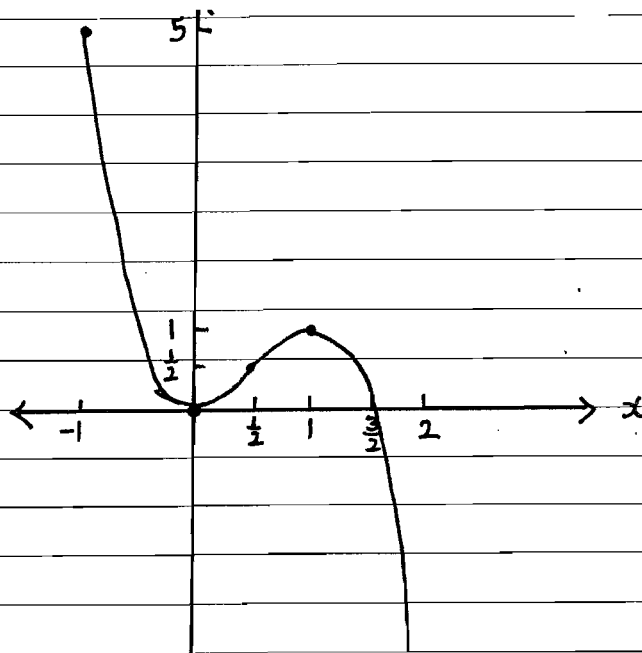
$$\text{ ii. Inflexion: } \frac{d^2y}{dx^2} = 0$$

$$6 - 12x = 0$$

$$12x = 6$$

$$x = \frac{1}{2}$$

$$\therefore \underline{\underline{\text{inflexion at } (\frac{1}{2}, \frac{1}{2})}}$$



$$\begin{aligned} \text{iv. } A &= \int_0^{\frac{3}{2}} 3x^2 - 2x^3 dx + \\ & \left| \int_{\frac{3}{2}}^2 3x^2 - 2x^3 dx \right| \\ &= \left[x^3 - \frac{x^4}{2} \right]_0^{\frac{3}{2}} + \left[x^3 - \frac{x^4}{2} \right]_{\frac{3}{2}}^2 \\ &= \left[\left(\frac{3}{2}\right)^3 - \frac{1}{2} \times \left(\frac{3}{2}\right)^4 - 0 \right] \\ & \quad + \left[\left[2^3 - \frac{2^4}{2} \right] - \left[\left(\frac{3}{2}\right)^3 - \frac{1}{2} \times \left(\frac{3}{2}\right)^4 \right] \right] \\ &= \frac{27}{32} + \left| -\frac{27}{32} \right| \end{aligned}$$

$$\therefore A = \underline{\underline{\frac{11}{16}}} u^2 \quad (1.6875)$$

b) $f'(x) = 0$ at $x = 1$ and 5
i.e. stationary points on $f(x)$

when $x < 1$, $f'(x) > 0$
i.e. increasing

when $1 < x < 5$, $f'(x) < 0$
i.e. decreasing

when $x > 5$, $f'(x) > 0$
i.e. increasing

\therefore min when $x = 5$

QUESTION 5

$$\begin{aligned} \text{a) i. } SA &= 2\pi r^2 + 2\pi rh \\ 600\pi &= 2\pi r^2 + 2\pi rh \\ 300 &= r^2 + rh \\ rh &= 300 - r^2 \\ h &= \frac{300 - r^2}{r} \end{aligned}$$

$$\therefore h = \frac{300 - r}{r}$$

$$\begin{aligned} \text{ii. } V &= \pi r^2 h \\ &= \pi r^2 (300 - r) \end{aligned}$$

$$\begin{aligned} \text{iii. } \frac{dV}{dr} &= 300\pi - 3\pi r^2 \\ \frac{d^2V}{dr^2} &= -6\pi r \end{aligned}$$

$$\text{max } V : \frac{dV}{dr} = 0$$

$$300\pi - 3\pi r^2 = 0$$

$$3\pi (100 - r^2) = 0$$

$$r = 10 \text{ only } (r > 0)$$

$$\text{when } r = 10, \frac{d^2V}{dr^2} < 0$$

$$\therefore \text{max } V \text{ when } r = 10$$

$$\therefore \text{height} = \frac{300}{10} - 10$$

$$\therefore h = \underline{\underline{20 \text{ cm}}}$$

$$\text{b) i. } y = x^3(1+x)^3$$

$$u = x^3 \quad v = (1+x)^3$$

$$u' = 3x^2 \quad v' = 3(1+x)^2$$

$$\begin{aligned} \frac{dy}{dx} &= 3x^2(1+x)^3 + 3x^3(1+x)^2 \\ &= 3x^2(1+x)^2[(1+x) + x] \\ &= 3x^2(1+x)^2(1+2x) \end{aligned}$$

$$\text{ii. } 0 = 3x^2(1+x)^2(1+2x)$$

$$\therefore \underline{\underline{x = 0, -1, -\frac{1}{2}}}$$