SYDNEY TECHNICAL HIGH SCHOOL

YEAR 12

ASSESSMENT TASK 2

MARCH 2008

MATHEMATICS

Time Allowed: 70 minutes

Instructions:

- Write your name and class at the bottom of this page
- Attempt all questions
- Show all necessary working
- Marks may be deducted for careless or badly arranged work
- Approved calculators may be used
- At the end of the examination hand in both the question paper and your answers
- Marks indicated are a guide only and may be varied if necessary
- Standard integrals are attached and may be removed for your convenience.

Name: ______________________________ Teacher: ______________________

Question 1	Question 2	Question 3	Question 4	Question 5	Total
/11	/10	/11	/11	/11	/54

QUESTION 1 (11 Marks)

i) Find the number of terms in the arithmetic sequence **2**

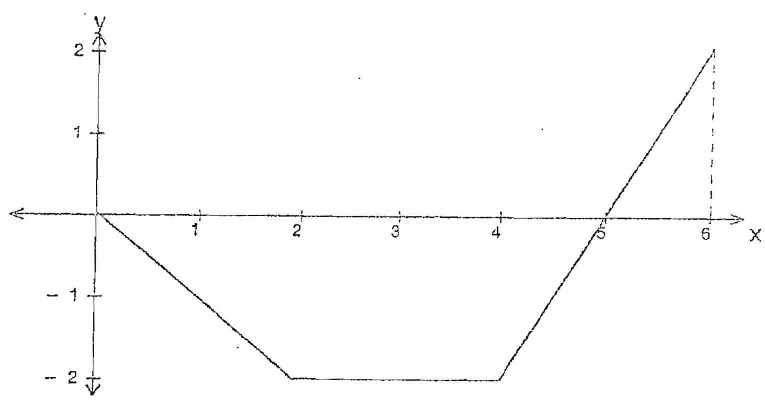
$10, 6, 2 \ldots -102$

ii) Differentiate $y = \frac{3x^2}{x+5}$ and express your answer in simplest form **3**

iii) Find the primitive function of **2**

$$2x^2 - \frac{1}{x^2}$$

iv) The diagram represents a function $y = f(x)$. **2**



Evaluate $\int_0^6 f(x)\, dx$.

2

v) Find the equation of the curve $y = f(x)$ given that $f'(x) = 2x + 1$ and that the curve passes through $(1, 4)$ **2**

QUESTION 2 (10 Marks) **Start a new page**

i) For a sequence it is given that

$$S_n = n^2 + 4n$$

a) Express S_{n-1} in terms of n **1**

b) Hence, or otherwise express T_n in terms of n **2**

c) Find the 10^{th} term of the sequence **1**

ii) A person saved \$1000 the first year and \$200 more each subsequent year.
How many years will it take to save \$58000? **4**

iii) Evaluate $32 + 24 + 18 + \ldots$ **2**

QUESTION 3 (11 Marks) **Start a new page**

A) Consider the function

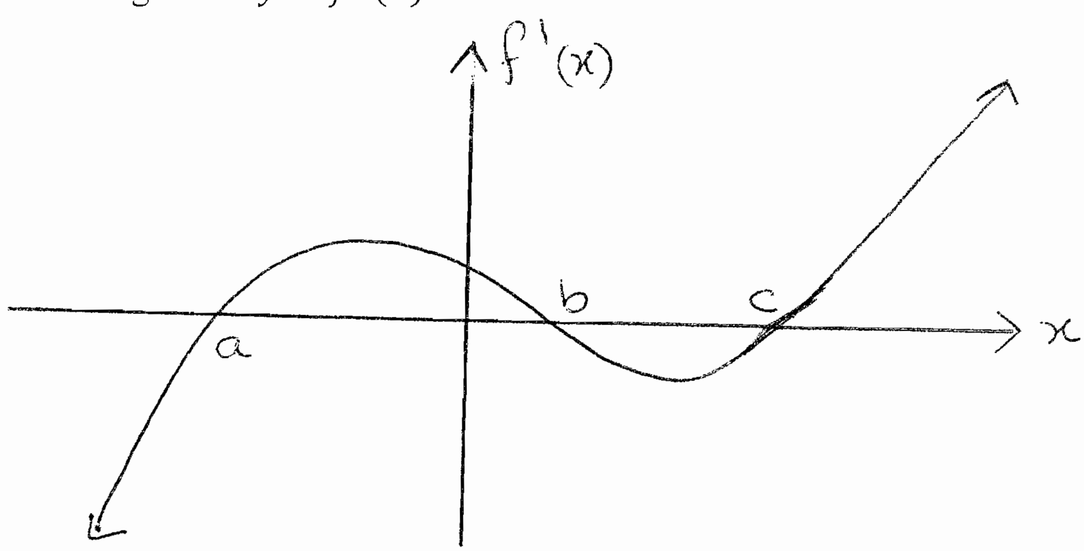
$$f(x) = x^3 + 9x^2 + 24x + 3$$

i) Find the co-ordinates of the stationary points of the curve $y = f(x)$ and determine their nature. **3**

ii) Sketch the curve, clearly labelling any stationary points and the $y-$intercept **2**

iii) For what vales of x is the curve decreasing? **1**

B) This is a diagram of $y = f'(x)$



i) Write down the x values of any stationary points on $y = f(x)$ **1**

ii) For what values of x is $y = f(x)$ increasing? **2**

iii) Sketch a possible graph of $y = f(x)$ given that $y = f(x)$ passes through $(0, 2)$ **2**

QUESTION 4 - (11 Marks) Start a new page

A) A couple borrow \$400,000 to purchase a house. They must repay the loan by equal quarterly instalments. Interest is charged at the rate of 8% p.a

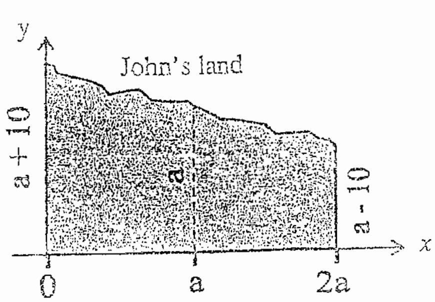
i) Write down the quarterly interest rate **1**

ii) Write an expression for A_1, the amount owing after the first quarterly repayment. Let M be the amount repaid at the end of each quarter. **1**

iii) Show that the amount owing at the end of the first year is given by
$400\,000\,(1.02)^4 - M\,(1 + 1.02 + 1.02^2 + 1.02^3)$ **2**

iv) Find the amount of each quarterly instalment if the loan is to be fully repaid in 12 years. (answer to the nearest dollar) **3**

B) The shaded area shown in the diagram below represents John's land. Its dimensions are given in terms of 'a'.



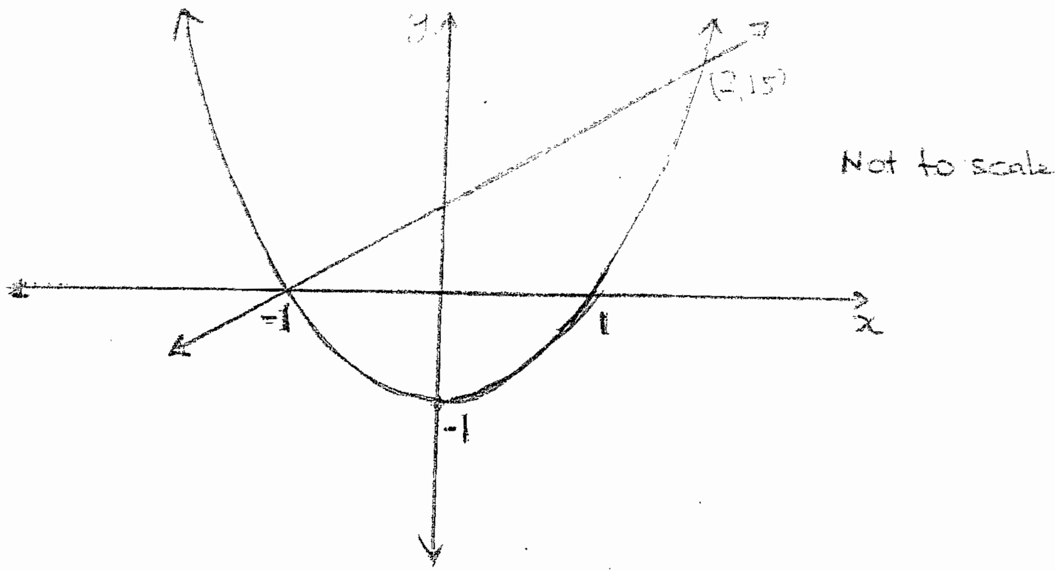
i) Complete the table: **1**

x	0	a	2a
y			

Given that the area of this land in 3200m^2, use Simpson's rule with 3 function values to find an estimate for the value of 'a' **3**

QUESTION 5 (11 Marks) Start a new page

A)



The diagram shows the curve $y = x^4 - 1$ and the line $y = 5x + 5$

(i) Show that the line and curve intersect at the points $(-1, 0)$ and $(2, 15)$ 2

(ii) Calculate the area between the curve and the line. 3

B) A piece of wire 24 cm long is cut into two pieces. Each is bent to form a square.

i) If one piece is x cm long, write an expression for the length of the other piece 1

ii) Show that the sum of the areas of the two squares is given by

$\left(\frac{x}{4}\right)^2 + \left(\frac{24-x}{4}\right)^2$ 2

iii) Find the minimum area of the two squares 3

END OF PAPER

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

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Question 1

(i) [illegible]

$d = -4$

$T_n = a + (n-1)d$

$-102 = 10 + (-4)(n-1)$

$= 10 - 4n + 4$

$\therefore 4n = 116$

$n = 29$ There are 29 terms

(ii) $y = \dfrac{3x^2}{x+5}$

Quotient

$\dfrac{u}{v}$ where $u = 3x^2$ $u' = 6x$

$v = x+5$ $v' = 1$

$$y' = \frac{u'v - v'u}{v^2}$$

$$= \frac{6x(x+5) - 1(3x^2)}{(x+5)^2}$$

$$= \frac{6x^2 + 30x - 3x^2}{(x+5)^2}$$

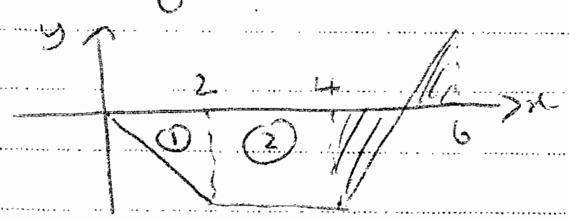
$$= \frac{3x^2 + 30x}{(x+5)^2}$$

$$= \frac{3x(x+10)}{(x+5)^2}$$

(iii) Primitive of $2x^2 - \dfrac{1}{x^2}$

is $\dfrac{2x^3}{3} + \dfrac{1}{x} + c$

(iv) $\int_0^6 f(x)\,dx$ is area of triangle (1) + area rectangle (2)



$= \frac{1}{2} \times 2 \times 2 + 2 \times 2$

$= -6$ (since below axis)

(v) $f'(x) = 2x + 1$ (1, 4) satisfies

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Question 2

(i) $S_n = n^2 + 4n$

a) $S_{n-1} = (n-1)^2 + 4(n-1)$

$= n^2 - 2n + 1 + 4n - 4$

$= n^2 + 2n - 3$

b) $T_n = S_n - S_{n-1}$

$= \cancel{n^2} + 4n - (\cancel{n^2} + 2n - 3)$

$= 2n + 3$

c) $T_{10} = 2 \times 10 + 3$

$= 23$

(ii) 1st yr saves \$1000

2nd yr \$1200 etc

AP with $a = 1000$

$d = 200$

Want $S_n = 58000$.

$S_n = \frac{n}{2}[2a + (n-1)d]$

$58000 = \frac{n}{2}\left[2000 + (n-1) \times 200\right]$

$580\cancel{0}\cancel{0} = \frac{n}{2} \times 1\cancel{0}\cancel{0}[20 + 2n - 2]$

$580 = n[9 + n]$

$\therefore n^2 + 9n - 580 = 0$

$(n + 29)(n - 20) = 0$

$n = -29$ or $n = 20$ (need n pos)

Will take 20 years

(iii) G.P. $a = 32$

$r = \frac{24}{32} = 3/4$

$S_\infty = \frac{a}{1-r}$

$= \frac{32}{1 - 3/4}$

$= 128$

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Question

A) $f(x) = x^3 + 9x^2 + 24x + 3$

(i) $f'(x) = 3x^2 + 18x + 24$

$= 3(x^2 + 6x + 8)$

$= 3(x+2)(x+4)$

$f'(x) = 0$ when $x = -2$ or $x = -4$

$y = -17$ $\quad$ $y = -13$

$f''(x) = 6x + 18$

$f''(-2) = 6(-2) + 18$

$= 6 > 0 \Rightarrow \min$

$f''(-4) = 6(-4) + 18$

$= -6 \Rightarrow \max$

y intercept is 3

(ii) $f''(x) = 0$ when

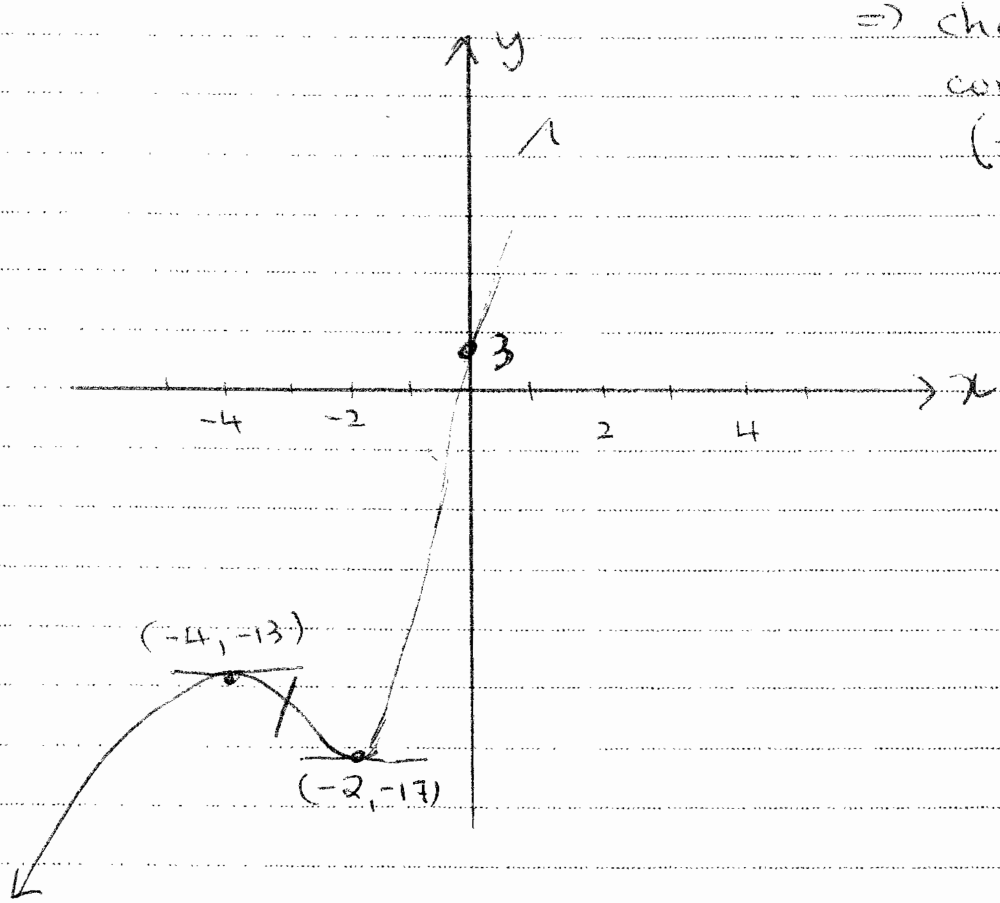
$6x + 18 = 0$

ie $x = -3$

and changes sign

$\Rightarrow$ change of concavity.

$(-3, -15)$



(iii) Decreasing for

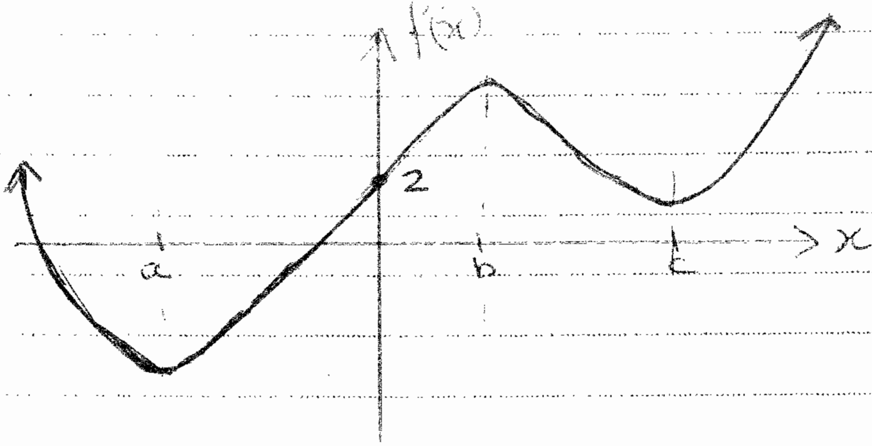
Teacher's Name ______ Student's Name/N°: ______

15) (i) Stationary at $x = a$, $x = b$, $x = c$

(ii) Increasing when $f'(x) > 0$

ie. $a < x < b$ and $x > c$

(iii)



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Question 4

A) (i) 2% per quarter

(ii) $A_1 = 400\,000(1.02)^1 - M$

(iii) $A_2 = [400\,000(1.02) - M](1.02) - M$

$= 400\,000(1.02)^2 - M(1.02 + 1)$

End of first year will be 4 payments

$\therefore$ Want A_4

$A_4 = 400\,000(1.02)^4 - M(1.02^3 + 1.02^2 + 1.02 + 1)$

$= 400\,000(1.02)^4 - M(1 + 1.02 + 1.02^2 + 1.02^3)$

(iv) 12 yrs quarterly $\Rightarrow$ 48 payments

and $A_{48} = 0$

$\therefore 0 = 400\,000(1.02)^{48} - M(1 + 1.02 + \cdots + 1.02^{47})$

$$M = \frac{400\,000(1.02)^{48}}{1 + 1.02 + \cdots + 1.02^{47}} \Longleftarrow \text{GP with } a = 1,\ r = 1.02,\ n = 48$$

$$S_{48} = \frac{1(1.02^{48} - 1)}{1.02 - 1}$$

$$\therefore M = \frac{400\,000(1.02)^{48}}{(1.02^{48} - 1)} \times 0.02$$

$= \$13041$ (nearest dollar)

B) i)

x	0	a	$2a$
y	$a+10$	a	$a-10$

ii) $3200 \doteqdot \frac{a}{3}\left[(a + \cancel{10}) + (a - \cancel{10}) + 4a\right]$

$9600 = a \times 6a$

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Question 5

$y = x^4 - 1 \qquad y = 5x + 5$

A) (i) Test $x = -1$ $\qquad y = (-1)^4 - 1$

$= 0$

$y = 5(-1) + 5$

$= 0$

Test $x = 2$ $\qquad y = 2^4 - 1$

$= 15$

$y = 5(2) + 5$

$= 15$

∴ Since $(-1, 0)$ and $(2, 15)$ satisfy each equation, these are the points of intersection

(ii) $\displaystyle\int_{-1}^{2} (5x + 5) - (x^4 - 1)\, dx$

$\displaystyle = \int_{-1}^{2} (-x^4 + 5x + 6)\, dx$

$\displaystyle = \left[-\frac{x^5}{5} + \frac{5x^2}{2} + 6x \right]_{-1}^{2}$

$\displaystyle = \left[-\frac{32}{5} + \frac{20}{2} + 12 \right] - \left[\frac{1}{5} + \frac{5}{2} - 6 \right]$

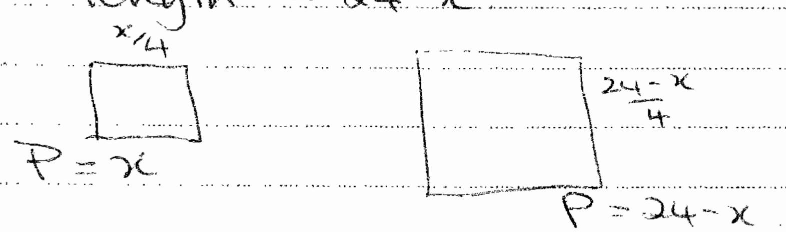
$\displaystyle = -\frac{33}{5} + 28 - \frac{5}{2}$

$= 18\frac{9}{10}\ u^2$

B) i) $x \qquad 24 - x$

length $= 24 - x$

ii)



$\displaystyle \text{Area} = \left(\frac{x}{4}\right)^2 + \left(\frac{24 - x}{4}\right)^2$

(iii) $\displaystyle \frac{dA}{dx} = 2\left(\frac{x}{4}\right) \times \frac{1}{4} + 2\left(\frac{24 - x}{4}\right) \times \left(-\frac{1}{4}\right)$

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$$\frac{dA}{dx} = \frac{x - 24 + x}{[illegible]}$$

$$= \frac{2x - 24}{8}$$

$$= \frac{x - 12}{4}$$

$\frac{dA}{dx} = 0$ when $\frac{x-12}{4} = 0$ ie $x = 12$

$\frac{d^2A}{dx^2} = \frac{1}{4} > 0 \Rightarrow$ minimum

$\therefore$ Minimum area when $x = 12$

Then

$$\text{Area} = \left(\frac{12}{4}\right)^2 + \left(\frac{24-12}{4}\right)^2$$

$$= 9 + 9$$

$$= 18 \ u^2$$