SYDNEY TECHNICAL HIGH SCHOOL

HSC Assessment Task 2
March 2009

Name: ____________________ Teacher: __________________

Mathematics

Time allowed —70 minutes

Instructions

- Approved calculators may be used.
- All necessary working must be shown. Marks may not be awarded for careless or badly arranged work.
- Marks awarded are shown on each question.
- Total marks — 50
- Attempt all questions.
- Start each question on a new page.

Question	1	2	3	4	5	Total /50	%
Marks /10							

Question 1 Marks 10

a) The curve $y = ax^2 + 4x - 5$ has a gradient of 10 when $x = 2$. **2**

Find the value of a.

b) Find the domain over which the curve $y = x^3 - x^2 - 8x + 4$ is increasing. **2**

c) Use calculus to identify and determine the nature of any stationary points **6**

and points of inflection of the function $y = x^3 - 12x$.

Hence, sketch the curve.

Question 2 Marks 10

a) (i) Determine whether $y = x^7$ is an odd or even function or neither. **1**

(ii) Hence evaluate $\int_{-3}^{3} x^7 dx$. **1**

b) The efficiency, E percent, of a particular spark plug when the gap is set to **4**

x mm, is given by $E = 800x - 1600x^2$.

Find the gap setting which gives maximum efficiency.

c) Two circles have radii a and b cm such that $a + b = 16$. **4**

Find the minimum sum of their areas.

Question 3 Marks 10

a) Find the following integrals:

(i) $\int(2x+3)^2\,dx$ 2

(ii) $\int\frac{1}{x^2}\,dx$ 2

b) (i) Consider a quadrant of a circle of radius 2 units. Dividing the area of the quadrant into five equal sub-intervals produced the following table of ordinates. 4

x	0	0.4	0.8	1.2	1.6	2.0
y	2	1.96	1.83	1.60	1.20	0

Use the Trapezoidal Rule to find an approximate area of the quadrant. (Write your answer to two decimal places.)

(ii) Calculate the area of a quadrant of a circle of radius 2 units using the formula $A=\pi r^2$. 1

(iii) Explain why your approximation of the area is an underestimate. 1

Question 4 Marks 10

a) Find the area between the curve $y = x^2 + 4$, the x-axis and the ordinates $x = 2$ and $x = 4$. 2

b) Calculate the area between the curve $y = x(x+1)(x-2)$, the x-axis and the ordinates $x = 1$ and $x = 3$. 4

c) What is the area between the curve $y = \sqrt{1-x}$ and the y-axis, between the ordinates where $x = 0$ and $x = \frac{3}{4}$? 4

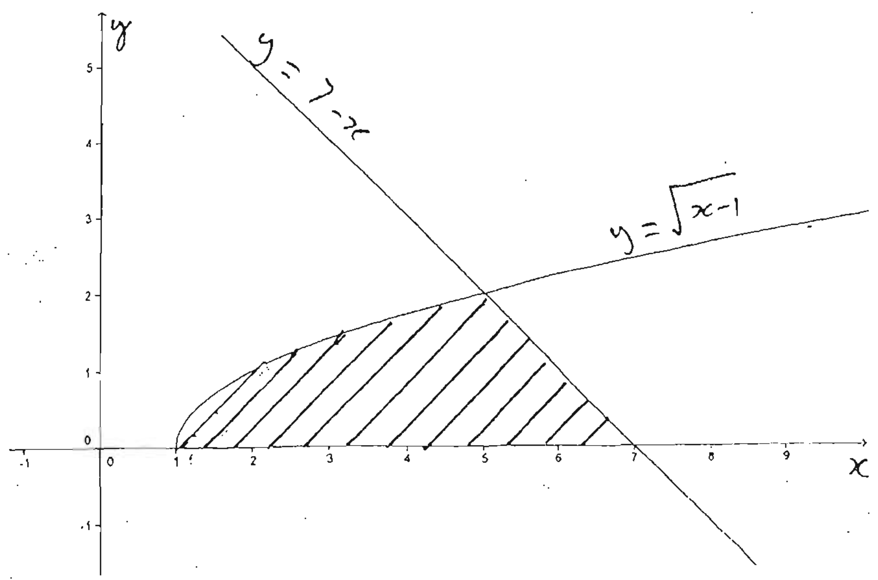
Question 5 Marks 10

a) Find the area of the region bounded by the graphs of $y = x^2$ and $y = x^3$. 3

b) A parabolic mirror is made by revolving the area bounded by the parabola $y = \frac{1}{2}x^2$, the y-axis and the line $y = 4$, about the y-axis. 3

What volume does it occupy?

c) Calculate the volume when the shaded area is revolved around the x-axis. 4



MATHEMATICS ASSESSMENT TASK 2 SOLUTIONS 2009

Question 1

a) $y = ax^2 + 4x - 5$

$\therefore \frac{dy}{dx} = 2ax + 4$ → (1)

gradient = 10 when $x = 2$

$\therefore 2a.2 + 4 = 10$

$\therefore 4a = 6$

$\therefore a = \frac{3}{2}$ → (1)

b) $y = x^3 - x^2 - 8x + 4$

$\therefore \frac{dy}{dx} = 3x^2 - 2x - 8$

$= (3x + 4)(x - 2)$

$= 0$ when $x = -\frac{4}{3}, 2$ → (1)

leading coeff of $\frac{dy}{dx} > 0$

$\therefore \frac{dy}{dx} > 0$ when $x < -\frac{4}{3}$ or $x > 2$ → (1)

c) $y = x^3 - 12x$

$\therefore y' = 3x^2 - 12$ → (1)

$= 3(x^2 - 4)$

$= 3(x - 2)(x + 2)$

$= 0$ when $x = +2, -2$ → (1)

$y = -16, 16$ → (1)

$y'' = 6x$

> 0 when $x = 2$

$\therefore$ min. at $(2, -16)$ → (1)

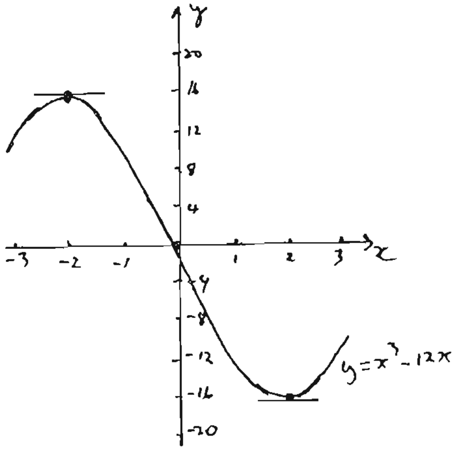
< 0 when $x = -2$

$\therefore$ max. at $(-2, 16)$ → (1)

$= 0$ when $x = 0$

$y' \neq 0$ when $x = 0$

$\therefore \exists$ a point of inflection at $(0,0)$ → (1)



Question 2

a) (i) let $f(x) = x^7$

$f(-x) = (-x)^7$

$= -x^7$

$= -f(x)$ → (1)

$\therefore f(x)$ is an ODD function.

(ii) $\int_{-3}^{3} x^7\, dx = 0$ → (1)

as $f(x)$ is ODD.

b) $E = 800x - 1600x^2$

$\therefore \frac{dE}{dx} = 800 - 3200x$ → (1)

$= 0$ when $3200x = 800$

$\therefore x = \frac{800}{3200}$

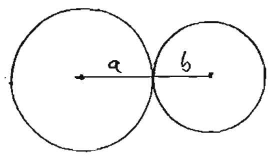
$= \frac{1}{4}$ → (1)

$\frac{d^2E}{dx^2} = -3200$

< 0 → (1)

$\therefore \exists$ a maximum in E when $x = \frac{1}{4}$ mm → (1)

c)



$a + b = 16$

$\therefore b = 16 - a$

$A = \pi a^2 + \pi b^2$

$= \pi a^2 + \pi(16 - a)^2$ → (1)

$= \pi[a^2 + 256 - 32a + a^2]$

$= \pi[2a^2 - 32a + 256]$

$= 2\pi[a^2 - 16a + 128]$

$\frac{dA}{da} = 2\pi(2a - 16)$

$= 4\pi(a - 8)$

$= 0$ when $a = 8$ → (1)

$\therefore b = 8$

$\frac{d^2A}{da^2} = 4\pi$

> 0

$\therefore \exists$ a minimum area when $a = 8$ and $b = 8$ cm. → (1)

$\therefore$ Minimum area $= 2 \times 64\pi$ → (1)

$= 128\pi$ cm^2

Question 3

a) (i) $\int (2x+3)^2\,dx = \frac{(2x+3)^3}{3\times 2} + C$ (1) (1)

$= \frac{1}{6}(2x+3)^3 + C$

(ii) $\int \frac{1}{x^2}\,dx = \int x^{-2}\,dx$ → (1)

$= \frac{x^{-1}}{-1} + C$ → (1)

$= \frac{-1}{x} + C$

b) (i) $A \approx \frac{h}{2}\left[(y_0+y_n) + 2(y_1+y_2+\dots+y_{n-1})\right]$ → (1)

(1) ← $\approx \frac{0.4}{2}\left[(2+0) + 2(1.96+1.83+1.60+1.20)\right]$ → (1)

$\approx 0.2\left[2 + 2\times 6.59\right]$

≈ 3.036

≈ 3.04 units^2 → (1)

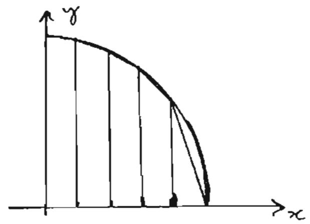
(ii) $A = \frac{\pi r^2}{4}$

$= \frac{4\pi}{4}$

(1) ← $= \pi$ units^2

$= 3.14159265\ldots\ldots$ units^2

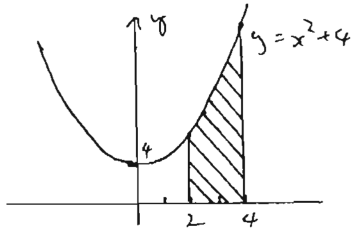
(iii)



(1) ← Approximation is an under-estimate because the trapezia are all within the quadrant. ∴ the sum of the trapezia is less than the area of the quadrant.

Question 4

a)



$A = \int_2^4 x^2+4\,dx$

$= \left[\frac{x^3}{3}+4x\right]_2^4$ → (1)

$= \left(\frac{64}{3}+16\right) - \left(\frac{8}{3}+8\right)$

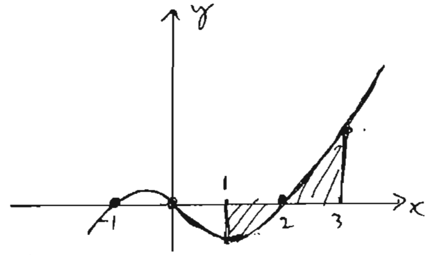
$= \frac{56}{3} + 8$

$= \frac{80}{3}$ units^2 } → (1)

$= 26\frac{2}{3}$ units^2

b) $y = x(x+1)(x-2)$

$= 0$ when $x = 0, -1, 2$ → (1)



When $x = 1$,

$\therefore y = 1\times 2\times -1$

< 0

When $x = 3$,

$y = 3\times 4\times 1$

> 0

$\therefore$ Area $= \left|\int_1^2 x(x+1)(x-2)\,dx\right| + \int_2^3 x(x+1)(x-2)\,dx$ → (1)

$= \left|\int_1^2 (x^3 - x^2 - 2x)\,dx\right| + \int_2^3 (x^3 - x^2 - 2x)\,dx$

$= \left|\left[\frac{x^4}{4} - \frac{x^3}{3} - x^2\right]_1^2\right| + \left[\frac{x^4}{4} - \frac{x^3}{3} - x^2\right]_2^3$ → (1)

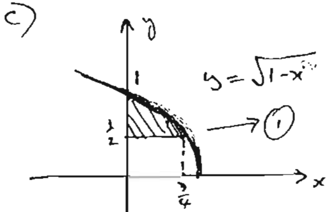
$= \left|\left[4 - \frac{8}{3} - 4\right] - \left[\frac{1}{4} - \frac{1}{3} - 1\right]\right| + \left[\frac{81}{4} - 9 - 9\right] - \left[4 - \frac{8}{3} - 4\right]$

$= \left|-\frac{8}{3} - \frac{1}{4} + \frac{1}{3} + 1\right| + \frac{9}{4} + \frac{8}{3}$

$= \left|-\frac{19}{12}\right| + \frac{59}{12}$ → (1)

$= \frac{78}{12} = 6.5$ units^2

c)



→ (1)

$y = \sqrt{1-x}$

$\therefore y^2 = 1-x$

$\therefore x = 1-y^2$

When $x = \frac{3}{4}$

$\frac{3}{4} = 1-y^2$

$\therefore y^2 = 1-\frac{3}{4}$

$= \frac{1}{4}$

$\therefore y = \frac{1}{2}$

$A = \int_{\frac{1}{2}}^{1} 1-y^2\,dy$ → (1) (limits → (1) both)

$= \left[y - \frac{y^3}{3}\right]_{\frac{1}{2}}^{1}$

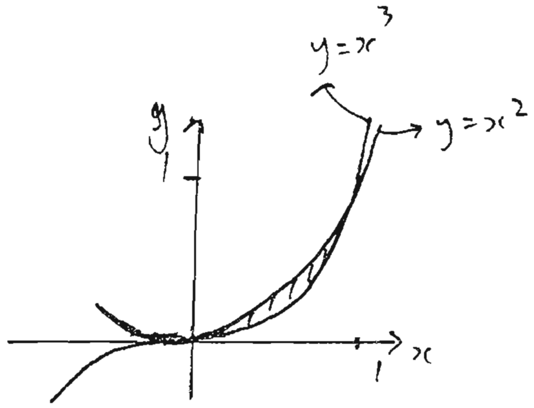
$= \left[\left(1-\frac{1}{3}\right) - \left(\frac{1}{2} - \frac{1}{24}\right)\right]$

$= 1 - \frac{1}{3} - \frac{1}{2} + \frac{1}{24}$

$= 5\ldots$ → (1)

Question 5

a) $y = x^3$ and $y = x^3$



let $x^3 = x^2$

$\therefore x^3 - x^2 = 0$

$\therefore x^2(x-1) = 0$

$\therefore x = 0, 1 \rightarrow (1)$

$A = \int_0^1 x^2\,dx - \int_0^1 x^3\,dx \rightarrow (1)$

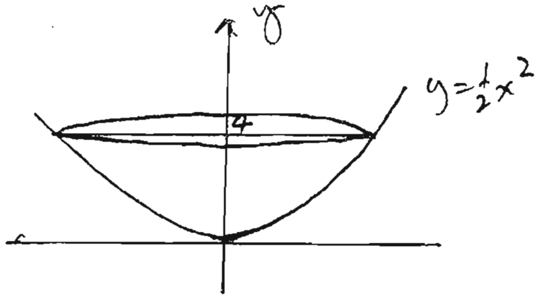
$= \int_0^1 (x^2 - x^3)\,dx$

$= \left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1$

$= \left(\frac{1}{3} - \frac{1}{4}\right) - (0)$

$= \frac{1}{12}$ unit^2 $\rightarrow (1)$

b)



$y = \frac{1}{2}x^2$

$\therefore x^2 = 2y$

$\therefore x = \sqrt{2y} \rightarrow (1)$

$\therefore V = \pi \int_0^4 (\sqrt{2y})^2\,dy \rightarrow (1)$

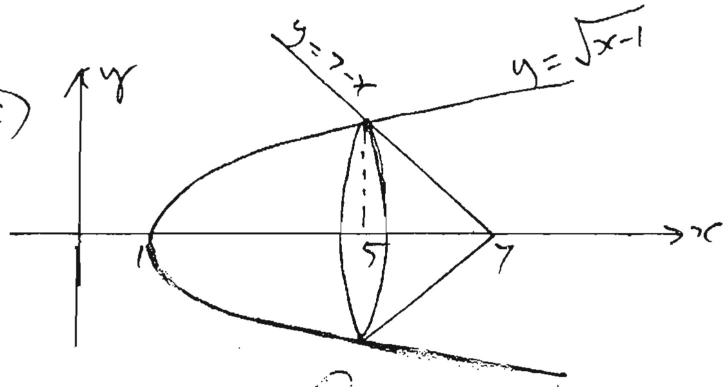
$= \pi \int_0^4 2y\,dy$

$= \pi \left[y^2\right]_0^4$

$= \pi(16 - 0)$

$= 16\pi$ units^3 $\rightarrow (1)$

c)



$V = \pi \int_1^5 (\sqrt{x-1})^2\,dx + \pi \int_5^7 (7-x)^2\,dx \rightarrow (1)$ $\rightarrow (1)$

$= \pi \int_1^5 (x-1)\,dx + \pi \int_5^7 (49 - 14x + x^2)\,dx$

$= \pi \left[\frac{x^2}{2} - x\right]_1^5 + \pi \left[49x - 7x^2 + \frac{x^3}{3}\right]_5^7 \rightarrow (1)$

$= \pi \left[\left(\frac{25}{2} - 5\right) - \left(\frac{1}{2} - 1\right)\right] + \pi \left[\left(343 - 343 + \frac{343}{3}\right) - \left(245 - 175 + \frac{125}{3}\right)\right]$

$= \pi [\ 8\ ] + \pi \left[\frac{343}{3} - \frac{335}{3}\right]$

$= \frac{32\pi}{3}$ units^3 $\rightarrow (1)$

≈ 33.51 u^3