SYDNEY TECHNICAL HIGH SCHOOL

MATHEMATICS

HSC ASSESSMENT TASK 2

MARCH 2010

Time Allowed: 70 minutes

Instructions:

- Write using blue or black pen.
- Approved calculators may be used.
- Attempt all questions.
- All necessary working must be shown. Marks may not be awarded for careless or badly arranged work.
- Marks indicated are a guide only and may be varied if necessary.
- Start each question on a new side of a page.
- A table of standard integrals is supplied.

Name:

Q1	**Q2**	**Q3**	**Q4**	**Q5**	**Total**
/10	/10	/9	/8	/10	/47

Question 1 **10 Marks**

(a) Is 102 a term of the series with nth term $T_n = 3n + 1$? Give reasons. **1**

(b) Which term of $2 + 5 + 8 + \ldots$ is equal to 320? **1**

(c) The 5th term of an arithmetic series is 14 and the 10th term is 59. Find the first term and the common difference of the series. **2**

(d) A brand of rechargeable batteries provides power for 20 hours when first purchased fully charged.

After its first recharge it only provides power for a further 18 hours.

After its second recharge it only supplies power for 16.2 hours.

Each subsequent recharging results in the battery having 90% of its previous power available.

(i) What is the power available after the third recharge? **1**

(ii) How many hours could you expect to get out of the battery? **2**

(e) Find all values of x for which the curve $y = 2x^2 - 8x + 3$ is increasing **1**

(f) Find, with reasons, the point of inflexion on the curve $y = 2x^3 - 12x^2 + 5$ **2**

Question 2 **10 Marks**

a) Shopping trolleys are 80 cm long. When two trolleys are pushed together the resulting pair is 100 cm long. When three are pushed together the resulting triple is 120 cm long.

(i) How long will a set of 10 trolleys pushed together be? **1**

(ii) For safety reasons trolley collectors are not allowed to push a set of trolleys more than 4.5metres long. What is the greatest number of trolleys that can be pushed? **1**

b) If $y = 8x^3 - 2x + 5$, find $\frac{dy}{dx}$ when $x = 2$. **1**

c) The tangent to a curve at point N has equation $5x - y - 1 = 0$.

i) If $\frac{dy}{dx} = 4x - 3$, for the curve, find the coordinates of N. **2**

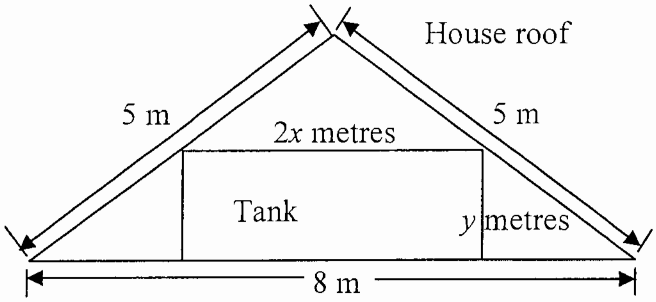
ii) Find the equation of the curve. **1**

d) Mark invests \$1 000 in a bank account that pays interest compounded quarterly. After four years he has \$1219.89 in the bank. What is the annual interest rate? **2**

e) The curve $y = ax^2 + bx - 1$ has a stationary point at (1,-5). Find the values of a and b **2**

Question 3 **9 Marks**

(a) The diagram below shows the cross section of a cylindrical hot-water tank, with diameter $2x$ metres and height y metres that fits exactly into the roof of a house. The cross section of the roof is an isosceles triangle with base 8 metres and equal sides 5 metres in length.



NOT TO SCALE

(i) Explain why the roof is 3 metres high. **1**

(ii) By using similar triangles, show that $y = \frac{3}{4}(4 - x)$. **2**

(iii) Show that the volume of the tank, V metres^3, is given by **1**

$$V = \frac{3\pi}{4}\left(4x^2 - x^3\right).$$

(iv) Use calculus to find the radius of the tank that gives it a maximum volume. **3**

(b) Evaluate $\sum_{n=2}^{4} 2^n$. **1**

(c) Find a primitive function of $5x^4 + 2x^3 - x^2 + 1$. **1**

Question 4 — 8 Marks

(a) Consider the function $f(x) = 2x^3 - 6x + 1$ in the domain $-2 \le x \le 3$.

(i) Find the coordinates of its turning points and determine their nature. **3**

(ii) Draw a neat sketch of the curve $y = f(x)$ clearly showing all essential features in the given domain. **1**

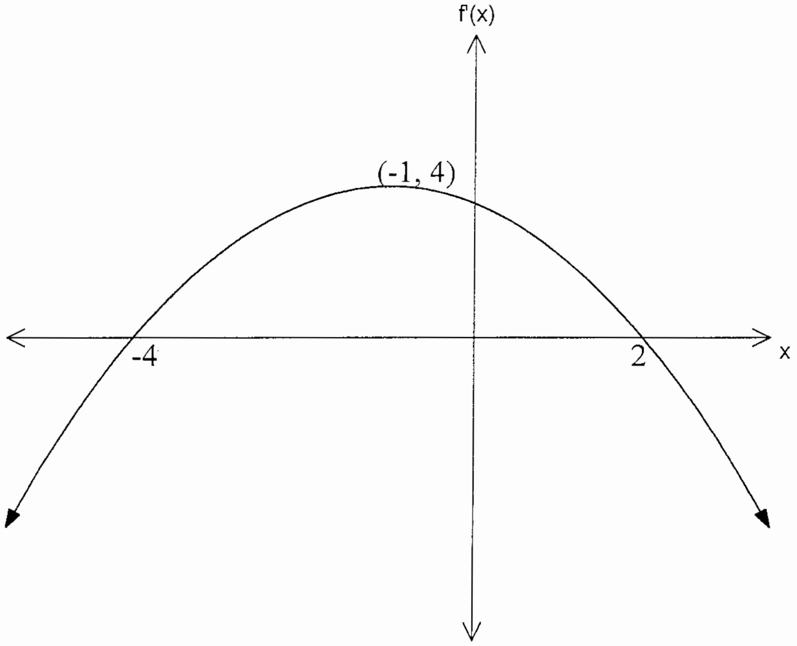
(iii) What are the greatest and least values of $f(x)$ in the domain $-2 \le x \le 3$? **2**

(b) Given $f'(x) = 6x^2 - 1$, and $f(0) = 4$, find $f(1)$. **2**

Question 5

10 Marks

(a) For a certain function $y = f(x)$, the sketch of $y = f'(x)$ is shown.



i) Sketch a possible *f(x)*. **2**

(b) A car dealership has a car for sale for a cash price of \$20 000. It can also be bought on terms over three years. The first six months are interest free and after that interest is charged at the rate of 1% per month on that months balance. Repayments are to be made in equal monthly instalments beginning at the end of the first month.

A customer buys the car on these terms and agrees to monthly repayments of \$$M$. Let \$$A_n$ be the amount owing at the end of the nth month.

i) Find an expression for A_6. **2**

ii) Show that $A_8 =$ □$(20\,000 - 6M)1\cdot01^2 - M(1 +$□□ $1\cdot01)$ **2**

iii) Find an expression for A_{36}. **2**

iv) Find the value of M. **2**

STHS H.S.C. Task 2 2010 mrl

Question 1

a) $102 = 3n + 1$

$101 = 3n$

$n = 33.\dot{6}$ $\therefore n$ not an integer

102 not a term.

b)

$320 = 2 + (n-1) \times 3$

$320 = 2 + 3n - 3$

$321 = 3n$

$\therefore n = 107$ $\quad \therefore T_{107} = 320$

c) $T_5 = 14$ $\therefore$ $14 = a + 4d$

$T_{10} = 59$ $\therefore$ $59 = a + 9d$

$45 = 5d$

$\therefore d = 9$ $\quad a = -22$

d) $20 + 18 + 16.2 + \dots$

i) $T_4 = 20 \times .9^3$

$T_4 = 14.58$

(3rd recharge $\Rightarrow T_4$)

ii) $S_\infty = \frac{20}{1 - .9}$

$= 200$ hrs

e) $y = 2x^2 - 8x + 3$

$\frac{dy}{dx} = 4x - 8$

$4x - 8 > 0$

$4x > 8$

$x > 2$

f) $y = 2x^3 - 12x^2 + 5$

$y' = 6x^2 - 24x$

$y'' = 12x - 24$ $\quad y'' = 0$ pt inf

$12x - 24 = 0$

$x = 2$

test concavity

x	1	2	3
y''	$-$	0	$+$

pt. inf

$(2, -27)$

Question 2

a) $T_1 = 80$

$T_2 = 100$

$T_3 = 120$ etc

AP $a = 80$ $d = 20$

i) $T_{10} = 80 + 9 \times 20$

$= 260$

ii) $450 = 80 + (n-1) \times 20$

$450 = 80 + 20n - 20$

$390 = 20n$

$19\frac{1}{2} = n$

$\therefore$ 19 trolleys

b) $y = 8x^3 - 2x + 5$

$\frac{dy}{dx} = 24x^2 - 2$

$x = 2$ $\quad \frac{dy}{dx} = 94$

c) $\frac{dy}{dx} = 4x - 3$

tang $5x - y - 1 = 0$

i) $y = 5x - 1$

$\therefore m = 5$

$5 = 4x - 3$

$8 = 4x$

$\therefore x = 2$ $\quad N(2, 9)$

ii) $y = 2x^2 - 3x + c$

$9 = 8 - 6 + c$

$c = 7$

$\therefore y = 2x^2 - 3x + 7$

d) $1219.89 = 1000(1 + r)^{16}$

$1.21989 = (1 + r)^{16}$

$r = 0.0125$ per quarter

$r = .05$ per year

$\therefore$ 5% p.a.

e) $y = ax^2 + bx - 1$

$\frac{dy}{dx} = 2ax + b$

at $x = 1$ $\quad \frac{dy}{dx} = 0$

$0 = 2a + b$ —— ①

at $(1, -5)$

$-5 = a + b - 1$ —— ②

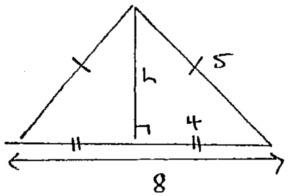
$2a + b = 0$

$a + b = -4$

$b = -8$ $\quad a = 4$

Question 3

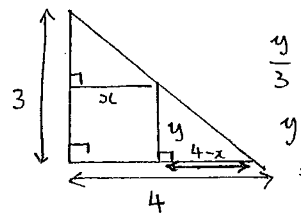
a) i)



$h = 3$

(pythag)

ii)



$\frac{y}{3} = \frac{4 - x}{4}$

$y = \frac{3}{4}(4 - x)$

iii) $V = \pi x^2 y$

$= \pi x^2 \cdot \frac{3}{4}(4 - x)$

$V = 3\pi x^2 - \frac{3}{4}\pi x^3$

iv) $\frac{dV}{dx} = 6\pi x - \frac{9}{4}\pi x^2$

$\frac{d^2V}{dx^2} = 6\pi - \frac{9}{2}\pi x$

$V' = 0$ $\quad 6\pi x - \frac{9}{4}\pi x^2 = 0$

$\pi x\left(6 - \frac{9}{4}x\right) = 0$

$x = 0$ $\quad \frac{9}{4}x = 6$

$x = \frac{24}{9}$

$x = \frac{8}{3}$

test max/min in $\frac{d^2V}{dx^2} = 6\pi - \frac{9}{2}\pi \cdot \frac{8}{3}$

$\frac{d^2V}{dx^2} < 0$ $\quad \therefore$ max

$\therefore$ max V if $x = 8/3$ m

b) $\sum_{n=2}^{4} 2^n = 2^2 + 2^3 + 2^4$

$= 28$

c) $\int (5x^4 + 2x^3 - x^2 + 1)\, dx$

$= x^5 + \frac{2x^4}{4} - \frac{x^3}{3} + x + c$

$= x^5 + \frac{x^4}{2} - \frac{x^3}{3} + x + c$

Question 4

a) i) $f(x) = 2x^3 - 6x + 1$

$\frac{dy}{dx} = 6x^2 - 6$

$\frac{d^2y}{dx^2} = 12x$

st pt $\frac{dy}{dx} = 0$ $\quad 6x^2 - 6 = 0$

$6(x-1)(x+1) = 0$

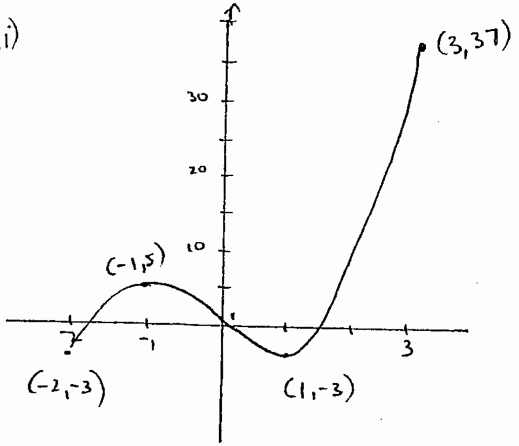
$x = \pm 1$

$(1, -3)$ $\quad y'' > 0$ min

$(-1, 5)$ $\quad y'' < 0$ max

end pts (3,37) (-2,-3)

ii)



iii) greatest value $y = 37$

least value $y = -3$

b) $f'(x) = 6x^2 - 1$

$f(x) = 2x^3 - x + c$

sub $f(0) = 4$

$\therefore 4 = 4$

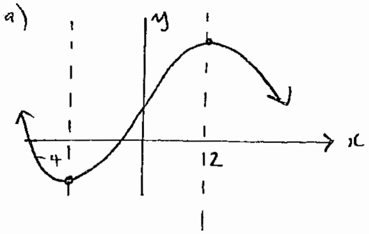
$f(x) = 2x^3 - x + 4$

$f(1) = 2 - 1 + 4$

$\underline{= 5}$

Question 5

a)



b)

b) $A_1 = 20{,}000$

i) $A_6 = 20{,}000 - 6M$

ii) $A_7 = (20{,}000 - 6M)1.01 - M$

$A_7 = 20{,}000(1.01)^1 - 6M(1.01) - M$

$A_8 = [20{,}000(1.01) - 6M(1.01) - M](1.01) - M$

$\underline{A_8 = 20000(1.01)^2 - 6M(1.01)^2 - M(1.01) - M}$

iii) $\therefore A_8 = (20000 - 6M)(1.01)^2 - M(1 + 1.01)$

$A_9 = [20{,}000(1.01)^2 - 6M(1.01)^2 - M(1.01) - M](1.01) - M$

$= 20{,}000(1.01)^3 - 6M(1.01)^3 - M(1.01)^2 - M(1.01) - M$

$\therefore A_{36} = 20{,}000(1.01)^{30} - 6M(1.01)^{30} - M(1.01)^{29} \ldots$

$\ldots\ldots - M$

$\underline{A_{36} = 20000(1.01)^{30} - 6M(1.01)^{30} - M(1 + 1.01 + \cdots 1.01^{29})}$

iv) $A_{36} = 0$

$6M(1.01)^{30} + M\left(\frac{1.01^{30} - 1}{1.01 - 1}\right) = 20{,}000(1.01)^{36}$

$M\left[6(1.01)^{30} + \frac{(1.01^{30} - 1)}{.01}\right] = 20{,}000(1.01)^{31}$

$\underline{M = \$628.78}$