HSC Assessment Task 2
March 2011

Mathematics

Time allowed – 70 minutes

Instructions

- Approved calculators may be used.
- All necessary working must be shown. Marks may not be awarded for careless or badly arranged work.
- Marks awarded are shown on each question.
- Total marks – 52
- Attempt all questions.
- Start each question on a new page.

Q1 /10	Q2 /10	Q3 /10	Q4 /12	Q5 10	Total /52

Question 1 — 10 Marks

a) Find the *16th* term of the series $6 + 10 + 14 + \ldots$

(2)

b) Evaluate $\sum_{n=1}^{\infty}\left(\frac{1}{5}\right)^n$

(2)

c) For all x in the domain $0 < x < 4$, a function $g(x)$ satisfies $g'(x)<0$ and $g''(x)<0$. Sketch a possible graph of $y = g(x)$ in this domain.

(2)

d) A closed box with a square base is to be made so that its volume is $100\ cm^3$.

i) Show that the surface area of the box is given by $A = 2x^2 + \frac{400}{x}$. (1)

ii) Find the minimum possible surface area of the box, to one decimal place.

(3)

Question 2 — 10 marks

a) The population *P*, of birds in a certain area is increasing over time *t*, but the rate of population growth is slowing.

Describe the sign of $\frac{dP}{dt}$ and $\frac{d^2P}{dt^2}$. (2)

b) The tangent to a curve at point *N* has equation *5x - y - 1 = 0.*

i) If $\frac{dy}{dx} = 4x - 3$, for the curve, find the coordinates of *N*. (2)

ii) Find the equation of the curve (2)

c) Which term of the series $\frac{7}{8} + \frac{1}{4} + \frac{1}{14} + \ldots$ is equal to $\frac{16}{117649}$? (2)

d) Find $\int \frac{3x^5 + 2x^2 - x}{x} dx$ (1)

e) Find $\int x(\sqrt{x} + 1)dx$ (1)

Question 3 — 10 Marks

a) For the function $y = x^6 - 6x^4$

i) Find the *x* coordinates of the points where the curve crosses the axes. (1)

ii) Determine the existence of any points of inflexion and state the coordinates. (2)

iii) Sketch the graph of $y = x^6 - 6x^4$ indicating clearly the intercepts, stationary points and points of inflexion. (3)

b) For the curve $y = 2x^3 - 12x^2 - 5x - 3$ find the equation of the normal at the point of inflexion. (2)

c) The sum to infinity of the series $x + \frac{2x}{3} + \frac{4x}{9} + \ldots$ is 15. Evaluate x. (2)

Question 4 — 12 Marks

a) Michael is in training for a local marathon. He is training by completing practice runs over the marathon course. So far he has completed three practice runs with times shown below.

Week 1	Week 2	Week 3
3 hours	2 hours 51 minutes	2 hours 42 minutes 27 seconds

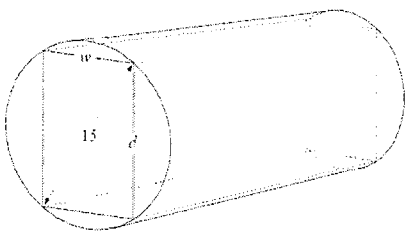
i) Show that these times form a geometric series with a common ratio $r = 0.95$. (1)

ii) If this series continues, what would be his expected time in *Week 5*, to the nearest second? (1)

iii) How many hours, minutes and seconds (to the nearest second) will he have run in total in his practice runs in these *5* weeks? (1)

iv) If the previous winning time for the marathon was *2 hours and 6 minutes*, how many weeks must he keep practising to be able to run the marathon in less than the previous winning time? (2)

b)



A rectangular beam of width w cm and depth, *d cm* is cut from a cylindrical pine log as shown.
The diameter of the cross section of the log (and hence the diagonal of the cross section of the beam) is *15cm*.
The strength *S* of the beam is proportional to the product of its width and the square of its depth, so that

$$S = kd^2 w$$

Question 4 (continued)

i) Show that $S = k(225w - w^3)$ (1)

ii) What numerical dimensions will give a beam of maximum strength? (2)

iii) A square beam with diagonal *15cm* could have been cut from the log. Show that the rectangular beam of maximum strength is more than *8%* stronger than this square beam.

(3)

b) Differentiate $3x^2(7x - 1)^5$. (1)

Question 5 — 10 Marks

a) (i) Grandma wants to invest a certain amount of money for her grandchild so that she will have *$5 000* in ten years time. If the bank account pays *5% p.a.* interest, compounding quarterly, how much will she need to invest now? (2)

b) Tiarn borrows *$500 000* to buy a house. An interest rate of *9% p.a.* compounded monthly is charged on the outstanding balance. The loan is to be repaid in equal monthly installments (R) over a *25* year period.

Show the amount owing after *3* months is:

i) $A_3 = 500000(1.0075)^3 - R\left[1+1.0075+1.0075^2\right]$ (1)

ii) Assuming this pattern continues the monthly repayment can be calculated using:

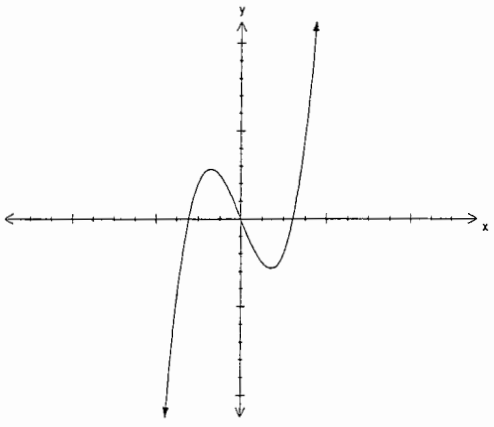
$$A_n = 500000(1.0075)^n - R\left[1+1.0075+1.0075^2+.....+1.0075^{n-1}\right]$$

How much should Tiarn be paying each month? (2)

iii) How much interest does Tiarn pay over the *25* years? (1)

iv) What is the equivalent simple interest rate of this loan? (1)

c) Below is a graph of $f'(x)$. Carefully copy the graph into your exam booklet and draw a neat sketch of the graph of $f(x)$ showing any significant points. (3)



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i) $6+10+14+\ldots$

AP $a=6$ $d=4$

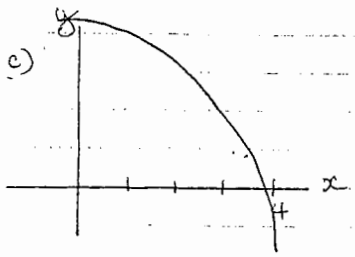
$T_{16} = a+(n-1)d$
$= 6+(15)4$
$= 66$

ii) $\sum_{n=1}^{\infty} \left(\frac{1}{5}\right)^n$ $S_\infty = \frac{a}{1-r}$

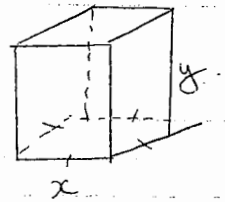
$= \frac{\frac{1}{5}}{\frac{4}{5}}$

$= \frac{1}{4}$

c)



d)



$V = x^2 y$

$100 = x^2 y$

$y = \frac{100}{x^2}$

i) $S.A = 2x^2 + 4xy$.

$= 2x^2 + 4x\frac{100}{x^2}$

$= 2x^2 + \frac{400}{x}$

ii) $A = 2x^2 + 400x^{-1}$

$\frac{dA}{dx} = 4x - 400x^{-2}$

$= 4x^3 - 400$

stat pt $\frac{dA}{dx} = 0$.

$4x^3 - 400 = 0$

$x^3 = 100$

$x \doteq 4.64\ldots$

check.

x	0	4.64	10
$f'x$	−	0	+

$\therefore x \doteq 4.64$ min

min SA $= 129.295\ldots$
$= 129.3$

Q2

(a) $\frac{dP}{dt} > 0$ $\frac{d^2P}{dt^2} < 0$

(b) $5x - y = 0$

(i) $y = 5x - 1$

$m = 5$

$4x - 3 = 5$

$x = 2$ $y = 9$ $N \Rightarrow (2,9)$

(ii) $\int 4x - 3\,dx = 2x^2 - 3x + C$

at $(2,9)$ $9 = 8 - 6 + C$

$C = 7$

$y = 2x^2 - 3x + 7$

Q2 (cont'd)

(c) $\frac{7}{8} + \frac{1}{4} + \frac{1}{14} + \ldots$

GP $r = 2/7$ $a = 7/8$

$T_n = ar^{n-1}$

$\frac{16}{117649} = \frac{7}{8} \times \left(\frac{2}{7}\right)^{n-1}$

$0.00155\ldots = \left(\frac{2}{7}\right)^{n-1}$

$\log \ldots = n-1 \log 2/7$

$n = 8$

(d) $\int \frac{3x^5 + 2x^2 - x}{x}\,dx$

$= \int 3x^4 + 2x - 1\,dx$

$= \frac{3x^5}{5} + x^2 - x + C$

(e) $\int x(x^{1/2} + 1)\,dx$

$= \int x^{3/2} + x\,dx$

$= \frac{2x^{5/2}}{5} + \frac{x^2}{2} + C$

$= \frac{2\sqrt{x^5}}{5} + \frac{x^2}{2} + C$.

Q3

(a) $y = x^6 - 6x^4$

at $y = 0$ $x^4(x^2 - 6) = 0$

$x = 0$, $x = \pm\sqrt{6}$

(ii) $y = x^6 - 6x^4$

$y' = 6x^5 - 24x^3$

stat pts at $y' = 0$

$6x^5 - 24x^3 = 0$

$x^3(6x^2 - 24) = 0$

$6x^2 = 24$

$x^2 = 4$

$x = \pm 2$.

x	−10	−2	−1	0	1	2	
$f'(x)$	−	0	+	0	−	0	−

$y'' = 30x^4 - 72x^2$

$30x^4 - 72x^2 = 0$

$x^2(30x^2 - 72) = 0$

$30x^2 = 72$

$x^2 = 12/5$

$x = \pm\sqrt{12/5}$

$\approx \pm 1.55$,

poss. horiz. pt infln at $x = 0$.

x	−1	0	+1
$f'(x)$	−	0	−

$x = 0$ not hor pt. infln

3* (cont'd)

t. points

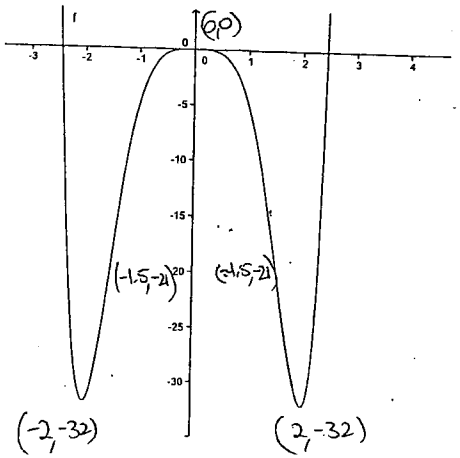
$(-2, -32)$ min

$(0, 0)$ max

$(2, -32)$ min

$\left(\sqrt{12/5}, -20\frac{92}{125}\right)$ pt inflex

$\left(-\sqrt{12/5}, -20\frac{92}{125}\right)$



(b) $y = 2x^3 - 12x^2 - 5x - 3$

$y' = 6x^2 - 24x - 5$

$y'' = 12x - 24$

pt inflex $12x - 24 = 0$

$x = 2$

x	1	2	3
$f'(x)$	−	0	+

at $x = 2$ $y = -45$

$m_\perp = \frac{1}{29}$

$(y + 45) = \frac{1}{29}(x - 2)$

$29y + 1305 = x - 2$

$29y = x - 1307$

$x - 29y - 1307 = 0$

(c) $x + \frac{2x}{3} + \frac{4x}{9} + \ldots$

$a = x$ $r = 2/3$

$S_\infty = \frac{a}{1-r}$

$15 = \frac{x}{1/3}$

$x = 5$

Q4

(i) $\frac{w2}{w1} = 19/20$

$\frac{w3}{w2} = 19/20$

$\therefore$ GP $a = 3$hrs $r = 0.95$

(ii) W4 − 2h 34m 19.65s

W5 − 2h 26m 36.67s

(iii) Tot − 13h 34m 23.32s

(iv) $T_n = ar^{n-1}$

$2\text{h } 6\text{m} = 3 \times 0.95^{n-1}$

$0.95^{n-1} = 42\text{m}$

$n - 1 = \frac{\log 42\text{m}}{\log 0.95}$

$n - 1 = 6.95\ldots$

$n = 7.95\ldots$

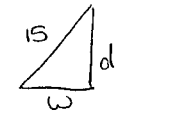
$n = 7$

$3\text{h} \times 0.95^7 = 2\text{h } 5\text{m } 42.04\text{s}$

(b) $S = kd^2w$

$S = k(225 - w^2)w$

$= k(225w - w^3)$



$w^2 + d^2 = 225$

$d^2 = 225 - w$

(ii) $\frac{dS}{dw} = 225k - 3kw^2$

stat pt $225k - 3kw^2 = 0$

$w^2 = 225/3$

$w = \sqrt{75}$

cannot be −ve.

$w \approx 8.66$

w		8.66	
$f'(w)$	+	0	−

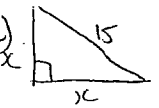
$\therefore$ 8.66 max.

$d = \sqrt{150}$

$= 12.247\ldots$

$= 12.25$

(iii)



$2x^2 = 15^2$

$x^2 = 112.5\ldots$

$x = 10.60$

$S \propto d^2w$

$\propto 150 \times 8.66\ldots$

$\propto 1299$

$S \propto d^2w$

$\propto 112.5 \times 10.6$

$\propto 1192.5$

strength $= 1299 - 1192.5$

$= 106.5$

strength $= \frac{106.5}{1192.5} \times 100$

Q4

(b) $3x^2(7x-1)^5$

$y' = 3x^2(5)(7x-1)^4(7) + (7x-1)^5 6x$

$= 105x^2(7x-1)^4 + (7x-1)^5 6x$

$= (7x-1)^4[105x^2 + (7x-1)6x]$

$= (7x-1)^4[105x^2 + 42x^2 - 6x]$

$= (7x-1)^4(147x^2 - 6x)$

Q5

(a) (i) $V = P(1+r)^n$

$5000 = P(1.0125)^{40}$

$P = \$3042.07$

(b) \$500,000 $i = \frac{9}{1200} = 0.0075$

$n = 25 \times 12 = 300$

i)

$r_1 = 500{,}000(1.0075) - M$

$A_2 = A_1(1.0075) - M$

$= [500{,}000(1.0075) - M](1.0075) - M$

$= 500{,}000(1.0075)^2 - M(1.0075 + 1)$

$A_3 = 500{,}000(1.0075)^3 - M(1.0075^2 + 1.0075 + 1)$

ii) $A_{300} = 500{,}000(1.0075^{300}) - M(1 + 1.0075 + \ldots 1.0075^{300})$

GP $S_n = \frac{a(r^n - 1)}{r - 1}$

$= \frac{1.0075^{300} - 1}{0.0075}$

(CONTD)

$0 = 500{,}000(1.0075)^{300} - M\left(\frac{1.0075^{300} - 1}{0.0075}\right)$

$M = 4195.9818 \ldots$

$M = \$4{,}195.98$

(iii) Int $= 4195.98 \times 300 - 500{,}000$

$= \$758{,}794$

(iv) $S.I = \frac{PRT}{100}$

$758{,}794 = \frac{500{,}000 \times R \times 25}{100}$

$R = 6.07\%$ p.a.

(c)