Name .... Teacher.................................

Sydney Technical High School

Mathematics
HSC Assessment Task 2
March 2012

General Instructions

- Reading Time - 5 minutes.
- Working Time – 70 minutes.
- Write using a blue or black pen.
- Board approved calculators may be used.
- A table of standard integrals is provided at the back of this paper.
- All necessary working should be shown for every question.
- Begin each question on a new page.

Total Marks 61

Section I

Total marks **(5)**

- Attempt Questions 1-5
- Answer on the Multiple Choice answer sheet provided.
- Allow about 5 minutes for this section

Section II

Total marks **(56)**

- Attempt questions 6 – 9
- Answer on the blank paper provided, unless otherwise instructed. Start a new page for each question.
- Allow about 65 minutes for this section

Section I **Total marks (5)**

Use the multiple choice answer sheet. Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

1. Determine the 14^{th} term of the geometric series: $6 + 12 + 24 + \ldots$

a) 12 288 b) 24 576

c) 49 152 d) 98 304

2. Evaluate $\sum_{k=3}^{7} 5(2)^k$

a) 600 b) 635

c) 1240 d) 1270

3. If x, 4, 8x are three consecutive terms in a geometric sequence, determine the values of x

a) ± 1 b) $\pm\sqrt{2}$

c) ± 2 d) $\pm 2\sqrt{2}$

4. $\int_0^3 (1+x)^2\,dx$ is equal to

a) 15 b) 21

c) $\frac{5}{8}$ d) $\frac{65}{3}$

5. If $y = ax^n$, then $\int y\,dx =$

a) $\int y\,dx = \frac{ax^{n+1}}{n+1}$ b) $\int y\,dx = \frac{ax^{n-1}}{n-1} + c$

c) $\int y\,dx = \frac{ax^{n+1}}{n} + c$ d) $\int y\,dx = \frac{ax^{n+1}}{n+1} + c$

Section II

Total Marks (56)
Attempt Questions 6-9
Allow about 65 minutes for this section.

Answer all questions, starting each question on a new sheet of paper.

Question 6 (Start a new page) **14 marks**

a) Find the primitive function of $4x^2 - 3 - \frac{3}{2x^2}$ **2**

b) John runs a marathon, which is a distance of 42 km. He runs the first kilometre in 4 minutes and from there on, he takes 5 seconds longer to run each successive kilometre. (i.e the 2nd kilometre takes 4 minutes and 5 seconds.)

(i) How long does he take to run the 16th kilometre? **1**

(ii) How long does it take to run the entire marathon? **1**

(iii) How far had he run after 2 hours 36 minutes and 15 seconds? **2**

c) Consider the curve given by $y = \frac{1}{4}x^4 - x^3$

i) Find any turning points and determine their nature. **3**

ii) Find any points of inflexion. **1**

iii) Sketch the curve for $-1.5 \leq x \leq 4.5$, indicating where the curve crosses the x- axis. **3**

iv) For what values of x is the curve concave down? **1**

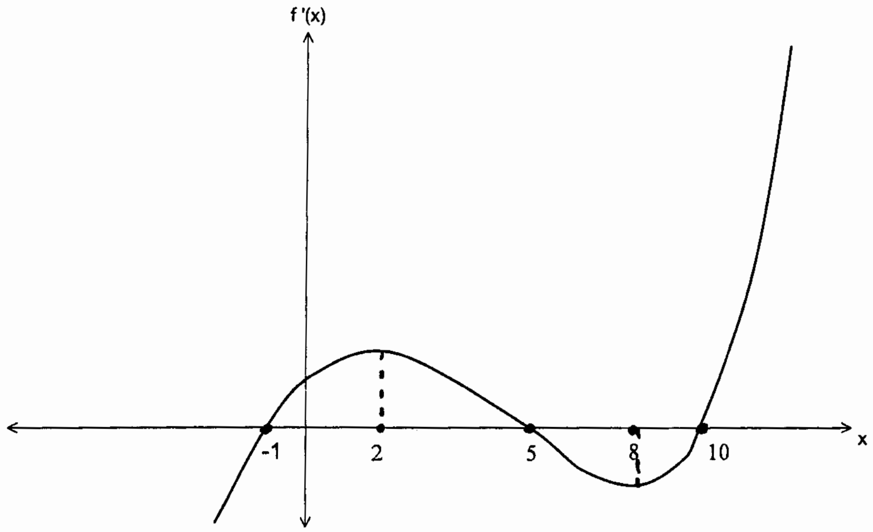
Question 7 (Start a new page) **14 marks**

a) Find

i) $\int (x-2)(x-5)dx$ **2**

ii) $\int \frac{x^2+3}{x^4} dx$ **2**

b) The graph of $y = f'(x)$ is shown below.



(i) Give the x values for the turning points of $y = f(x)$. **1**

(ii) Draw a sketch of $y = f(x)$. **2**

(iii) Draw a sketch of $y = f''(x)$. **2**

c) On the first day of each month for 12 months, starting 1st January 2012, Eleni deposited \$200 into a special savings account, which was paying 6% per annum interest, compounded monthly.

i) What was the monthly rate of interest **1**

ii) What was the value of the \$200 she deposited at 31st December 2012? **2**

iii) What was the total of her savings account at 31st December 2012? **2**

Question 8 (Start a new page) **14 marks**

a) Find

i) $\int_1^4 x^{\frac{1}{2}} + x^{-\frac{1}{2}}\,dx$ **2**

ii) $\int_0^1 (2-5x)^4\,dx$ **2**

b) If $f'(x) = 10(3x-2)^4$ and $f(1) = 2$, find $f(x)$ **2**

c) Nectarios invests $50 000 in an account which earns 8% interest, compounded annually. He intends to withdraw $M at the end of each year, immediately after the interest has been paid. He wishes to be able to do this for exactly 20 years, so that the account will then be empty.

i) Write an expression in terms of M for the amount still in the account immediately after he has made his first withdrawal? **1**

ii) Write an expression in terms of M for the amount of money in the account immediately after his 20th withdrawal. **2**

iii) Calculate the value of M which leaves his account empty after the 20th withdrawal. **2**

iv) Suppose Nectarios wished to be able to withdraw $8000 per year for the 20 years. Find, to the nearest percent, the interest rate he would then need to earn on his account. **1**

d) The world's population P is increasing at an increasing rate over time t. What does this say about the sign of $\frac{dP}{dt}$ and $\frac{d^2P}{dt^2}$ **2**

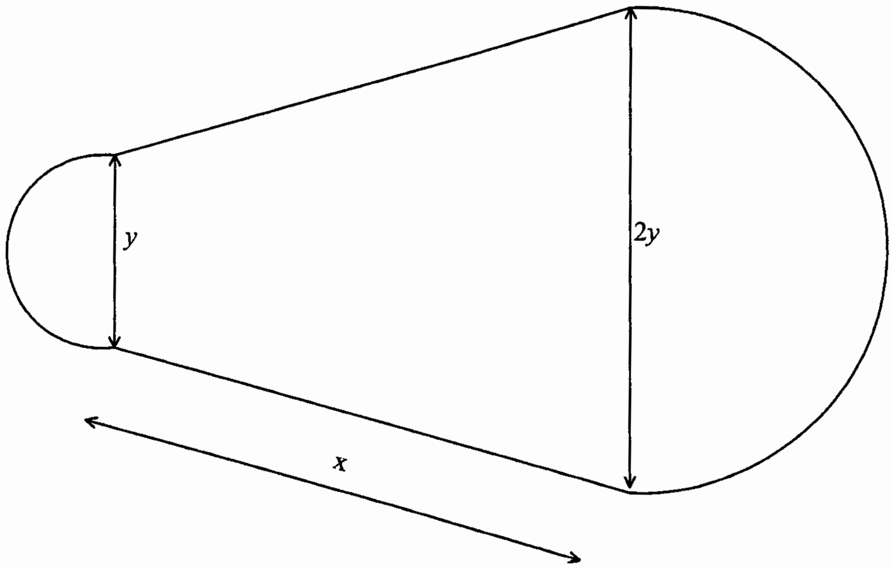
Question 9 **(Start a new page)** **14 marks**

a) $\int_4^a (2x+3)\,dx = 12$ find a **3**

b) Find the equation of the normal to the curve $y = x^2 + x$ at the point where $x = 1$ **3**

c) Evaluate w if $5 + 5w^2 + 5w^4 + \ldots = \frac{49}{8}$ **3**

d) The track for a race is in the shape shown, with two semicircular curves, one whose diameter is twice that of the other. It also has two straights which are equal in length. The total length of the track is 400 m.



(i) Using x m to represent the length of the straight and y m for the diameter of the smaller semicircle, show that $y = \dfrac{800-4x}{3\pi}$. **2**

(ii) The average speed that a vehicle can attain on a lap of the track depends on the length of the straights. Given that the average speed that a certain vehicle can attain on the track is given by:

$$\text{Speed} = 100 - \left(\left(\frac{x}{30}\right)^3 + \frac{\pi}{6}(y)\right)$$

find the length of straight which maximizes the speed of this vehicle. **3**

END OF EXAM

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Mathematics - HSC Assessment Task 2 March 2012

Multiple Choice

1. C
2. C
3. B
4. B
5. D

Question 1

a) $\frac{4x^3}{3} - 3x + \frac{3}{2}x^{-1} + c$

bi) $T_n = a + (n-1)d$
$= 240 + 15 \times 5$
$= 315$ seconds
$= 5$ min 15 sec

ii) $S_{42} = \frac{42}{2}[2 \times 240 + (42-1) \times 5]$
$= 21[480 + 41 \times 5]$
$= 14\,385$ sec
$= 3$ hours 59 min 45 seconds

iii) 2 hours 36min 15sec = 9375 sec

$9375 = \frac{n}{2}[480 + 5(n-1)]$
$18750 = n(475 + 5n)$
$18750 = 475n + 5n^2$
$5n^2 + 475n - 18750 = 0$
$n^2 + 95n - 3750 = 0$

$n = \frac{-95 \pm \sqrt{9025 + 15000}}{2}$

$n = 30$ or -125

c) i) $y = \frac{1}{4}x^4 - x^3$
$y' = x^3 - 3x^2$
$y'' = 3x^2 - 6x$

Stationary Points occur when $y' = 0$
$x^3 - 3x^2 = 0$
$x^2(x-3) = 0$
$x = 0 \quad x = 3$
$y = 0 \quad y = -\frac{27}{4}$

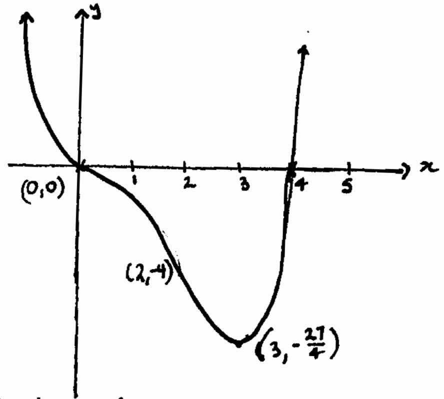
When $x = 3$
$y'' = 9 > 0 \quad \therefore$ min $(3, -\frac{27}{4})$

When $x = 0$
$y'' = 0$

x	-1	0	1
y'	$-$	0	$-$

$\therefore$ Inflexion Point at $(0,0)$

ii) Points of Inflexion occurs when $y'' = 0$
$3x^2 - 6x = 0$
$3x(x-2) = 0$
$x = 0 \quad x = 2$
$y = 0 \quad y = -4$



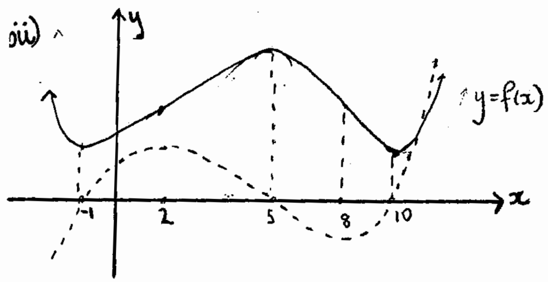
d) Values for which concavity is downwards
$y'' < 0 \quad 3x(x-2) < 0$
$0 < x < 2$

Question 7

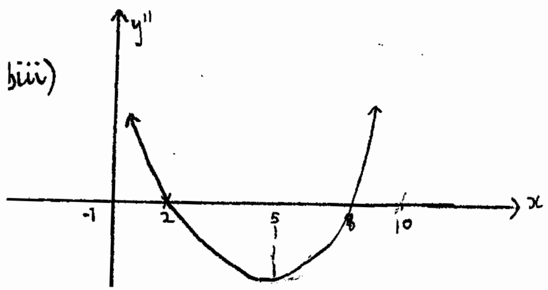
ai) $\int (x-2)(x-5)\,dx = \int x^2 - 7x + 10\,dx$
$= \frac{x^3}{3} - \frac{7x^2}{2} + 10x + c$

bi) $\int \frac{x^2+3}{x^4}\,dx = \int x^{-2} + 3x^{-4}\,dx$
$= -\frac{1}{x} - \frac{1}{x^3} + c$

bii)



biii)



bi) Turning points occur when $x = -1, 5, 10$

c i) monthly interest rate
$0.06 \div 12 = 0.005$

ii) $200(1 + 0.005)^{12} = \$212.34$

iii) $200(1.005)^{12} + 200(1.005)^{11} + 200(1.005)$
$+ \ldots + 200(1.005)^2 + 200(1.005)^1$

$= 200[1.005 + 1.005^1 + 1.005^2 + \ldots + 1.005^{12}]$

$= 200\left[\frac{1.005(1.005^{12} - 1)}{1.005 - 1}\right]$

$= \$$ ~~2497.45~~

Question 8

ai) $\int_1^4 x^{\frac{1}{2}} - x^{-\frac{1}{2}}\,dx = \left[\frac{x^{3/2}}{3/2} + \frac{x^{1/2}}{1/2}\right]_1^4$
$= (\frac{16}{3} + 4) - (\frac{2}{3} + 2)$
$= 6\frac{2}{3}$

ii) $\int_0^1 (2-5x)^4\,dx = \left[\frac{(2-5x)^5}{5 \times -5}\right]_0^1$
$= \frac{243}{25} + \frac{32}{25})$
$= \frac{275}{25}$
$= 11$

b) $f'(x) = 10(3x-2)^4$
$f(x) = \frac{10(3x-2)^5}{5 \times 3}$

$f(x) = \frac{2}{3}(3x-2)^5 + c$

$f(1) = \frac{2}{3}(3 \times 1 - 2)^5 + c = 2$

$\frac{2}{3} + c = 2$
$c = 4/3$

$\therefore f(x) = \frac{2}{3}(3x-2)^5 + 4/3$

c)i) $A_1 = P(1.08) - M$
$= 50\,000(1.08) - M$

ii) $A_2 = [50\,000(1.08) - M]1.08 - M$
$= 50\,000(1.08)^2 - 1.08M - M$

iii) $A_3 = 50\,000(1.08)^3 - M[1 + 1.08 + 1.08^2]$

$A_{20} = 50\,000(1.08)^{20} - M[1 + 1.08 + 1.08^2 + \dots + 1.08^{19}]$

Account is empty when $A_{20} = 0$

$0 = 50\,000(1.08)^{20} - M\left[\frac{1.08^{20} - 1}{0.08}\right]$

$M = \frac{50\,000(1.08)^{20} \times 0.08}{1.08^{20} - 1}$

$M = \$5092.61$

ii) 15%

d) $\frac{dP}{dt} > 0$

$\frac{d^2P}{dt^2} > 0$

Question 9

a) $\int_4^a (2x + 3) = 12$

$[x^2 + 3x]_4^a = 12$

$(a^2 + 3a) - (4^2 + 12) = 12$
$a^2 + 3a - 28 = 12$
$a^2 + 3a - 40 = 0$
$(a + 8)(a - 5) = 0$
$a = -8 \qquad a = 5$
No Solution ↑   ↖ only solution

b) $y = x^2 + x$
$\frac{dy}{dx} = 2x + 1$

When $x = 1 \quad y = 2$
$m_1 = 3$
$m_2 = -\frac{1}{3}$

$y - y_1 = m(x - x_1)$
$y - 2 = -\frac{1}{3}(x - 1)$
$3y - 6 = -x + 1$
$x + 3y - 7 = 0$

c) $S + Sw^2 + Sw^4 + \dots = \frac{49}{8}$
$S(1 + w^2 + w^4 + \dots) = \frac{49}{8}$

$1 + w^2 + w^4 + \dots = \frac{49}{40}$

$\frac{1}{1 - w^2} = \frac{49}{40}$
$40 = 49(1 - w^2)$
$40 = 49 - 49w^2$
$9 = 49w^2$
$w^2 = \frac{9}{49}$
$w = \pm\frac{3}{7}$

di) $P = 2x + \frac{\pi y}{2} + \pi y$

$P = 400$

$400 = 2x + \frac{\pi y}{2} + \pi y$

$800 = 4x + \pi y + 2\pi y$

$800 = 4x + 3\pi y$

$3\pi y = 800 - 4x$

$y = \frac{800 - 4x}{3\pi}$

dii) $\text{Speed} = 100 - \left[\left(\frac{x}{30}\right)^3 + \frac{\pi}{6}(y)\right]$

$S = 100 - \left(\frac{x}{30}\right)^3 - \frac{\pi}{6}\left(\frac{800 - 4x}{3\pi}\right)$

$S = 100 - \frac{x^3}{27\,000} - \frac{800 - 4x}{18}$

$\frac{dS}{dx} = \frac{-3x^2}{27\,000} + \frac{4}{18}$

Stationary points occur when $\frac{dS}{dx} = 0$

$\frac{-3x^2}{27\,000} + \frac{4}{18} = 0$

$\frac{3x^2}{27\,000} = \frac{4}{18}$

$x^2 = \frac{4}{18} \times \frac{27\,000}{3}$

$x^2 = 2000$
$x = \sqrt{2000}$
$x = 44.7 \quad (1\,dp)$

$\frac{d^2S}{dx^2} = \frac{-6x}{27\,000} - \frac{x}{45000}$

When $x = 44.7$
$\frac{d^2S}{dx^2} = \frac{-44.7}{4500}$
$= -0.009$

$\therefore$ max occurs when $x = 44.7$