Name:____________________ Teacher:______________________

SYDNEY TECHNICAL HIGH SCHOOL

MATHEMATICS

March 2014

Time Allowed – 70 minutes

DIRECTION TO CANDIDATES:

- All questions may be attempted.
- All questions are not of equal value. The marks indicated are only a guide and may be changed.
- Full marks may not be awarded for careless or badly arranged work, including illegible writing.
- Approved calculators may be used.
- Diagrams are not drawn to scale.
- All necessary working should be shown in every question.
- **Each question attempted is to be started ON A NEW PAGE, clearly marked with the number of the question and your name on the top right hand side of the page.**

SECTION 1

1. Given that the curve $y = ax^2 - 8x - 8$ has a stationary point at $x = 2$, find the value of a

A. $a = \frac{1}{2}$ B. $a = 2$ C. $a = 6$ D. $a = -2$

2. An infinite geometric series has a first term of 8 and a limiting sum of 12. What is the common ratio?

A. $\frac{1}{6}$ B. $\frac{1}{4}$ C. $\frac{1}{3}$ D. $\frac{1}{2}$

3. The equation of the directrix of the parabola $y^2 = -8x$ is

A. $x = 2$ B. $y = 2$ C. $x = -2$ D. $y = -2$

4. $2x + 5,\ 3x$ and m form a geometric sequence with a common ratio of 4. The value of m is

A. -4 B. -12 C. -16 D. -48

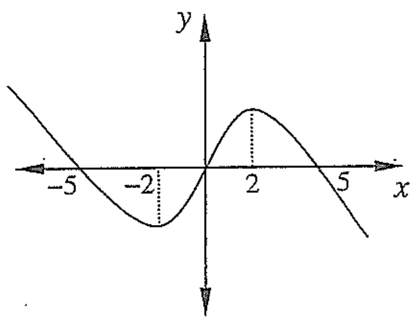
5.

Consider a curve with the following properties:

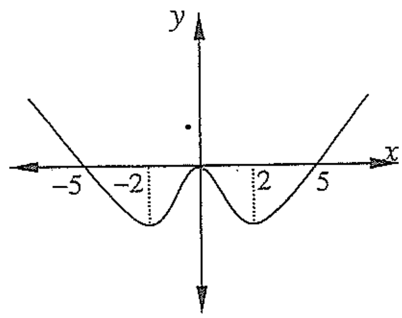
$g(x)$ is odd.
$g(5)=0$ and $g'(2)=0$.
$g'(x)>0$ for $x>2$.

Which of the following could be the graph of $y=g(x)$?

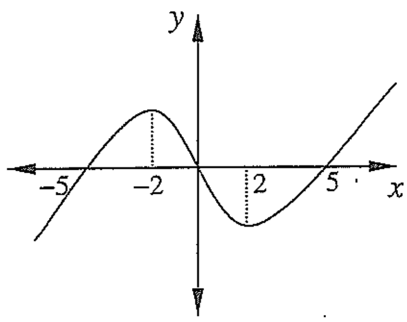
(A)



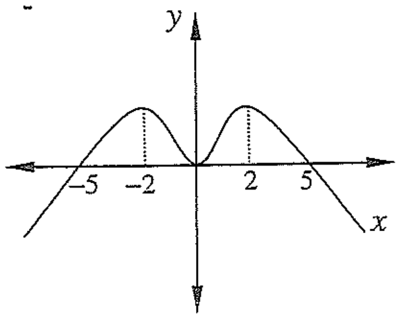
(B)



(C)



(D)



SECTION 2

Question 6 (12 Marks) Start a New page

(a). Find the value of $\sum_{k=0}^{5}\left(k^2+1\right)$ (1)

(b). If α and β are the roots of the equation

$2x^2 - 6x - 3 = 0$, find the value of

(i) $2\alpha\beta$ (1)

(ii) $(\alpha + \beta)^2$ (1)

(iii) $\frac{1}{\alpha} + \frac{1}{\beta}$ (1)

(iv) $\alpha^2 + \beta^2$ (1)

(c). For the parabola $4y = x^2 + 4x + 12$

(i) Find the co-ordinates of the vertex (2)

(ii) Find the focal length (1)

(iii) Sketch the parabola showing the vertex, focus and directrix (2)

(d) The second term of a geometric series is $\frac{3}{8}$ and the seventh term is 12. Find the 14^{th} term. (2)

Question 7 (12 marks) Start a New page

(a) The first 3 terms of an arithmetic series are 62, 56 and 50.

(i) Write down a formula for the nth term (1)
(ii) If the last term is -88, how many terms are there in the series? (2)
(iii) Find the sum of the series. (2)

(b) Find the value of k in the equation
$x^2 - (k+3)x + (k+6) = 0$ if it has no real roots. (2)

(c) Find the equation of the locus of a point $P(x, y)$ which moves so that line PA is perpendicular to the line PB where $A = (1,5)$ and $B = (-2,-3)$ (3)

(d) Solve the equation $3^{2x} + 2.3^x - 15 = 0$ (2)

Question 8 (12 marks) Start a New page

(a) Consider the curve given by $y = -x^3 + 6x^2 - 9x - 1$

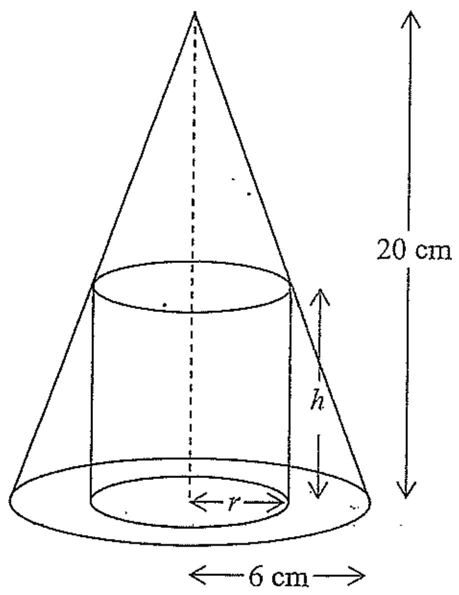
(i) Find the co-ordinates of any stationary points and determine their nature (3)

(ii) Prove a point of inflexion exists and find its co-ordinates (2)

(iii) Sketch the curve for $x \geq 0$, clearly indicating all significant points. (2)

Question 8 Continued

(b)



NOT TO SCALE

A cylinder of radius r cm and height h cm is inscribed in a cone with base radius 6cm and height 20cm as in the diagram.

(i) Show, using similar triangles, that $h = \frac{10(6-r)}{3}$ (1)

(ii) Show that the volume of the cylinder is given by (1)

$$V = \frac{10\pi r^2(6-r)}{3}$$

(iii) Hence find the values of r and h for the cylinder which has maximum value (3)

Question 9 (14 marks) Start a New Page

(a) Find the primitives of

(i) $8x + 3x^2 - 4x^3$ (ii) $(2x-1)^3$

(b) Find the equation of the curve passing through the point (2,5) with gradient function $f'(x) = 3x^2 - 4x + 1$.

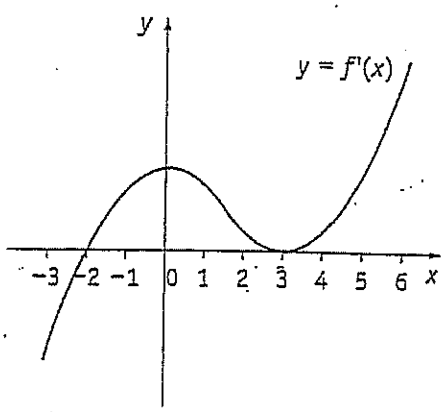
(c) The diagram shows the derivative of y=f(x).

(i) Write down the x co-ordinate of the turning point on y=f(x) and state whether it is a maximum or minimum turning point. (2)

(ii) At what x value on y=f(x) is there a horizontal point of inflexion? (1)

(iii)Where is the function y=f(x) increasing?(1)

(iv) Sketch a possible graph of y=f(x) (1)



(d) On their son Geoffrey's 11th birthday, Mr and Mrs Shum deposited $600 into an account earning 5% p.a. interest compounded annually. They will continue to deposit $600 on each of his successive birthdays, up to and including his 21st, giving him the accumulated funds as a present on his 21st birthday.

(i) Show that the amount of Geoffrey's 21st birthday present was $8524 (to the nearest dollar) (3)

(ii) What single deposit on Geoffrey's 11th birthday would have, under the same conditions, provided the same 21st birthday present? (2)

1. B
2. C
3. A
4. D
5. C.

SECTION 2

1)(a) $= 1+2+5+10+17+26$
$= 61$

(b) $\alpha\beta = \frac{c}{a} = \frac{-3}{2}$
$\alpha+\beta = \frac{-b}{a} = \frac{6}{2} = 3$

(i) $2\alpha\beta = 2\times -\frac{3}{2} = -3$

(ii) $(\alpha+\beta)^2 = 3^2 = 9$

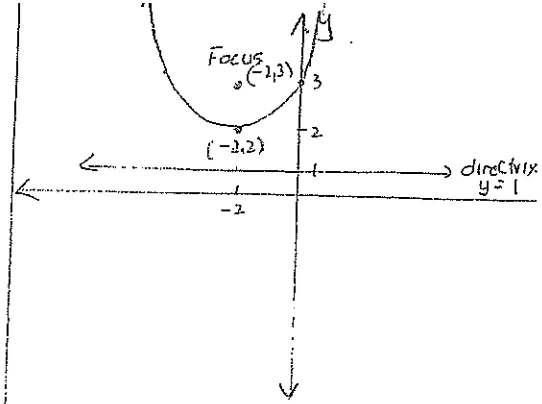
(iii) $\frac{1}{\alpha}+\frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{3}{-3/2} = -2$

(iv) $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$
$= 3^2 - 2\times -\frac{3}{2}$
$= 12$

(c) $4y-12+4 = x^2+4x+4$
$4y-8 = (x+2)^2$
$4(y-2) = (x+2)^2$

(i) vertex $(-2,2)$

(ii) Focal length $4a=4$
$a=1$



(d) $t_2 = ar = \frac{3}{8}$ — ①
$t_7 = ar^6 = 12$ — ②

② ÷ ① $\frac{ar^6}{ar} = \frac{12}{3/8}$
$r^5 = 32$
$r = 2$
$a = 3/16$
$t_{14} = ar^{13}$
$= 1536$

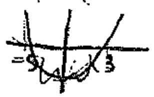
2(a) 62, 56, 50,
$a=62 \quad d=-6$

(i) $T_n = 62+(n-1)-6$
$= 62 - 6n + 6$
$T_n = 68-6n$

(ii) $68-6n = -88$
$-6n = -156$
$n = 26$
∴ 26 terms

(iii) $S_{26} = \frac{26}{2}[2\times 62 + 25\times -6]$
$= -338$

(b) $\Delta = b^2-4ac$
No real roots $\Delta < 0$
$(k+3)^2 - 4.1.(k+6) < 0$
$k^2+6k+9-4k-24<0$
$k^2+2k-15<0$
$(k+5)(k-3)<0$



$-5<k<3$

(c) $m_{PA} = \frac{-1}{m_{PB}}$

$m_{PB} = \frac{y+3}{x+2}$

$m_{PA} = \frac{y-5}{x-1}$

∴ $\frac{y-5}{x-1} = \frac{-1}{\frac{y-3}{x+2}}$

$\frac{y-5}{x-1} = \frac{-(x+2)}{y+3}$

$(y-5)(y+3) = -(x+2)(x-1)$
$y^2-2y-15 = -(x^2+x-2)$

$y^2-2y-15 = -x^2-x+2$
$x^2+y^2-2y+x-17=0.$

(d) $3^{2x}+2.3^x-15=0$
Let $y=3^x$
$y^2+2y-15=0$
$(y+5)(y-3)=0$
$y=-5 \quad y=3$
$3^x=-5 \quad 3^x=3$
No SOLN $\quad x=1$

(a) $y = -x^3 + 6x^2 - 9x - 1$

(i) $y' = -3x^2 + 12x - 9$

$= -3(x^2 - 4x + 3)$

$= -3(x-3)(x-1)$

For st pts let $y' = 0$

$x = 3, \quad 1$

$y = -1 \quad -5$

$(3,-1) \quad (1,-5)$

$y'' = -6x + 12$

When $x = 3$ $y'' = -6 < 0$ Max $(3,-1)$

When $x = 1$ $y'' = 6 > 0$ Min $(1,-5)$

(ii) $y'' = -6x + 12$

For pt of inflexion

$y'' = -6x + 12 = 0$

$-6x = -12$

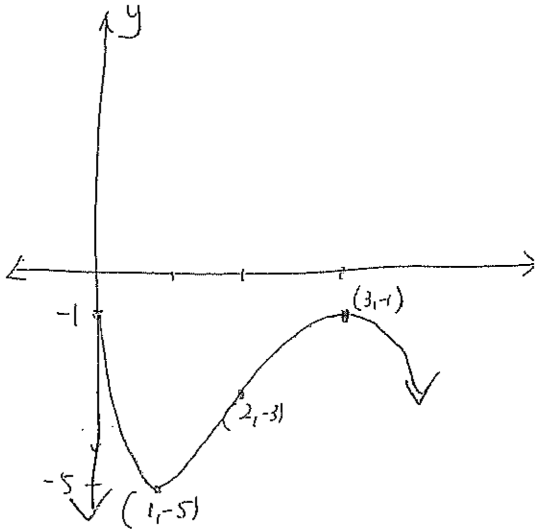
$x = 2$

$y = 3.$

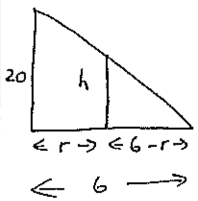
x	1	2	3
y''	6	0	-6

$(2,-3)$ is a pt of inflexion

(iii)



(b)(i) $V = \pi r^2 h$



From similar Δ's

$\frac{h}{20} = \frac{6-r}{6}$

$h = \frac{20(6-r)}{6}$

$h = \frac{10(6-r)}{3}$

$\therefore \quad V = \pi r^2 \times \frac{10(6-r)}{3}$

$V = \frac{10\pi r^2(6-r)}{3}$

(ii) $\frac{dv}{dr} = \frac{10\pi}{3} \times \frac{d}{dr}(6r^2 - r^3)$

$= \frac{10\pi}{3} \times 12r - 3r^2$

$= \frac{10\pi}{3} \times 3r(4-r)$

$= 10\pi r(4-r)$

$\frac{dv}{dr} = 0$ when ~~$r = 0$~~ (not) and $r = 4$

$\frac{dv}{dr} = 40\pi r - 10\pi r^2$

$\frac{d^2v}{dr^2} = 40\pi - 20\pi r$

when $r = 4$ $\frac{d^2v}{dr^2} < 0$

$\therefore$ Max when $r = 4$

and $h = \frac{10(6-4)}{3}$

$h = \frac{20}{3}$

) 11th bday $A_1 = 600 \times 1.05^{10}$

12th " $A_2 = 600 \times 1.05^9$

: :

20th " $A_{10} = 600 \times 1.05^1$

21st " $A_{11} = 600$

otal $= 600(1.05^{10} + 1.05^9 + \dots + 1)$

$= 600(1 + 1.05 + \dots + 1.05^{10}$

$a = 1,\ n = 11,\ r = 1.05$

$= 600 \times \frac{(1.05^{11} - 1)}{0.05}$

$= \$8524$

) $8524 = x \times 1.05^{10}$

$x = \$5233$

) ∘

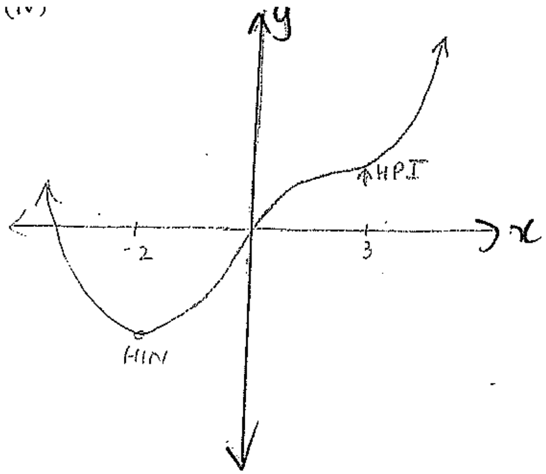
(i) $x = -2$

minimum

(ii) $x = 3$

iii) $-2 < x < 3$ and $x > 3$

(iv)



(c) (i) $\frac{8x^2}{2} + \frac{3x^3}{3} - \frac{4x^4}{4} + C$

$= 4x^2 + x^3 - x^4 + C$

(ii) $\frac{(2x-1)^4}{2.4} + C$

$= \frac{(2x-1)^4}{8} + C$

(d) $f'(x) = 3x^2 - 4x + 1$

$f(x) = x^3 - 2x^2 + x + C$

At (2, 5)

$5 = 8 - 8 + 2 + C$

$C = 3$

$\therefore\ f(x) = x^3 - 2x^2 + x + 3$