Name: .... Maths Class: ....

SYDNEY TECHNICAL HIGH SCHOOL

Year 12

Mathematics

HSC Course

Assessment 2

March, 2015

Time allowed: 70 minutes

General Instructions:

- Marks for each question are indicated on the question.
- Approved calculators may be used
- All necessary working should be shown
- Full marks may not be awarded for careless work or illegible writing
- *Begin each question on a new page*
- Write using black or blue pen
- All answers are to be in the writing booklet provided
- A set of Standard Integrals is provided at the rear of this Question Booklet, and may be removed at any time.

Multiple Choice
Questions 1-5
5 Marks

Questions 6-10
45 Marks

Question 1

Which expression is used to find the interest earned when $\$P$ is invested for n years and interest of 10% p.a. is compounded twice yearly?

A. $I = P(1.1)^n - P$

B. $I = P(1.05)^n - P$

C. $I = P(1.05)^{2n} - P$

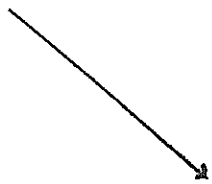
D. $I = P(1.05)^{n/2} - P$

Question 2

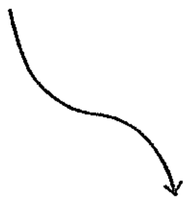
Which decreasing function shows positive second derivative for all x values?

A.

C.



B.



D.

Question 3

For which x values is the curve $y = \frac{1}{3}x^3 - 4x + 5$ increasing?

A. $-2 < x < 2$

B. $x < -2$ and $x > 2$

C. $x > 2$

D. $x < -2$

Question 4

Identify which type of stationary point occurs at (3,2) on a curve where $\frac{dy}{dx} = (6-2x)^2(x+5)\sqrt{x}$.

A. maximum turning point

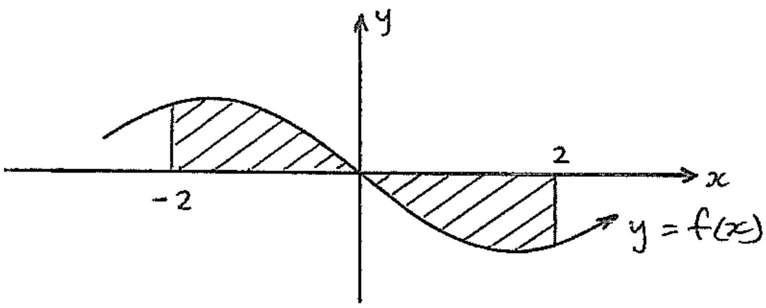
B. minimum turning point

C. horizontal point of inflexion on a rising curve

D. horizontal point of inflexion on a falling curve

Question 5

Which expression does not correctly evaluate the shaded area for the odd function below?



A. $-2\int_0^2 f(x)\,dx$

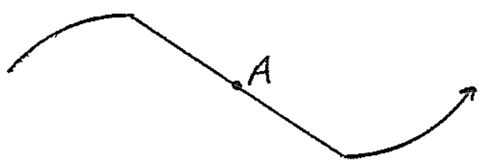
B. $-2\int_0^{-2} f(x)\,dx$

C. $2\int_{-2}^0 f(x)\,dx$

D. $2\int_0^2 f(x)\,dx$

Question 6 (8 marks) Start on a new page.

a) Select two statements that are true for the point A shown on the function below? 1



1. $f'(x) < 0$ 2. $f'(x) > 0$ 3. $f'(x) = 0$

4. $f''(x) < 0$ 5. $f''(x) > 0$ 6. $f''(x) = 0$

b) A curve has gradient function $x + 5$. If the curve passes through (4,0) :

i) find the equation of the curve. 1

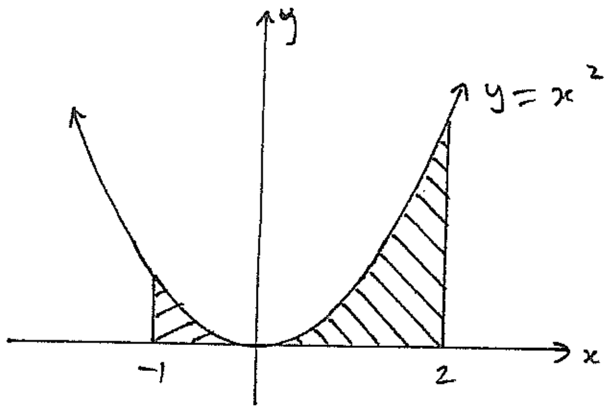
ii) find the equation of the tangent to the curve at this point. 1

c) Find :

i) $\int (2x+5)^6 \, dx$ 1

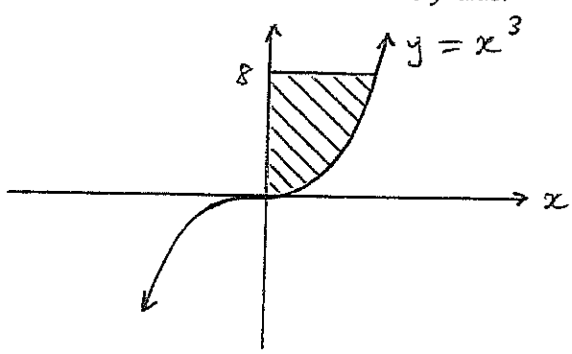
ii) $\int \frac{(x-1)(x+1)}{x^2} \, dx$ 2

d) Find the exact shaded area below : 2



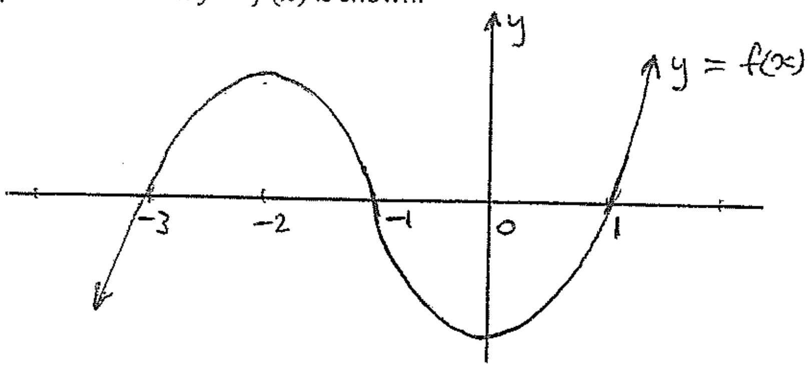
Question 7 (10 marks) Start on a new page.

a) Find the shaded area with reference to the y-axis. 3



b) Find the shaded area in b) above using an alternative method. 2

c) The graph of a function $y = f(x)$ is shown.



i) For which x values does the curve have :

$\alpha)$ $f'(x) > 0$? 1

$\beta)$ $f''(x) < 0$? 1

ii) On a separate diagram, neatly sketch a possible graph of $y = f'(x)$. 2

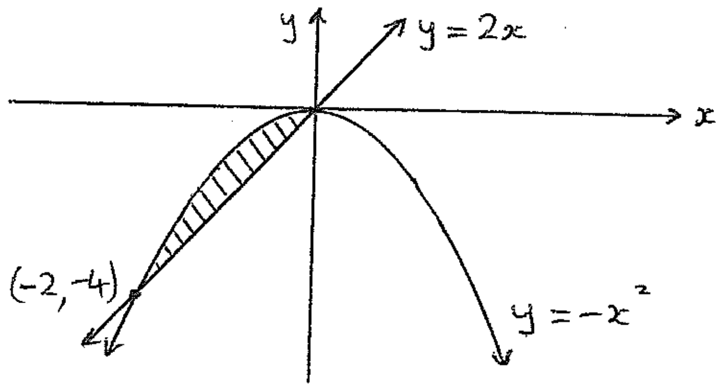
Show relevant x values.

iii) On a separate diagram, neatly sketch a possible graph of $y = f''(x)$. 1

Show relevant x values.

Question 8 (9 marks) Start on a new page.

a) Find the shaded area between the two curves as shown. 3



b) Twenty-five kangaroos were released on an island. The population P of kangaroos on the island t years after release is given by $P = -t^3 + 6t^2 + 25$, for $0 \leq t \leq 6$.

i) After how many years was the population a maximum? 2

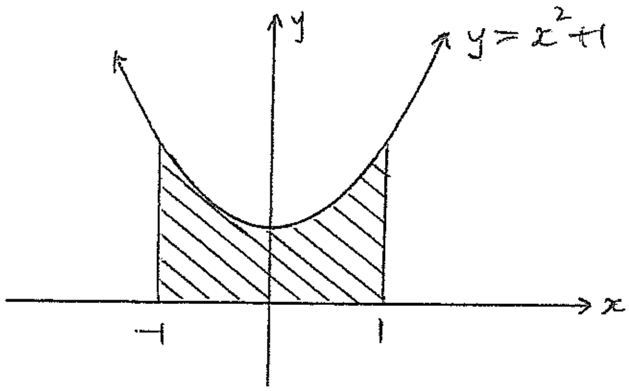
ii) Determine that the graph of P has a point of inflexion. 2

iii) Sketch the curve $P = -t^3 + 6t^2 + 25$, for $0 \leq t \leq 6$. 2

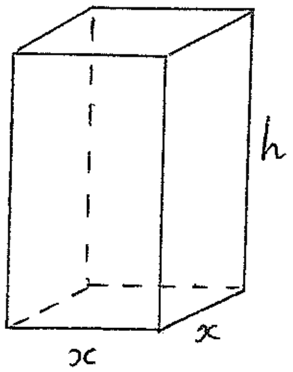
Question 9 (8 marks) Start on a new page.

a) The shaded area below is rotated about the x-axis. 3

Find, in terms of π, the volume of the generated solid.



b) A closed square prism box has width x cm, length x cm and height h cm as shown.



The sum of width + length + height is to remain constant at 20 cm.

i) Express h in terms of x. 1

ii) Show that the box has surface area A given by $A = 80x - 6x^2$ 1

iii) Find, and prove, the maximum surface area of the box. 3

Question 10 (10 marks) Start on a new page.

a) The curve $y = ax^3 + x^2 + bx$ has a horizontal point of inflexion when $x = 1$. 2

Find the values of a and b.

b) A new car has a cash price of $50,000 but is bought using time payment.

The interest rate is 9% p.a. and monthly repayments R are made for 4 years.

A repayment R is made immediately after that month's interest is charged.

Let A_n be the amount still owing on the loan after n months.

i) Write a simple expression for A_1 1

ii) Show that $A_2 = 50000(1.0075)^2 - R(1 + 1.0075)$ 1

iii) Write an expression for A_{48} . Simplify your answer. 2

iv) Show that $R = \$1244$ (to the nearest dollar). 1

v) Find the amount still owing after 24 months (to the nearest dollar). 1

vi) After 24 months, the car buyer increases her repayments to $2200 per month in order to pay off the loan more quickly. 2

Using v) above, find how many remaining months n are now required to pay off the loan. Give your answer to the nearest whole value.

END OF TEST

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}}\,dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}}\,dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

2U SOLUTIONS (2015 – TASK 2)

① C ② A ③ B ④ C ⑤ D

⑥ a) 1 and 6

b) i) $y = \frac{x^2}{2} + 5x + c$

$x = 4, y = 0 \Rightarrow 0 = 8 + 20 + c$

$c = -28$

$\therefore y = \frac{x^2}{2} + 5x - 28$

ii) $m_T = 9 \Rightarrow$ tangent is $y - 0 = 9(x - 4)$

$\therefore y = 9x - 36$

c) i) $\frac{(2x+5)^7}{14} + c$

ii) $\int \frac{x^2 - 1}{x^2}\,dx = \int (1 - x^{-2})\,dx$

$= x + \frac{1}{x} + c$

d) $A = \int_{-1}^{2} x^2\,dx$

$= \left[\frac{x^3}{3}\right]_{-1}^{2}$

$= \frac{8}{3} - \left(-\frac{1}{3}\right)$

$= 3\ u^2$

7) a) $x = y^{\frac{1}{3}}$

$$A = \int_0^8 y^{\frac{1}{3}}\,dy$$

$$= \left[\frac{3}{4} y^{\frac{4}{3}}\right]_0^8$$

$$= \frac{3}{4}\left(8^{\frac{4}{3}} - 0\right)$$

$$= \frac{3}{4} \times 16$$

$$= 12\ u^2$$

b)

$$A = (8 \times 2) - \int_0^2 x^3\,dx$$

$$= 16 - \left[\frac{x^4}{4}\right]_0^2$$

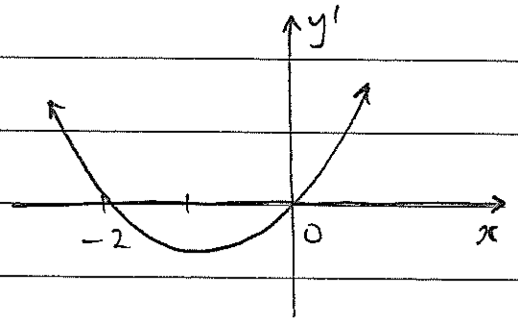
$$= 16 - (4 - 0)$$

$$= 12\ u^2$$

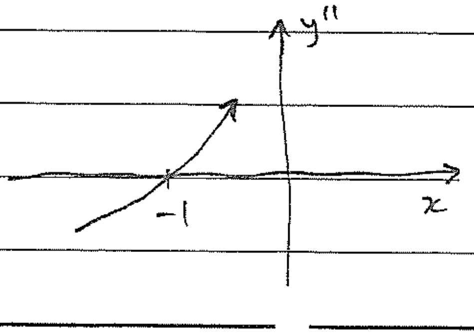
c) i) α) $x < -2,\ x > 0$

β) $x < -1$

ii)



iii)



8)

a) $A = \left|\int_{-2}^{0} (2x + x^2)\,dx\right|$

$$= \left|\left[x^2 + \frac{x^3}{3}\right]_{-2}^{0}\right|$$

$$= \left|0 + 0 - \left(4 - 2\tfrac{2}{3}\right)\right|$$

$$= 1\tfrac{1}{3}\ u^2$$

b) i) max P when $\frac{dP}{dt} = 0$

$$\frac{dP}{dt} = -3t^2 + 12t = 0$$

$$-3t(t-4) = 0$$

$$\therefore t = 0, 4$$

Test: $P'' = -6t + 12$

When $t = 0$, $P'' > 0$

When $t = 4$, $P'' < 0$

$\therefore$ max. P. when $t = 4$ years

ii) P. of I. when $P'' = 0$ and changes sign

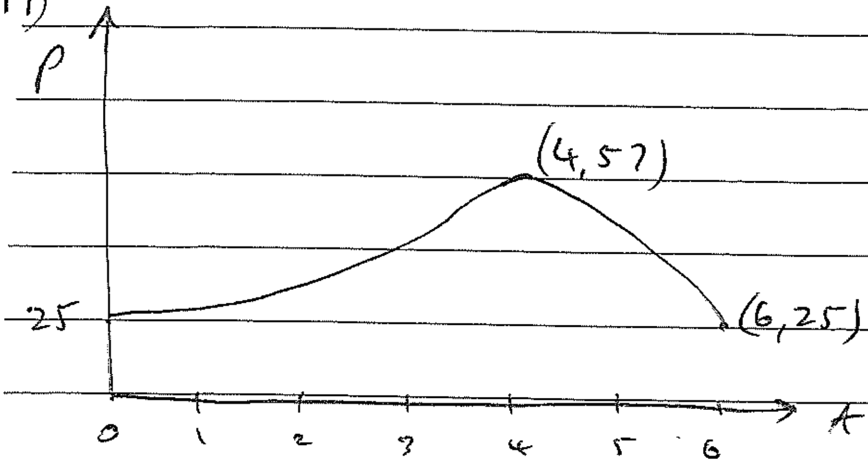
$$P'' = -6t + 12 = 0$$

$$\therefore t = 2$$

t	1.9	2	2.1
P''	+	0	−

$\therefore$ concavity changes

$\therefore$ P. of I. when $t = 2$ years iii)



(9) a)

$$V = 2\pi \int_0^1 (x^2+1)^2 \, dx$$

$$= 2\pi \int_0^1 (x^4 + 2x^2 + 1) \, dx$$

$$= 2\pi \left[\frac{x^5}{5} + \frac{2x^3}{3} + x \right]_0^1$$

$$= 2\pi \left[\left(\frac{1}{5} + \frac{2}{3} + 1 \right) - 0 \right]$$

$$= \frac{56\pi}{15} \, u^3$$

b) i) $x + x + h = 20$

$\therefore h = 20 - 2x$

ii) $A = 2x^2 + 4xh$

$= 2x^2 + 4x(20 - 2x)$

$= 2x^2 + 80x - 8x^2$

$= 80x - 6x^2$ as req'd.

iii) ~~minimum~~ A when $\frac{dA}{dx} = 0$

$\frac{dA}{dx} = 80 - 12x = 0$

$\therefore x = 6\frac{2}{3}$

and prove max : $A'' = -12 \ (<0)$

$\therefore$ max. Area is proved for $x = 6\frac{2}{3}$

and $A_{max} = 80 \times 6\frac{2}{3} - 6 \times (6\frac{2}{3})^2$

$= 266\frac{2}{3} \ u^2$

(10) a) Given $f'(1) = 0$ and $f''(1) = 0$

Now, $f'(x) = 3ax^2 + 2x + b \implies 3a + 2 + b = 0$ ①

$f''(x) = 6ax + 2 \implies 6a + 2 = 0$

$\therefore a = -\frac{1}{3}$

Sub in ①, $\therefore b = -1$

b) i) $A_1 = 50000 \times 1.0075 - R$

ii) $A_2 = A_1 \times 1.0075 - R$

$= (50000 \times 1.0075 - R) \times 1.0075 - R$

$= 50000 \times 1.0075^2 - 1.0075R - R$

$= 50000 \times 1.0075^2 - R(1 + 1.0075)$ as reqd.

iii) $A_{48} = 50000 \times 1.0075^{48} - R(1 + 1.0075 + 1.0075^2 + \ldots + 1.0075^{47})$

$= 50000 \times 1.0075^{48} - \frac{R(1.0075^{48} - 1)}{0.0075}$

iv) $A_{48} = 0 \implies \frac{R(1.0075^{48} - 1)}{0.0075} = 50000 \times 1.0075$

$\therefore R = \frac{50000 \times 1.0075^{48} \times 0.0075}{1.0075^{48} - 1}$

$= \$1244$ as reqd.

v) owing $= A_{24} = 50000 \times 1.0075^{24} - \frac{1244(1.0075^{24} - 1)}{0.0075}$

$= \$27242$

vi) $2200 = \frac{27242 \times 1.0075^n \times 0.0075}{1.0075^n - 1}$ (from iv)

$2200 \times 1.0075^n - 2200 = 204.315 \times 1.0075^n$

$1995.685 \times 1.0075^n = 2200$

$\therefore 1.0075^n \doteqdot 1.10$

$\therefore n \doteqdot 13$ months to pay off the loan.