TRINITY GRAMMAR SCHOOL
MATHEMATICS DEPARTMENT

YEAR 12

1998 APRIL SCHOOL EXAMINATIONS

MATHEMATICS

2/3 UNIT (COMMON)

Time Allowed - Three Hours
(Plus 5 minutes reading time)

DIRECTIONS TO STUDENTS:

1. Attempt ALL questions.
2. Show all necessary working.
3. Begin each question on a new sheet of paper.
4. Place your name, class and teacher at the top of each page.
5. Mark values are shown at the side of each question.
6. Non-programmable calculators are permitted.
7. Standard integrals are printed on the last page.

Question 1. **Start each question on a new page.** **Marks**

a) (i) Write the basic numeral for 4.17×10^{-3}. 1

(ii) Express $\sqrt{0.003615}$ correct to 3 significant figures. 1

b) Simplify $\sqrt{12} + \sqrt{27} - \sqrt{3}$ 2

c) Solve: 2

$$x + 2y = 1$$
$$3x + y = 13$$

d) Find the exact value of $\sin 120^\circ + \cos 300^\circ$ 2

e) Solve: $|x - 4| > 2$ 2

f) The retail price of a T.V. is increased by 15% to \$1104.
What was the price before the increase? 2

Question 2. **Start each question on a new page.** **Marks**

a) Simplify $(x - y)^2 - (x + y)^2$ 2

b) Express $\dfrac{2}{3 + \sqrt{7}}$ with a rational denominator and simplify. 2

c) Factorise completely $8y^3 + 8$. 2

d) If $v^2 = u^2 - 2as$, find the value of s if $v = 3.26$, $u = 1.73$ and $a = -4.6$
(Give your answer correct to 2 decimal places.) 2

e) Find the value of $\lim\limits_{m \to -2} \dfrac{m + 2}{m^2 - 4}$ 2

f) Solve: $x + \dfrac{4 - x}{3} = 12$ 2

Question 3. Start each question on a new page. **Marks**

The points A, B, and C have co-ordinates (0,4), (2,0) and (1,-6) respectively.

a) Find the length of AB and the gradient of AB. 2

b) Show that the equation of the line l , drawn through C parallel to AB 2

is $2x + y + 4 = 0$.

c) Find the coordinates of D, the point where the line l intersects the x-axis. 2

d) M is the midpoint of AC. Find the coordinates of the point M. 1

e) Find the distance of the point A from the line l . 2

f) Find the area of the quadrilateral ABCD. 3

Question 4. Start each question on a new page. **Marks**

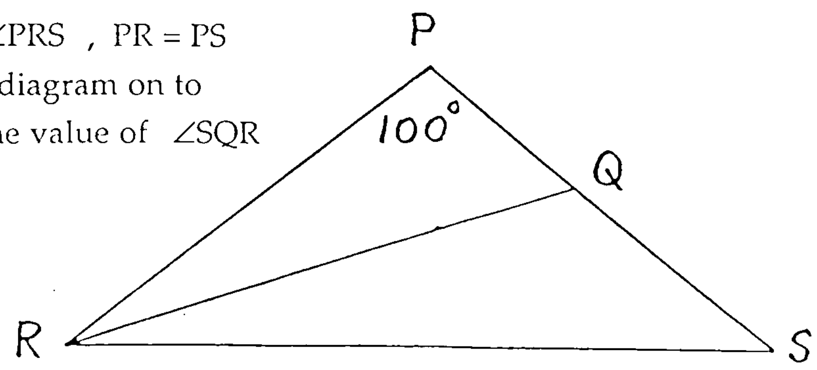
a) Show that the expression $2x^2 - 4x + 5$ is positive definite. 2

b) If α and β are the roots of the equation $3x^2 + 2x - 6 = 0$, find the value of:

(i) $\alpha^2\beta^2$ ii) $\alpha^2 + \beta^2$ 3

c) On a number plane shade the region satisfied simultaneously by $x + 2y \geq 2$ and $y \leq 4 - x^2$. 3

d) In the diagram QR bisects $\angle PRS$, PR = PS and $\angle RPS = 100°$. Copy the diagram on to your exam paper and find the value of $\angle SQR$ 2



e) Evaluate $\sum_{k=1}^{30}(2k-1)$ 2

Question 5. **Start each question on a new page.** **Marks**

a) Differentiate :

(i) $(x+3)(x^2-1)$ (ii) $\sqrt[3]{x}+3x$ 3

b) Solve $5^{2x-1}=1$ 2

c) In the triangle ABC, the length of AB is 14.1 cm; BC = 19.2 cm and $\angle BAC = 83°$.

i) Find the size of $\angle BCA$ correct to the nearest degree. 2

ii) Calculate the area of ΔABC. 2

d) Find the equation of the normal to the curve $y=\sqrt{x}$ at the point where x = 4. 3

Question 6. **Start each question on a new page.** **Marks**

a) State the domain and range of the function $y=\sqrt{x-3}$ 2

b) Simplify $(1-\sin^2\theta)(\tan^2\theta+1)$ 2

c) Solve the equation: $2\sin\theta+1=0$ for $0° \le \theta \le 360°$. 2

d) i) On the same coordinate axes draw the graphs of $y=\sin 2x$ and $y=\cos x$ for $0° \le x \le 360°$. 2

ii) Find the number of solutions to the equation $\sin 2x = \cos x$ in the domain $0° \le x \le 360°$. 1

e) If $f(x)=4x^3-3x^2+6x-1$, find $f''(x)$ and hence evaluate $f''(-1)$ - $f'(1)$. 3

Question 7. **Start each question on a new page.** **Marks**

a) Which term of the sequence -5, 3, 11, 19, . . . , is 283? 2

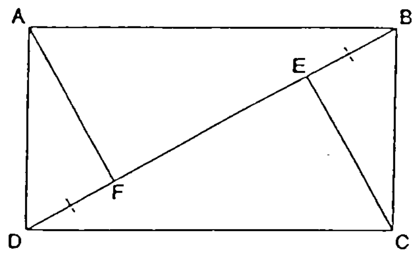
b) For the sequence $\frac{2}{3}+\frac{4}{9}+\frac{8}{27}+\ldots$ find :

i) the common ratio. 1

ii) the limiting sum . 1

c) Find the area enclosed by the curves $y=x+2$ and $y=x^2$. 4

d) ABCD is a rectangle. E and F are two points on the diagonal BD such that BE=DF.



Copy the diagram on to your exam paper. 4

i) Prove that $\Delta AFD \equiv \Delta CEB$ ii) Hence prove $\angle DAF = \angle BCE$.

Question 8. **Start each question on a new page.** **Marks**

a) Find the values of k for which the equation $x^2-(k+3)x+(k+6)=0$ has equal roots. 2

b) A battleship A is 230 km due west of a lighthouse L. It travels a distance of 110 km on a bearing of N62°E to a position C. Calculate the distance from the lighthouse to the battleship's position at C to the nearest kilometre. 3

c) For the curve $y=2x^3-6x^2-18x+1$ 7

(i) Find the stationary points and determine their nature.

(ii) Find the coordinates of any points of inflexion.

(iii) For what values of x is the curve increasing?

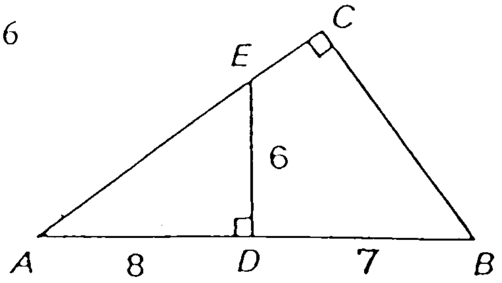
(iv) Sketch the curve in the domain $-2 \le x \le 5$.

(v) For what values of x is the curve concave down?

Question 9. **Start each question on a new page.** **Marks**

a) Find: i) $\int \frac{x^3+6}{x^2}\,dx$ ii) $\int_3^5 (2x+1)\,dx$ 4

b) $ED \perp AB$, $BC \perp AC$, $AD = 8$, $BD = 7$, $DE = 6$



i) Prove $\Delta ABC \,|||\, \Delta AED$ 2

ii) Hence or otherwise, find the lengths of BC and EC. 3

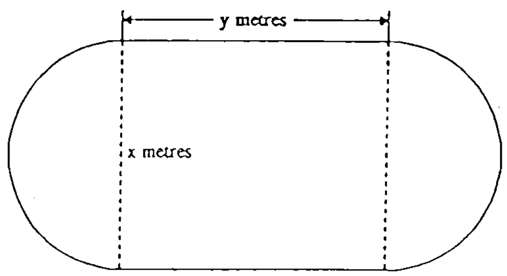
c) If $A(x-1)(x-2) + B(x-1) + C \equiv 2x^2 - 3x + 5$, find A, B, and C. 3

Question 10. **Start each question on a new page.** **Marks**

a) The gradient function of a curve is given by $\frac{dy}{dx} = 2x - \frac{4}{x^2}$. The curve passes through the point (2,3). Find the equation of the curve. 3

b) For what values of x is $x^2 \geq (x+1)(x+2)$. 2

c) A railway enthusiast designs a miniature railway of length 1000 metres. The route consists of two semi-circles at opposite ends of a rectangle. 7



i) If the rectangle has a length of y metres and its width is x metres, show that $y = 500 - \frac{\pi x}{2}$.

ii) Show that the area, A, enclosed by the railway track is given by $A = \frac{2000x - \pi x^2}{4}$.

iii) Find the maximum area enclosed by the railway track.

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

Question one

A I) 0·00417 (1)

II) $0{\cdot}0601248 = 6{\cdot}01 \times 10^{-2}$ (1)

B) $\sqrt{4}\sqrt{3} + \sqrt{9}\sqrt{3} - \sqrt{3}$

$= 2\sqrt{3} + 3\sqrt{3} - \sqrt{3}$

$= 4\sqrt{3}$ (2)

c) $x + 2y = 1$

$x = 1 - 2y$ ①

$3x + y = 13$ ②

sub ① into ②

$3(1-2y) + y = 13$

$3 - 6y + y = 13$

$-5y = 10$

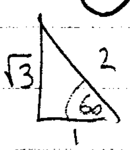
$y = -2$

sub $y = -2$ into ①

$x = 1 - 2(-2)$

$= 5$ (2)

$\therefore (5, -2)$



d) $\sin 120 + \cos 300$

$+ \sin 60 + \cos 60$

$\frac{\sqrt{3}}{2} + \frac{1}{2}$

$(\sqrt{3}+1)/2$ (2)

e) $|x - 4| > 2$

$x - 4 > 2$ $\quad -(x-4) > 2$

$x > 6$ $\quad x - 4 < -2$

$x < 2$ (2)

F) $115\% = 1104$

$1\% = 9{\cdot}6$

$100\% = 960$

∴ originally \$960

Question Two

A) $x^2 - 2xy + y^2 - (x^2 + 2xy + y^2)$

$x^2 - 2xy + y^2 - x^2 - 2xy - y^2$

$-4xy$ (2)

B) $\frac{2}{3+\sqrt{7}} \times \frac{3-\sqrt{7}}{3-\sqrt{7}} = \frac{6-2\sqrt{7}}{9-7}$

$= \frac{2(3-\sqrt{7})}{2}$

$= 3 - \sqrt{7}$ (2)

c) $8y^3 + 8 = 8(y^3 + 1)$

$= 8(y+1)(y^2 + y + 1)$ (2)

d) $V^2 = u^2 - 2as$

$3{\cdot}26^2 = 1{\cdot}73^2 - 2 \times {}^-4{\cdot}6 \times S$

$7{\cdot}6347 = 9{\cdot}2S$

$S = 0{\cdot}8298$

$= 0{\cdot}83$ (2)

e) $\lim_{m \to -2} \frac{(m+2)}{(m+2)(m-2)} = L \frac{1}{m-2}$

$= \frac{1}{-2-2}$

$= -\frac{1}{4}$ (2)

f) $x + \frac{4-x}{3} = 12$ $\quad (\times 3)$

$3x + (4 - x) = 36$

$2x = 32$

$x = 16$ (2)

Question Three

A) $d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$= \sqrt{(2-0)^2 + (0-4)^2}$

$= \sqrt{20}$

$= 2\sqrt{5}$ (1)

$m = (y_2 - y_1)/(x_2 - x_1)$

$= \frac{0 - {}^-4}{2 - 0}$

$= -2$ (1)

B) $y - y_1 = m(x - x_1)$

$y - {}^-6 = -2(x - 1)$

$y + 6 = -2x + 2$

$2x + y + 4 = 0$ (2)

c) cuts x-axis when $y = 0$

$2x + 0 + 4 = 0$

$x = -2$

D ∴ $(-2, 0)$ (2)

D) $MP = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$M = \left(\frac{0+1}{2}, \frac{4-6}{2}\right)$

$= \left(\frac{1}{2}, -1\right)$ (1)

E) $d = \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}}\right|$

$= \left|\frac{2(0) + 4 + 4}{\sqrt{2^2 + 1^2}}\right|$

$= \frac{8}{\sqrt{5}}\ u$

$= \frac{8\sqrt{5}}{5}$ (2)

F) $DC = \sqrt{(-2-1)^2 + (0 - {}^-6)^2}$

$DC = \sqrt{45}$

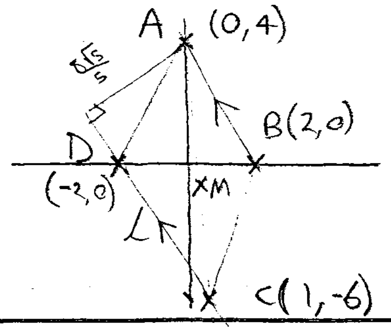
$= 3\sqrt{5}$

$A = \frac{h}{2}(a+b)$

$= \frac{8}{2\sqrt{5}} \times (2\sqrt{5} + 3\sqrt{5})$

$= \frac{40\sqrt{5}}{2\sqrt{5}}$

$= 20\ u^2$ (3)



Question 4

A) $\Delta = b^2 - 4ac$

$= (-4)^2 - 4(2)(5)$

$= 16 - 40$

$= -24 \quad a > 0$

∴ positive definite ②

B) I) $\alpha^2\beta^2 = (\alpha\beta)^2$

$= \left(\frac{c}{a}\right)^2$

$= \left(\frac{-6}{3}\right)^2$

$= 4$ ①

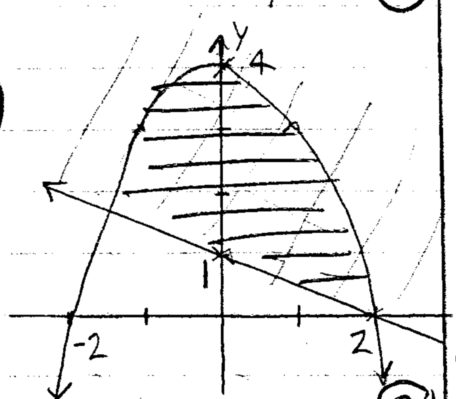
II) $(\alpha^2+\beta^2) = (\alpha+\beta)^2 - 2\alpha\beta$

$= \left(\frac{-b}{a}\right)^2 - 2\times -2$

$= \left(\frac{-2}{3}\right)^2 + 4$

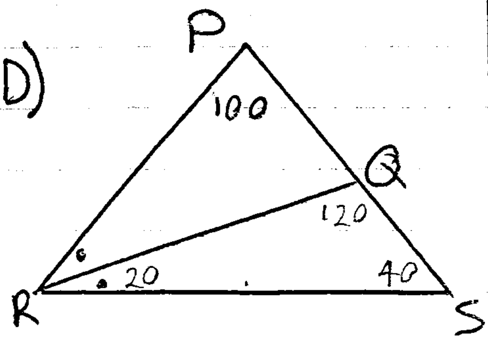
$= 4\frac{4}{9}$ ②

C)



③

D)



$\angle S = \angle PRQ = x$ ∠'s opp = sides are =

$2x + 100 = 180$ ∠ sum of △PRS

$x = 40°$

$\angle PRQ = \angle SRQ$ RQ bisects ∠PRS

$\angle QRS = 20°$

$\angle RQS + 20 + 40 = 180$ ∠ sum of △QRS

$\angle RQS = 120°$ ②

E) $1 + 3 + 5 + \ldots + 59$

$a = 1 \quad d = 2 \quad n = 30 \quad l = 59$

$S_n = \frac{n}{2}(a + l)$

$= \frac{30}{2}(1 + 59)$

$= 900$ ②

Question Five

A I) $y = (x+3)(x^2-1)$

$= x^3 + 3x^2 - x - 3$

$y' = 3x^2 + 6x - 1$ ①

II) $y = x^{\frac{1}{3}} + 3x$

$y' = \frac{1}{3}x^{-\frac{2}{3}} + 3$

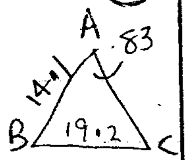
$= \frac{1}{3\sqrt[3]{x^2}} + 3$ ②

B) $5^{2x-1} = 1$

$5^{2x-1} = 5^0$

$2x - 1 = 0$

$x = \frac{1}{2}$ ②



C I) $\frac{\sin C}{c} = \frac{\sin A}{a}$

$\frac{\sin C}{14.1} = \frac{\sin 83}{19.2}$

$\sin C = \frac{14.1 \sin 83}{19.2}$

$C = 47°$ ②

II) $\angle B = 180 - 83 - 47$

$= 50°$

$A = \frac{1}{2}ac\sin B$

$= \frac{1}{2} \times 14.1 \times 19.2 \times \sin 50$

$= 103.69\text{cm}^2$ ②

D) $y = x^{\frac{1}{2}}$

$y' = \frac{1}{2}x^{-\frac{1}{2}}$

when $x = 4$

$y = 2 \qquad y' = \frac{1}{2} \times \frac{1}{\sqrt{4}}$

$m = \frac{1}{4}$

$y - 2 = \frac{1}{4}(x - 4)$

$4y - 8 = x - 4$

$x - 4y + 4 = 0$ ③

Question six

A) D: $x \geq 3$

R: $y \geq 0$ ②

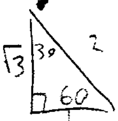
B) $(1 - \sin^2\theta)(\tan^2\theta + 1)$

$= \cos^2\theta \times \sec^2\theta$

$= \frac{\cos^2\theta}{1} \times \frac{1}{\cos^2\theta}$

$= 1$ ②

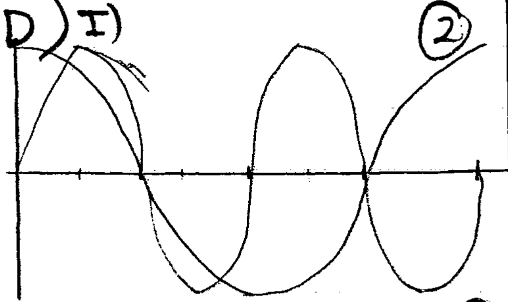
C) $2\sin\theta + 1 = 0$



$\sin\theta = -\frac{1}{2}$

$\theta = 210°, 330°$ ②

D) I) ②



II) 5 solutions ①

E) $f(x) = 4x^3 - 3x^2 + 6x - 1$

$f'(x) = 12x^2 - 6x + 6$

$f''(x) = 24x - 6$

$f''(-1) = 24(-1) - 6$

$= -30$

$f'(+1) = 12(+1)^2 - 6(+1) + 6$

$= 12 - 6 + 6$

$= 12$

$f''(-1) - f'(1) = -30 - 12$

$= -42$ ③

Question Seven

A) $a = -5 \quad d = 8 \quad n = ?$

$T_n = a + (n-1)d$

$283 = -5 + (n-1) \times 8$

$288 = (n-1) \times 8$

$n - 1 = 36$

$n = 37$ (2)

B) I) $\frac{4}{9} \div \frac{2}{3} = \frac{2}{3}$ (1)

II) $S_\infty = \frac{a}{1-r}$

$= \frac{2/3}{1 - 2/3}$

$= 2$ (1)

C) $x + 2 = x^2$

$0 = x^2 - x - 2$

$= (x-2)(x+1)$

$x = -1$ or 2

$A = \int_{-1}^{2} (x+2) - x^2 \, dx$

$= \int_{-1}^{2} x + 2 - x^2 \, dx$

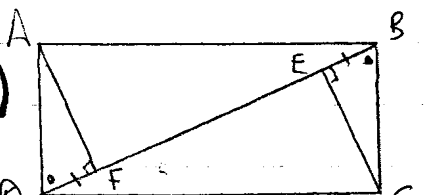
$= \left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_{-1}^{2}$

$= \left(\frac{4}{2} + 4 - \frac{8}{3}\right) - \left(\frac{1}{2} - 2 - \frac{-1}{3}\right)$

$= 3\frac{1}{3} - -1\frac{1}{6}$

$= 4\frac{1}{2} u^2$ (4)

D)



I) $DF = BE$ Given

$\angle ADF = \angle EBC$ alt ∠'s $AD \parallel BC$

$AD \parallel BC$ opp sides of rect. are ∥

$AD = BC$ opp sides of rect. are =

$\therefore \triangle AFD \equiv \triangle CEB$ SAS (3)

II) $\angle DAF = \angle BCE$ corresp ∠'s of ≡ △'s (1)

Question Eight

A) $\Delta = b^2 - 4ac = 0$

$(k+3)^2 - 4.1(k+6) = 0$

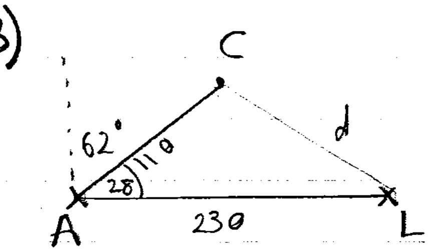
$k^2 + 6k + 9 - 4k - 24 = 0$

$k^2 + 2k - 15 = 0$

$(k-5)(k+3) = 0$

$k = 5$ or -3

B)



$d^2 = c^2 + l^2 - 2cl\cos A$

$d^2 = 110^2 + 230^2 - 2 \times 110 \times 230 \times \cos 28$

$d^2 =$

$d =$ (3)

C) I) $y = 2x^3 - 6x^2 - 18x + 1$

$y' = 6x^2 - 12x - 18$

$y'' = 12x - 12$

stat. point when $y' = 0$

$0 = 6(x^2 - 2x - 3)$

$0 = (x-3)(x+1)$

$x = -1$ or 3

Test using y'' or $f''(x)$

$F''(-1) = 12(-1) - 12 \qquad F''(3) = 12 \times 3 - 12$

$= -24 \qquad = 24$

$\therefore$ max $\qquad \therefore$ min

Find y-values

$F(-1) = 2(-1)^3 - 6(-1)^2 - 18(-1) + 1$ (2)

$= 11$

$\therefore$ max at $(-1, 11)$

$F(3) = 2(3)^3 - 6(3)^2 - 18(3) + 1$

$54 - 54 - 54 + 1$

$= -53$

$\therefore$ min at $(3, -53)$

II) Pt of inflex when $y'' = 0$

$0 = 12x - 12$

$x = 1$

Test			
x	-1	1	3
y''	-24	0	24

$\therefore$ change in concavity

$F(1) = 2(1)^3 - 6(1)^2 - 18(1) + 1$

$2 - 6 - 18 + 1$

$= -21$ (1)

$\therefore$ Pt of inflex at $(1, -21)$

III) increasing when $y' > 0$

$\therefore x < -1, \; x > 3$ (1)

IV) $F(-2) = 2(-2)^3 - 6(-2)^2 - 18(-2) + 1$

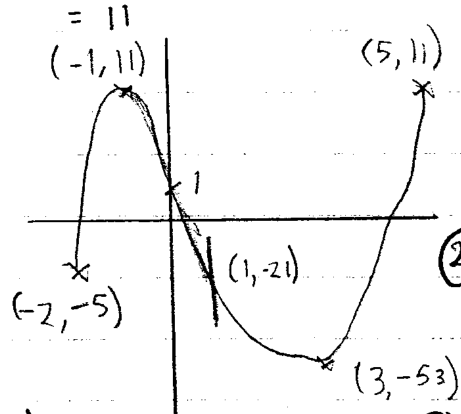
$-16 - 24 + 36 + 1$

$= -5$

$F(5) = 2(5)^3 - 6(5)^2 - 18(5) + 1$

$250 - 150 - 90 + 1$

$= 11$



(2)

V) $x < 1$ (1)

Question 9

A I) $\int \frac{x^3 + 6}{x^2} \, dx = \int x + 6x^{-2} \, dx$

$= \frac{x^2}{2} - \frac{6x^{-3}}{3} + C$

$= \frac{x^2}{2} - \frac{2}{x^3} + C$

II) $\int_3^5 (2x+1) \, dx = \left[x^2 + x\right]_3^5$

$= (5^2 + 5) - (3^2 + 3)$

$= 30 - 12$

$= 18$

Question nine

B) I) $\angle C = \angle ADE$ given
$\angle A$ is common (2)
$\therefore \triangle ABC \parallel\!\parallel \triangle AED$ AAA

II) $AE = 10$ pythagoras
$BC/AB = DE/AE$ corresp. sides of ||| Δ's
$BC/15 = 6/10$
$BC = 9$ (2)
$AC = 12$ pythag
$EC = 12 - 10$
$= 2$ (1)

c) $2x^2 - 3x + 5 \equiv$
$A(x-1)(x-2) + B(x-1) + C$ (3)
$\equiv Ax^2 - 3Ax + 2A + Bx - B + C$
$\equiv Ax^2 + x(B-3A) + (2A - B + C)$
$\therefore A = 2$ | $B - 3A = -3$ | $2A - B + C = 5$
| $B - 6 = -3$ | $4 - 3 + C = 5$
| $B = 3$ | $C = 4$

Question Ten

A) $y' = 2x - 4x^{-2}$
$y = x^2 + 4/x + C$
$3 = 2^2 + 4/2 + C$
$3 = 6 + C$
$C = -3$ (3)
$y = x^2 + \frac{4}{x} - 3$

B) $x^2 \geq (x+1)(x+2)$
$x^2 \geq x^2 + 3x + 2$
$0 \geq 3x + 2$
$-2 \geq 3x$
$-2/3 \geq x$ (2)
$x \leq -2/3$

C) I) $P = 2Y + 2\pi r$
$1000 = 2y + x\pi$
$2y = 1000 - x\pi$
$y = 500 - \pi x/2$ (1)

II) $A = xy + \pi r^2$
$A = x(500 - \pi x/2) + \pi\left(\frac{x}{2}\right)^2$
$= 500x - \pi x^2/2 + \pi x^2/4$
$= 500x - \pi x^2/4$
$\therefore = \frac{2000x - \pi x^2}{4}$ (2)

III) $A = 500x - \pi x^2/2$
$A' = 500 - \pi x$
$A'' = -\pi$

Let $A' = 0$ for max
$0 = 500 - \pi x$
$\pi x = 500$
$x = 500/\pi$

As $A'' < 0$ area is max

$A = 500 \times \frac{500}{\pi} - \frac{\pi}{2} \times \left(\frac{500}{\pi}\right)^2$
$= 250000/\pi - 250000\,\frac{\pi}{2\pi^2}$
$= 250000/2\pi$ m^2 (4)