Trinity 2007. - Half yearly

Question 1 (Start a new page) **12 marks**

(a) Simplify : $\frac{2}{3}-\frac{x-1}{2}$ **2**

(b) Express $\frac{1}{1-2\sqrt{3}}$ in the form $a+b\sqrt{3}$, where a and b are rational numbers. **2**

(c) Find the values of x that satisfy the inequality $9-2x<17$ **2**

Integration

(d) Find a primitive function of $6x-x^{-\frac{1}{2}}$ **2**

(e) If $\tan x=\frac{2}{3}$ find the exact value of $\cos x$ if x is acute. **2**

(f) Solve $2^{3x}=3$ correct to 2 decimal places. **2**

Question 2 (start a new page) **12 marks**

(a) Let α and β be the roots of the equation
$$x^2 - 5x + 2 = 0$$

Find the values of the following:

i) $\alpha + \beta$ **1**

ii) $\alpha\beta$ **1**

iii) $(\alpha + 1)(\beta + 1)$ **2**

(b) For what values of m will the equation $x^2 - mx + (8 + m) = 0$
Have:

i) Real and unequal roots? **2**

ii) Roots which are reciprocals of each other? **2**

(c) A parabola has equation $(x-2)^2 = -16(y-3)$

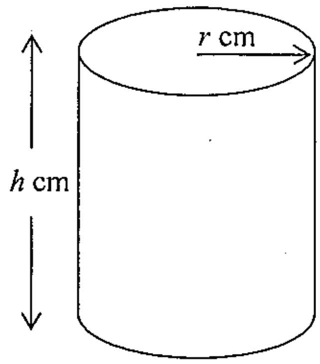
i) Find the coordinates of its vertex **1**

ii) Find the coordinates of its focus **1**

iii) Sketch the parabola clearly showing the position and equation of its directrix. **2**

Question 3 (start a new page) **12 marks**

(a) A can of baked beans is in the shape of a closed cylinder with height h cm and radius r cm, as shown in the diagram below. The volume of the can is 500cm^3.



i) Show that the surface area, $S\,\text{cm}^2$, of the can is given by **2**

$$S = 2\pi r^2 + \frac{1000}{r}$$

ii) If the surface area, S, of metal used to make the can is to be minimised, find the radius, r, of the can. **3**

(b) For the function $f(x) = x^3 - 3x^2 - 9x + 15$,

i) Show that $\dfrac{dy}{dx} = 3(x-3)(x+1)$ **1**

ii) Find the coordinates of any stationary points and determine their nature. **2**

iii) Find the coordinates of any point(s) of inflexion. **2**

iv) Sketch the curve $y = f(x)$ showing all important features. **2**

I ♡ Maths

Question 4 (start a new page) **12 marks**

(a) Differentiate the following with respect to x:

i) $\dfrac{1}{x^4}$ **2**

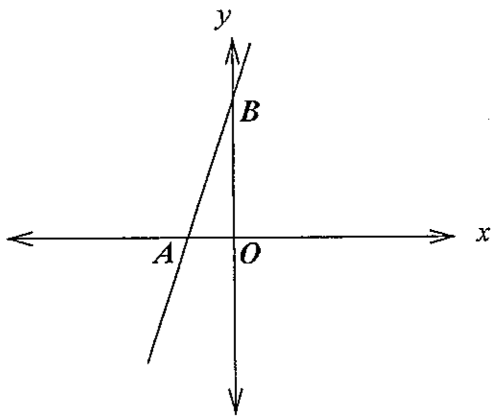
ii) $(3x^2+4)^5$ **2**

iii) $\dfrac{4-x}{1+x^2}$ **2**

(b) i) Show that the equation of the tangent to the curve $y=4-x^2$ at $x=-1$ has equation $2x-y+5=0$ **2**

ii) The tangent $2x-y+5=0$ cuts the x and y axis at A and B respectively. **2**

Calculate the exact area of ΔAOB where O is the origin.



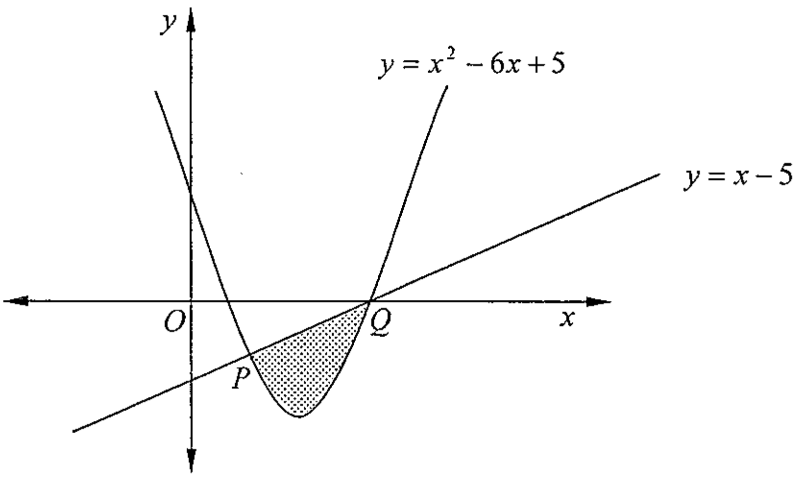
(c) Find the equation of the curve $g(x)$, given the curve passes through the point (-1, 2) and the gradient function is given by $g'(x)=3x^2-2x+1$. **2**

Question 5 (start a new page) **12 marks**

(a) Find $\int (1+2x)^3\,dx$ **1**

(b) Evaluate $\displaystyle\int_1^3 \frac{x^3+2x^2+x}{x}\,dx$ **2**

(c)



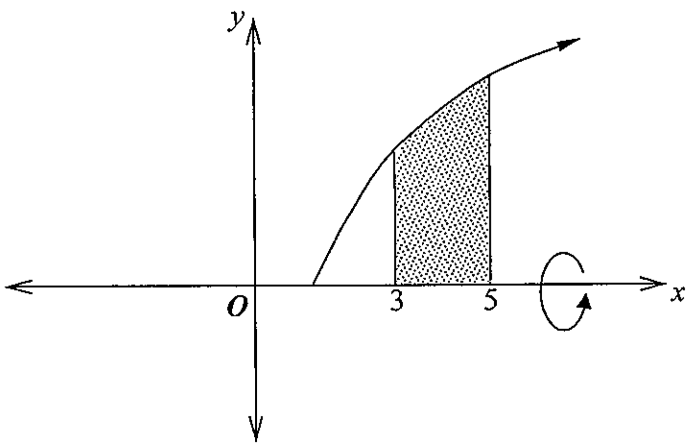
The diagram shows the graphs of the functions $y = x^2 - 6x + 5$ and $y = x - 5$ where P and Q are the points of intersection.

i) Find the x values of P and Q. uhh? **2**

ii) Calculate the exact area of the shaded region. **2**

Question 5 (continued)

(d) Find the volume of the solid generated when the area between the curve $y = \sqrt{x^2 - 3}$ and the x-axis bounded by the ordinates $x = 3$ and $x = 5$ is rotated about the x-axis. Write your answer in terms of π. **2**

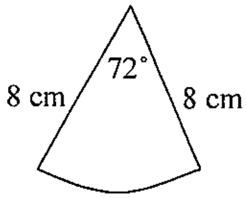


(e) Use the trapezoidal Rule with 3 subintervals to find an approximation for: **3**

$$\int_0^3 x(4-x)^2 \, dx$$

Question 6 (start a new page) **12 marks**

(a) The figure shows a sector of a circle with radius 8 cm. Find the area of the sector to correct 2 decimal places **2**



(b) Differentiate with respect to x:

i) $x\sin 2x$ **2**

ii) $4\tan\dfrac{x}{3}$ **2**

(c) Find $\int (1+\sin 2x)\,dx$ **2**

(d) Find the exact value of $\displaystyle\int_{\frac{\pi}{8}}^{\frac{\pi}{6}} \sec^2 2x \; dx$ **2**

(e) Find the equation of the normal to the curve $y = 3\sin 4x$ **2**

at the point $\left(\dfrac{\pi}{2}, 0\right)$

Question 7 (start a new page) **12 marks**

(a) Simplify the following: **2**

$$\log_9 25 - \log_3 5$$

(b) Differentiate the following:

i) $(\log 2x)^3$ **2**

ii) $xe^{2x} + x$ **2**

(c) Evaluate $\int \frac{x^3 - 1}{x^4 - 4x}\, dx$ **2**

(d) i) Sketch the graph $y = e^{2x}$ for $-2 \leq x \leq 2$ **2**

ii) Calculate the area under the curve $y = e^{2x}$ for $-2 \leq x \leq 2$ **2**

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Question one.

a) $\frac{2}{3} - \frac{(x-1)}{2}$.

$= \frac{4-3(x-1)}{6}$ 1

$= \frac{7-3x}{6}$. 1

b) $\frac{1}{1-2\sqrt{3}} \times \frac{1+2\sqrt{3}}{1+2\sqrt{3}}$ 1

$= \frac{1+2\sqrt{3}}{1-12}$.

$= \frac{1+2\sqrt{3}}{-11}$ $\therefore$ $a = \frac{-1}{11}$ $b = \frac{2}{-11}$ 1

c) $9 - 2x < 17$.

$-2x < 8$ 1

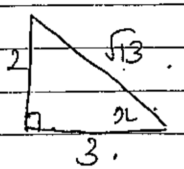
$x > -4$. 1

d) $\int 6x - x^{-\frac{1}{2}}\ dx$

$= \frac{6x^2}{2} - \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C$

$= 3x^2 - 2x^{1/2} + C$

e) $\tan x = \frac{2}{3}$.



$y^2 = 2^2 + 3^2$.

$y^2 = 4 + 9$

$y = \sqrt{13}$. 1

$\cos x = \frac{3}{\sqrt{13}}$. 1

f) $2^{3x} = 3$.

$\log 2^{3x} = \log 3$.

$3x \log 2 = \log 3$.

$3x = \frac{\log 3}{\log 2}$

$x = 0.53$.

or.

$\log_2 3 = 3x$.

1 $\frac{\log 3}{\log 2} = 3x$.

1 $0.53 = x$.

Question two.

a) $x^2 - 5x + 2 = 0$.

i) $\alpha + \beta = \frac{-b}{a}$

$= 5$ 1

ii) $\alpha\beta = \frac{c}{a}$

$= 2$ 1

iii) $(\alpha+1)(\beta+1)$

$= \alpha\beta + \alpha + \beta + 1$ 1

$= 5 + 2 + 1$

$= 8$. 1

b) $x^2 - mx + (8+m) = 0$.

i) For real and unequal roots

$\Delta > 0$.

$\Delta = b^2 - 4ac$

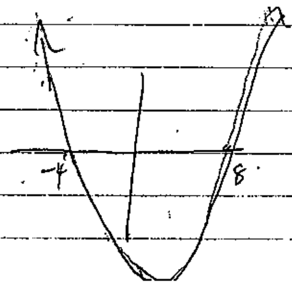
$= m^2 - 4(1)(8+m)$

$= m^2 - 32 - 4m$. 1

$m^2 - 4m - 32 > 0$.

$(m-8)(m+4) > 0$.

$m < -4$, $m > 8$. 1



ii) let the roots be α and $\frac{1}{\alpha}$.

$\therefore \alpha \times \frac{1}{\alpha} = \frac{c}{a}$. 1

$1 = \frac{8+m}{1}$

$m = -7$ 1

c) $(x-2)^2 = -16(y-3)$

i) vertex $(2,3)$ 1

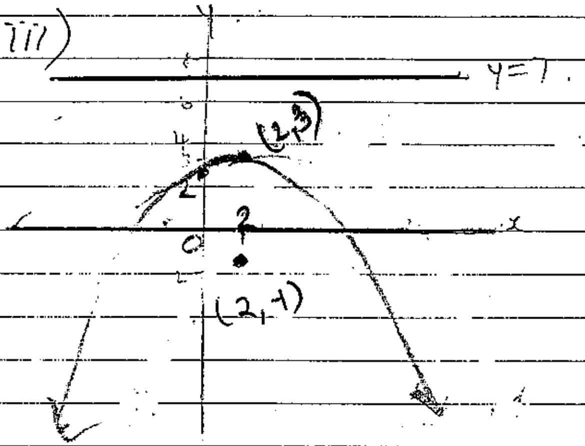
ii) Focal length $4a = 16$

$a = 4$

$\therefore$ Focus has co-ordinates:

$(2,-1)$ 1

iii)



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Question Three

a)

Volume of cylinder given by:

$V = \pi r^2 h$

$500 = \pi r^2 h$

$h = \frac{500}{\pi r^2}$. 1

$\therefore SA = 2\pi r^2 + 2\pi r h$

$= 2\pi r^2 + 2\pi r\left(\frac{500}{\pi r^2}\right)$

$= 2\pi r^2 + \frac{1000}{r}$. 1

ii) $SA = 2\pi r^2 + \frac{1000}{r}$.

$SA' = 4\pi r - \frac{1000}{r^2}$. 1

For minima $SA' = 0$.

$4\pi r - \frac{1000}{r^2} = 0$.

$4\pi r^3 - 1000 = 0$.

$4\pi r^3 = 1000$

$r^3 = 79.5774\ldots$

$r = 4.3$. 1

$SA'' = 4\pi + \frac{2000}{r^3}$.

at $r = 4.3$

$= 37.72$

> 0 $\therefore$ concave up 1

$\therefore r = 4.3$ gives a minimum Surface Are b)

$f(x) = x^3 - 3x^2 - 9x + 15$.

i) $f'(x) = 3x^2 - 6x - 9$
$= 3(x^2 - 2x - 3)$
$= 3(x-3)(x+1)$ 1

ii) stationary points occur when
$f'(x) = 0$.
$3(x-3)(x+1) = 0$
$x = 3, -1$
$\therefore (3, -12)$ $(-1, 20)$ 1

check $x = 3$

x	2	3	4
$f'(x)$	-ve	0	+ve

$\therefore x = 3$ is a minimum. 1/2

check $x = -1$

x	-2	-1	0
$f'(x)$	+ve	0	-ve

$\therefore x = -1$ is a maximum. 1/2

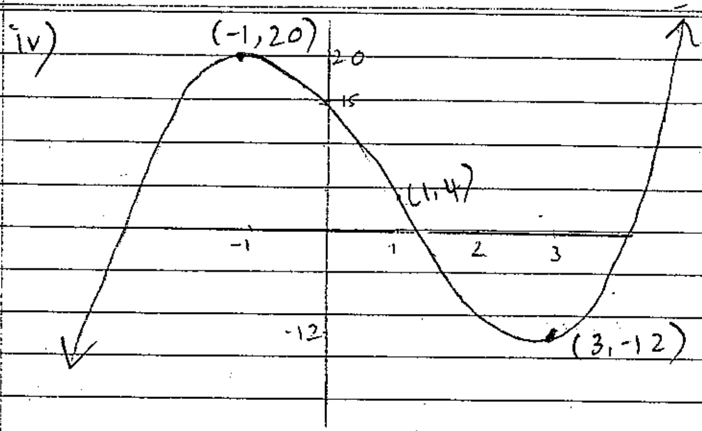
iii) inflexion points occur when $f''(x) = 0$

$f''(x) = 6x - 6$.
$6x - 6 = 0$.
$6x = 6$
$x = 1$.
$(1, 4)$ 1

x	0	1	2
$f''(x)$	-ve	0	+ve

Change of concavity
$\therefore (1, 4)$ is a point of inflexion 1

iv)



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Question four

a)

i) let $y = \dfrac{1}{x^4}$

$y = x^{-4}$ 1

$\dfrac{dy}{dx} = -4x^{-5}$ 1

$= \dfrac{-4}{x^5}$.

ii) let $y = (3x^2+4)^5$

$\dfrac{dy}{dx} = 5(6x)(3x^2+4)^4$ 1

$= 30x(3x^2+4)^4$. 1

iii) let $y = \dfrac{4-x}{1+x^2}$.

$u = 4-x$
$u' = -1$
$v = 1+x^2$
$v' = 2x$.

$\dfrac{dy}{dx} = \dfrac{vu' - uv'}{v^2}$

$= \dfrac{-(1+x^2) - 2x(4-x)}{(1+x^2)^2}$ 1

$= \dfrac{x^2 - 8x - 1}{(1+x^2)^2}$ 1

b)

i) $y = 4 - x^2$ when $x = -1$

$\frac{dy}{dx} = -2x$. $y = 3$.

at $x = -1$

$= 2$

gradient at $x = -1$ is 2. 1

$y - y_1 = m(x - x_1)$

$y - 3 = 2(x + 1)$

$y - 3 = 2x + 2$.

$2x - y + 5 = 0$. 1

ii) Area $= \frac{1}{2} bh$

Find co-ordinates A and B.

let $x = 0$.

$-y + 5 = 0$. B.

$y = 5$.

let $y = 0$.

$2x + 5 = 0$

$x = \frac{-5}{2}$. A.

Area $= 1/2 \times 5/2 \times 5$

$= \frac{25}{4}$ units^2 2

c) $g'(x) = 3x^2 - 2x + 1$

$\int 3x^2 - 2x + 1 \, dx$.

$= \frac{3x^3}{3} - \frac{2x^2}{2} + x + c$

$g(x) = x^3 - x^2 + x + c$. 1

$g(-1) = 2$.

$2 = (-1)^3 - (-1)^2 + (-1) + c$.

$c = 5$

$\therefore g(x) = x^3 - x^2 + x + 5$ 1

Question FIVE

a) $\int (1+2x)^3 \, dx$.

$= \frac{(1+2x)^4}{4 \times 2} + c$

$= \frac{(1+2x)^4}{8} + c$. 1

b) $\int_1^3 \frac{x^3 + 2x^2 + x}{x} \, dx$

$= \int_1^3 x^2 + 2x + 1 \, dx$ 1

$= \left[\frac{x^3}{3} + x^2 + x \right]_1^3$

$= \left[\frac{3^3}{3} + 3^2 + 3\right] - \left[\frac{1}{3} + 1 + 1\right]$

$= 18\,^2/_3$

c)

i) Solve simultaneously.

$x^2 - 6x + 5 = x - 5$

$x^2 - 7x + 10 = 0$

$(x-5)(x-2) = 0$

$x = 5, 2.$

ii) $\int_2^5 (x-5) - (x^2 - 6x + 5)\,dx$

$= \int_2^5 -x^2 + 7x - 10\,dx$

$= \left[\frac{-x^3}{3} + \frac{7x^2}{2} - 10x\right]_2^5$

$= \left[\frac{-5^3}{3} + \frac{7(5)^2}{2} - 10(5)\right] - \left[\frac{-2^3}{3} + \frac{7(2)^2}{2} - 10(2)\right]$

$= -4\,^1/_6 - (-8\,^2/_3)$

$= 4\,^1/_2$ units^2.

d) $V = \pi \int y^2\,dx$

$y = \sqrt{x^2 - 3}$

$y^2 = (\sqrt{x^2-3})^2$

$= x^2 - 3$

$V = \pi \int_3^5 x^2 - 3\,dx$

$= \pi \left[\frac{x^3}{3} - 3x\right]_3^5$

$= \pi \left[\frac{5^3}{3} - 15\right] - \left[\frac{3^3}{3} - 9\right]$

$= \frac{80\pi}{3}$ units^3.

e)

x	0	1	2	3
y	0	9	8	3
w	1	2	2	1
wy	0	18	16	3

$\Sigma wy = 37$

$\Sigma wy \times \frac{h}{2}$ $\quad h = \frac{1}{2}$

$37 \times \frac{1}{2}$

$= 18\frac{1}{2}$

Question six

a) change 72° into radians.

$\frac{72 \times \pi}{180} = \frac{2\pi}{5}$. 1

$\therefore$ Area $= \frac{1}{2} r^2 \theta$

$= \frac{1}{2} \times 8^2 \times \frac{2\pi}{5}$.

$= 40.21 cm^2$ 1

b)

i) let $y = x \sin 2x$.

$\frac{dy}{dx} = vu' + uv'$ 1

$= \sin 2x + 2x \cos 2x$. 1

ii) let $y = 4 \tan \frac{x}{3}$.

$\frac{dy}{dx} = \frac{4}{3} \sec^2 \frac{x}{3}$. 2.

c) $\int (1 + \sin 2x)\, dx$.

$= x - \frac{1}{2} \cos 2x + c$. 2.

d) $\int_{\pi/8}^{\pi/6} \sec^2 2x \, dx$

$= \left[\frac{1}{2} \tan 2x\right]_{\pi/6}^{\pi/6}$

$= \left[\frac{1}{2} \tan \frac{\pi}{3}\right) - \left[\frac{1}{2} \tan \frac{\pi}{4}\right]$ 1

$= \left[\frac{1}{2} \times \sqrt{3}\right] - \left[\frac{1}{2} \times 1\right]$.

$= \frac{\sqrt{3}}{2} - \frac{1}{2}$.

$= \frac{\sqrt{3} - 1}{2}$. 1

e) $y = 3 \sin 4x$.

$\frac{dy}{dx} = 12 \cos 4x$.

at $x = \frac{\pi}{2}$

$= 12 \cos 2\pi$

$= 12(1) = 12$.

12 is the gradient of the tangent to the curve.

The gradient of the normal is $\frac{-1}{12}$. 1

$\therefore y - y_1 = m(x - x_1)$

$y - 0 = \frac{-1}{12} \left(x - \frac{\pi}{2}\right)$

$12(y) = -x + \frac{\pi}{2}$.

$x + 12y - \frac{\pi}{2} = 0$. 1

$2x + 24y - \pi = 0$.

Question seven

a)

$\log_9 25 - \log_3 5 = \frac{\log 25}{\log 9} - \frac{\log 5}{\log 3}$ 1

$= \frac{\cancel{2}\log 5}{\cancel{2}\log 3} - \frac{\log 5}{\log 3}$

$= 0.$ 1

b)

i) let $y = (\log 2x)^3$

$\frac{dy}{dx} = 3 \times \left(\frac{2}{2x}\right)(\log 2x)^2$ 1

$= \frac{3}{x}(\log 2x)^2.$ 1

ii) let $y = xe^{2x} + x.$

$\frac{dy}{dx} = e^{2x} + 2xe^{2x} + 1$ 1

$= e^{2x}(1+2x) + 1.$ 1

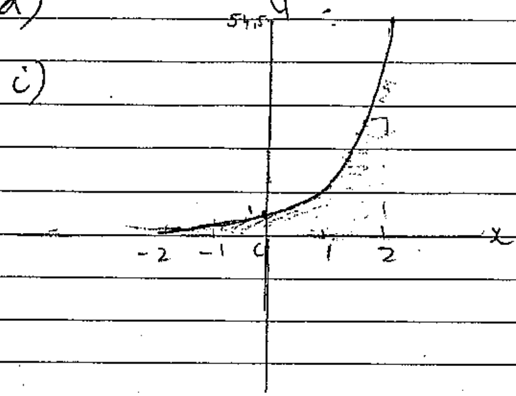
c) $\int \frac{x^3-1}{x^4-4x}\,dx.$

$= \frac{1}{4}\int \frac{4(x^3-1)}{x^4-4x}\,dx.$

$= \frac{1}{4}\log_e(x^4-4x) + c$ 2.

d)

i)



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ii) $\int_{-2}^{2} e^{2x}\,dx.$

$= \left[\frac{1}{2}e^{2x}\right]_{-2}^{2}$ 1

$= \left[\frac{1}{2}e^{4}\right] - \left[\frac{1}{2}e^{-4}\right]$ 1

$= \frac{1}{2}(e^4 - e^{-4})$