TRINITY GRAMMAR SCHOOL
MATHEMATICS DEPARTMENT

YEAR 12 2009 ASSESSMENT TASK 2

MATHEMATICS

(2 UNIT/EXTENSION 1)

Time Allowed – ***one hour***

(2 UNIT/EXTENSION 1)

Weighting 20% towards final result

Outcomes referred to: P2, P4, P5, P6, P7, P8, H1, H2, H4, H5, H6, H7, H8, H9

INSTRUCTIONS:

1. Attempt ALL questions.
2. Show all necessary working.
3. Begin each question on a new page.
4. Each question is of equal value. Mark values are shown beside each part.
5. Non-programmable silent Board of Studies approved calculators are permitted.
6. If requested, additional writing sheets may be obtained from the examinations supervisor upon request.
7. A double sided A4 page of notes is permitted to be referred to throughout this task.

BEGIN A **NEW QUESTION** ON A **NEW PAGE** **TOTAL: 58 MARKS**

Question 1: A(2,-2), B(-2,-3) and C(0,2) are the vertices of a triangle ABC.

(a)

(i)	Draw a sketch diagram of the triangle.	
(ii)	Find the length of AC and the gradient of AC.	2
(iii)	Find the equation of AC in the general form.	2
(iv)	Calculate the perpendicular distance of B from the side AC.	2
(v)	Hence, or otherwise, find the area of ΔABC.	1

(b) Find the co-ordinates of the point on the curve $y = 3x^2 - 2x + 1$ where the tangent is parallel to the straight line $4x - y - 1 = 0$. **3**

Question 2:

(a) For the curve $f(x) = x^4 - 4x^3$, find the values of x for which the curve is concave up. **2**

(b) Consider the curve $y = 3x^4 - 8x^3 + 6x^2$.

(i)	Find the co-ordinates of its stationary points.	3
(ii)	Use the second derivative to determine their nature.	3
(iii)	Find any points of inflexion.	2
(iv)	Sketch the curve for the domain $-1 \le x \le 2$	3

Question 3:

(a) Find the primitive functions of the following:

(i) $3x^5 - 2x^2 - x$ **2**

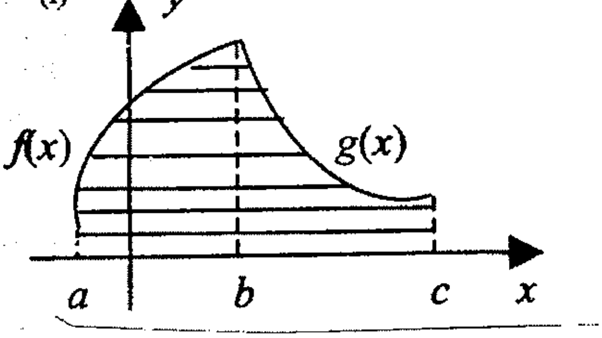
(ii) $3(3x-6)^4$ **2**

(iii) $\dfrac{2}{\sqrt{x}} + 2\sqrt{x} - 3$ **3**

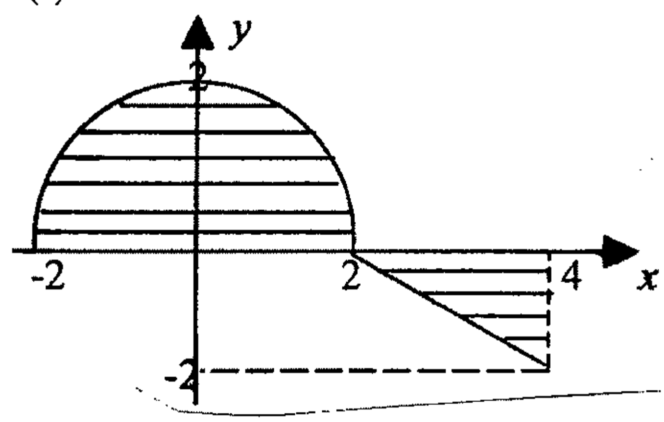
(b) Express y in terms of x, given that $\dfrac{d^2y}{dx^2} = 12(x-1)^2$ and that at $x=1$, $\dfrac{dy}{dx} = 3$ and $y = 2$. **3**

(c) Express the shaded area as the sum or difference of two integrals. DO NOT find the areas.

(i)



(ii) **4**



(d) Find the area enclosed between $y = x^3 - 6x^2 + 8x$ and the x-axis. **3**

Question 4:

(a) Find the volume of the solid generated when the semicircle $y = \sqrt{9 - x^2}$ is rotated about the x - axis. **3**

(b) (i) Show that the points A(0,3) and B(2,5) lie on the curve $y = 3 + \sqrt{2x}$. **2**

(ii) The part of the curve between A and B, is rotated about the y - axis. Find the volume of the solid generated. **4**

Question 5:

(a) Evaluate the definite integral $\int_0^4 x(\sqrt{x} + x^2)dx$. **3**

(b) A rectangular pyramid is inscribed in a cylinder. The diameter of the base of the cylinder is 12cm and its height is 18cm. The sides of the rectangles are a cm and b cm.

(i) Show that the volume of the pyramid is $6\sqrt{144a^2 - a^4}$ **2**

(ii) Show that the maximum volume of the pyramid and the volume of the cylinder are in the ratio $2:3\pi$. **4**

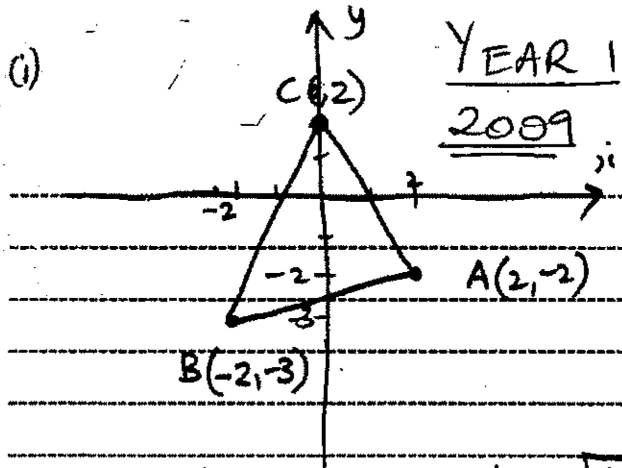
Q1 (i)

YEAR 12: MATHEMATICS TASK 2

2009

Student Number: SOLUTIONS

70



(ii) Length AC $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(2-0)^2+(-2-2)^2}$

$= \sqrt{4+16}$

$\boxed{d = \sqrt{20} \text{ units}}$ ✓ (1) $\sqrt{20} = 2\sqrt{5}$ $\therefore d = 2\sqrt{5}$

Gradient AC $m = \frac{y_2-y}{x_2-x_1}$

$= \frac{-2-2}{2-0}$

$= \frac{-4}{2}$

$\boxed{\therefore m = -2}$ ✓ (1)

(iii) Eqn AC $y - y_1 = m(x-x_1)$

$y - -2 = -2(x-2)$ (1) ✓

$y+2 = -2x+4$

$\boxed{2x+y-2=0}$ (1) ✓

(iv) $D_P = \left|\frac{Ax_1+By_1+C}{\sqrt{A^2+B^2}}\right|$ $A=2$, $B=1$, $C=-2$; Point $B(x_1, y_1) = (-2,-3)$

$= \left|\frac{2.-2+1.-3+-2}{\sqrt{2^2+1^2}}\right|$ (1) ✓

$= \left|\frac{-4-3-2}{\sqrt{5}}\right|$

$= \left|\frac{-9}{\sqrt{5}}\right|$

$= \frac{9}{\sqrt{5}}\ u^2.$ (1) ✓

(v) $A = \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times \sqrt{20} \times \frac{9}{\sqrt{5}}$ ✓

$= \frac{1}{2} \times 2\cancel{\sqrt{5}} \times \frac{9}{\cancel{\sqrt{5}}}$

$\boxed{A = 9u^2}$ (1)

7

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Student Number: ____________

Q1 (b) $y = 3x^2 - 2x + 1$

Eqn of tangent $\frac{dy}{dx} = 6x - 2$ straight line $y = 4x - 1$

here $m = 4$ (1)

$\therefore$ when $y' = 4$,
that is the point where the gradients are parallel.

$\therefore$ $4 = 6x - 2$ (1)

$x = 1$ sub into $y = 3x^2 - 2x + 1$.

$y = 3 - 2 + 1$
$= 2$ Point where tangent is parallel is $(1, 2)$ (1)

3

QUESTION 2:

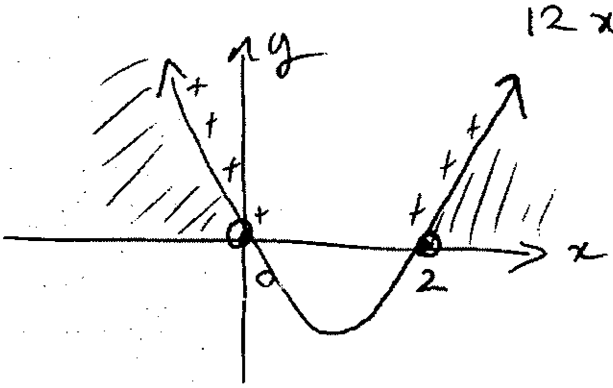
(a) $f'(x) = 4x^3 - 12x^2$

$f''(x) = 12x^2 - 24x$. 1

$\therefore$ For concave up, $f''(x) > 0$

$\therefore 12x^2 - 24x > 0$

$12x(x-2) > 0$ 1



$\therefore x > 2$

$x < 0$

(b) (i) $y = 3x^4 - 8x^3 + 6x^2$

$\frac{dy}{dx} = 12x^3 - 24x^2 + 12x$. 1

let $\frac{dy}{dx} = 0$ for st. pts.

$0 = 12x(x^2 - 2x + 1)$

$0 = 12x(x-1)(x-1)$

$\therefore x = 0, \ x = 1$

when $x = 0, \quad y = 0$ 1

$x = 1, \quad y = 1$ 1

(ii) $\dfrac{d^2y}{dx^2} = 36x^2 - 48x + 12$ 1

$\dfrac{d^2y}{dx^2} = 12(3x^2 - 4x + 1)$

Test $x = 0$, $\dfrac{d^2y}{dx^2} = 12$ 1 min.

Test $x = 1$, $\dfrac{d^2y}{dx^2} = 0$ 1 horizontal pt of inflex.

(iii) let $\dfrac{d^2y}{dx^2} = 0$

$0 = 12(3x - 1)(x - 1)$

$\therefore \; x = 1$ and $x = \frac{1}{3}$. 1

For $x = 1$

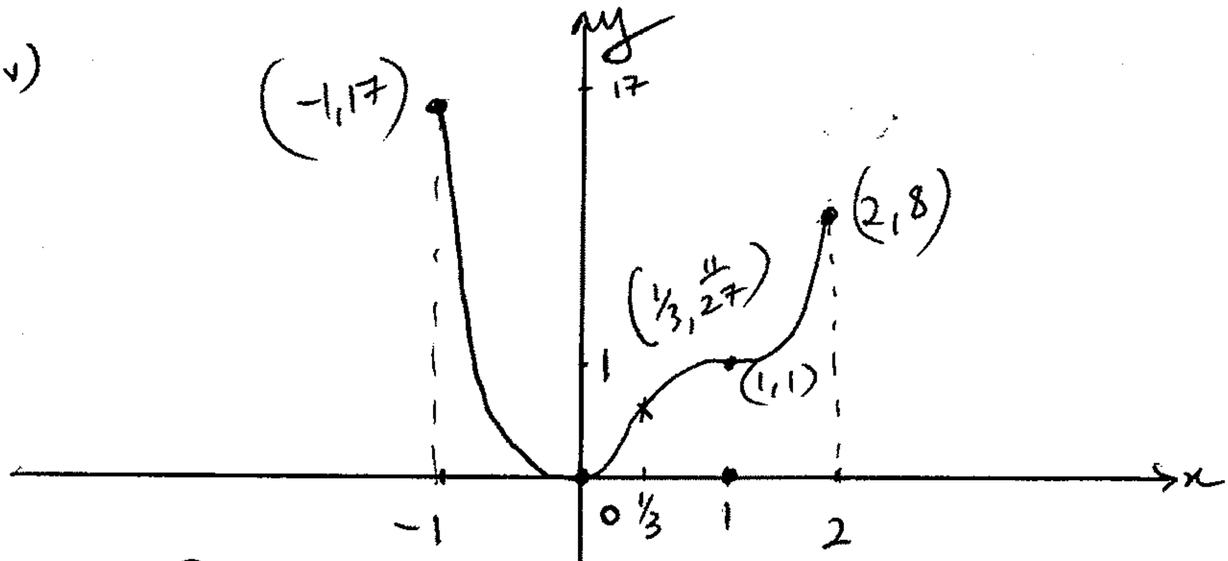
x	$\frac{1}{2}$	1	2
y''	$-$	0	$+$

1

For $x = \frac{1}{3}$

x	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$
y''	$+$	0	$-$

(iv)



end pts ①

showing st. pts ①

general shape showing pt of inflexion ①

QUESTION 3

(a) (i) $\dfrac{3x^6}{6} - \dfrac{2x^3}{3} - \dfrac{x^2}{2} + c$ ①

$= \dfrac{x^6}{2} - \dfrac{2}{3}x^3 - \dfrac{x^2}{2} + C$ ①

(ii) $\dfrac{3(3x-6)^5}{5 \times 3}$ ①

$= \dfrac{(3x-6)^5}{5} + C$ ①

(iii) $\frac{d}{dx} = 2(x)^{-\frac{1}{2}} + 2x^{\frac{1}{2}} - 3$ (1)

$$= \frac{2x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{2x^{\frac{3}{2}}}{\frac{3}{2}} - 3x + C \quad (1)$$

$$= 4\sqrt{x} + \frac{4x\sqrt{x}}{3} - 3x + C \quad (1)$$

(b) $$\frac{dy}{dx} = \frac{12(x-1)^3}{3} + c$$

$$\frac{dy}{dx} = 4(x-1)^3 + C \qquad \text{when} \begin{cases} x = 1 \\ \frac{dy}{dx} = 3 \end{cases}$$

$$3 = 4(1-1)^3 + c \quad (1)$$

$$3 = c$$

$$\frac{dy}{dx} = 4(x-1)^3 + 3 \quad (1)$$

$$y = \frac{\cancel{4}(x-1)^4}{\cancel{4}} + 3x + c \qquad \text{when} \begin{cases} x = 1 \\ y = 2 \end{cases}$$

$$2 = (1-1)^4 + 3 + c$$

$$-1 = c$$

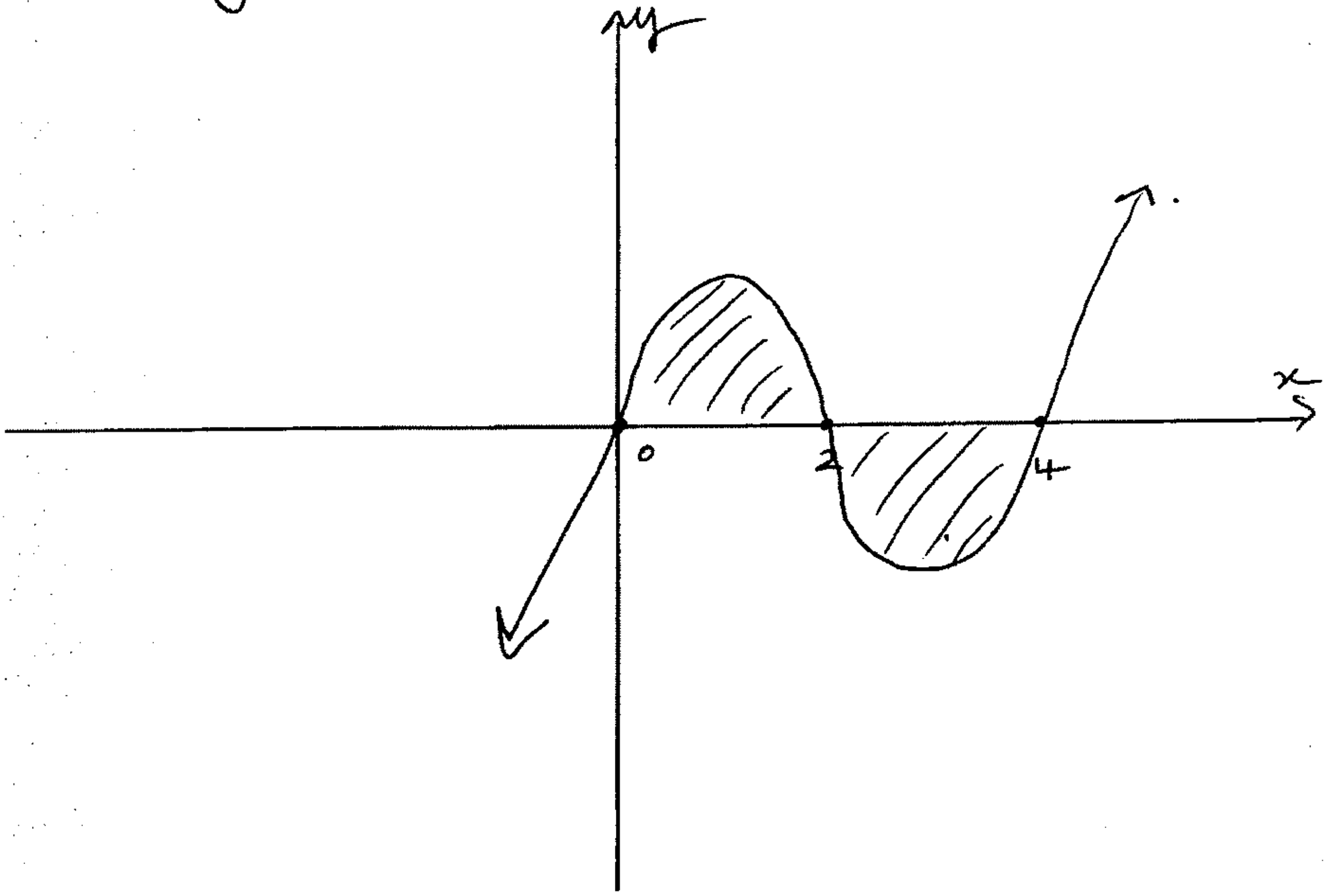
$$\therefore \quad y = (x-1)^4 + 3x - 1 \quad (1)$$

(c) (i) $A = \int_a^b f(x)\,dx + \int_b^c g(x)\,dx$ ① ①

(ii) $A = \int_{-2}^{2} \sqrt{4 - x^2}\,dx + \left| \int_2^4 -x + 2\,dx \right|$ ① ①

(d) $y = x(x^2 - 6x + 8)$

$y = x(x - 2)(x - 4)$



1 $$A = \int_0^2 x^3 - 6x^2 + 8x \, dx + \left| \int_2^4 x^3 - 6x^2 + 8x \, dx \right|$$

$$A = \left[\frac{x^4}{4} - \frac{\overset{2}{\cancel{6}}x^3}{\cancel{3}} + \frac{\overset{4}{\cancel{8}}x^2}{\cancel{2}} \right]_0^2 + \left| \left[\frac{x^4}{4} - \frac{\overset{2}{\cancel{6}}x^3}{\cancel{3}} + \frac{\overset{4}{\cancel{8}}x^2}{\cancel{2}} \right]_2^4 \right|$$

$$A = \left[\left(\frac{16}{4} - 2 \times 8 + 16 \right) - \left(0 \right) \right] + \left| \left[\left(\frac{256}{4} - \frac{128}{1} + 64 \right) - \left(\frac{16}{4} - 16 + 16 \right) \right] \right|$$

1

$$A = 4 + \left| \left[(0) - (4) \right] \right|$$

$$A = 4 + 4$$

$$A = 8 \text{ units}^2$$

1

QUESTION 4:

(a)

$$V = \pi \int_{-3}^{3} \left(\sqrt{9-x^2}\right)^2 dx \quad 1$$

$$V = \pi \int_{-3}^{3} 9-x^2 \, dx$$

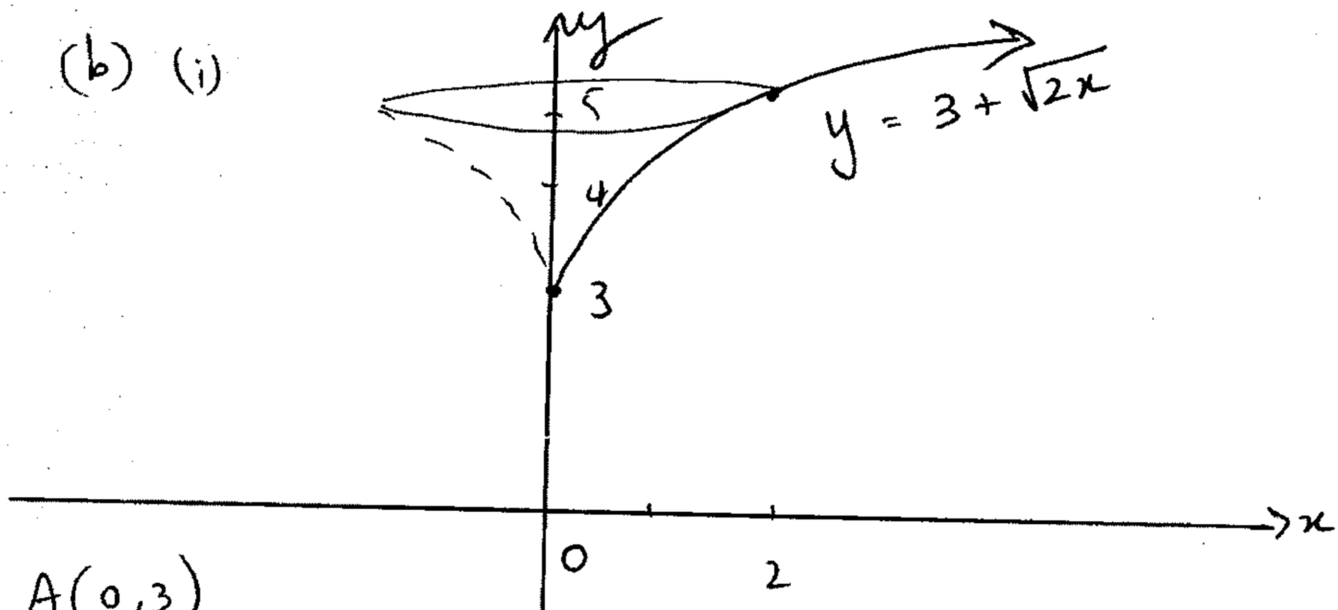
$$V = \pi \left[9x - \frac{x^3}{3} \right]_{-3}^{3} \quad 1$$

$$V = \pi \left[\left(27 - \frac{27}{3}\right) - \left(-27 + \frac{27}{3}\right) \right]$$

$$V = \pi \left[18 - (-18) \right]$$

$$V = 36\pi \text{ units}^3 \quad 1$$

(b) (i)



$A(0,3)$

$3 = 3 + \sqrt{0}$

$3 = 3$ ✓ (2)

$B(2,5)$

$5 = 3 + \sqrt{2 \times 2}$

$5 = 3 + 2$ ✓

$y - 3 = \sqrt{2x}$

$(y-3)^2 = 2x$

$\dfrac{(y-3)^2}{2} = x$

(ii) $$V = \pi \int_3^5 \frac{(y-3)^4}{4} \, dy$$ (1)

$$V = \pi \left[\frac{(y-3)^5}{20} \right]_3^5$$ (1)

$$V = \pi \left[\frac{2^5}{20} - \frac{0}{20} \right] = \pi \left[\frac{32}{20} \right] = \frac{8}{5}\pi \, u^3$$ (1)

QUESTION 5:

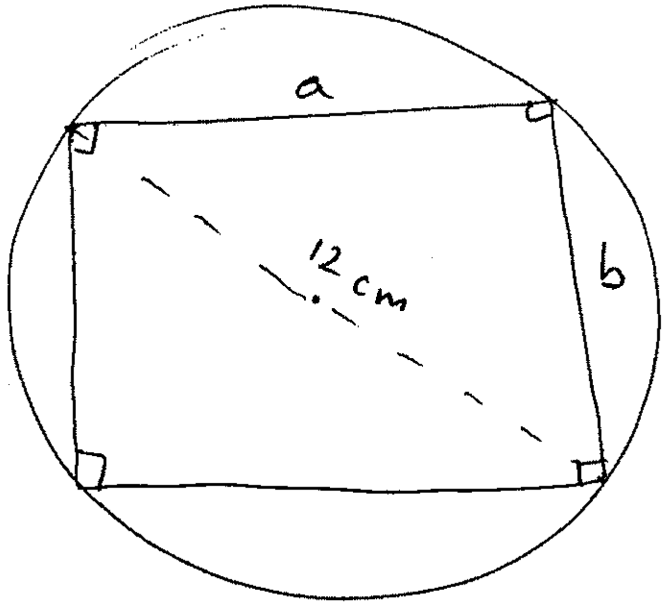
(a) $\displaystyle\int_0^4 x^{3/2} + x^3 \, dx$ (1)

$$= \left[\frac{2x^{5/2}}{5} + \frac{x^4}{4} \right]_0^4 \quad (1)$$

$$= \left[\left(\frac{64}{5} + 64 \right) - (0) \right]$$

$$= \frac{384}{5} \text{ OR } 76\frac{4}{5}. \quad (1)$$

(b) (i)



$$a^2 + b^2 = 144$$

$$b^2 = 144 - a^2$$

$$b = \sqrt{144 - a^2} \quad 1$$

$$V \text{ of pyramid} = \frac{a \times b \times 18}{3}$$

$$V = \frac{a \times \sqrt{144 - a^2} \times 18}{3}$$

$$V = 6a\sqrt{144 - a^2}$$

$$V = 6\sqrt{a^2(144 - a^2)} \quad 1$$

$$V = 6\sqrt{144a^2 - a^4}$$

(ii) $$\frac{dV}{da} = \frac{1}{2} \times 6\left(144a^2 - a^4\right)^{-\frac{1}{2}} \times \left(288a - 4a^3\right)$$

let $\frac{dV}{da} = 0$

$$0 = \frac{3\left(288a - 4a^3\right)}{\left(144a^2 - a^4\right)^{1/2}}$$

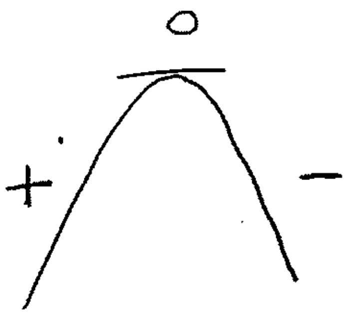
$$0 = 3\left(288a - 4a^3\right) \quad 1$$

$$0 = 4a\left(72 - a^2\right)$$

$$0 = 4a\left(6\sqrt{2} - a\right)\left(6\sqrt{2} + a\right)$$

$$\therefore \quad \boxed{a = 6\sqrt{2}} \quad 1$$

a	8	$6\sqrt{2}$	9
$\frac{dV}{da}$	$+$	0	$-$



1

$$V \text{ of pyramid} = \frac{6\sqrt{2} \times 6\sqrt{2} \times 18}{3}$$

$$= 432 \text{ cm}^3$$

V of cylinder $= \pi \times 6^2 \times 18$

$= 648\pi \ cm^3$

$\therefore \quad 432 : 648\pi$

$2 : 3\pi$. 1