TRINITY GRAMMAR SCHOOL
MATHEMATICS DEPARTMENT

YEAR 12 2010 ASSESSMENT TASK 2

MATHEMATICS

(2 UNIT/EXTENSION 1)

Time Allowed – *one hour*

WEIGHTING 20% towards final result

Outcomes referred to: P2, P4, P5, P6, P7, P8, H1, H2, H4, H5, H6, H7, H8, H9

INSTRUCTIONS:

1. Attempt **ALL** questions.
2. Show all necessary working.
3. **Begin** each question on a **new page**.
4. Each question is of equal value. Mark values are shown besides each part.
5. Non-programmable silent Board of Studies approved calculators are permitted.
6. If requested, additional writing sheets may be obtained from the examinations supervisor upon request.
7. A double sided A4 page of notes is permitted to be referred to throughout this task.

Question 1: Start question on a new page.

(a) Find the primitives of:

(i) 3 **1**

(ii) $x^3 - 3x^2$ **2**

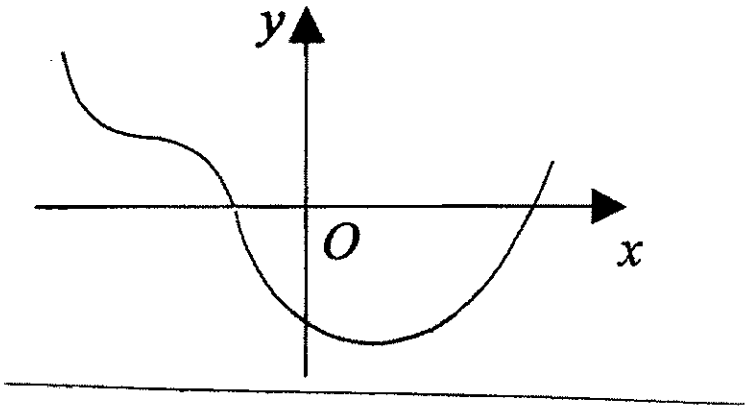
(iii) $(3x + 5)^3$ **2**

(iv) $\frac{x+1}{\sqrt{x}}$ **3**

(b) Evaluate $\int_2^4 (x + \frac{1}{x})^2 \, dx$ **4**

Question 2: Start question on a new page.

(a) The graph shows the graph of $y = f(x)$.



(i) Copy this diagram onto your answer sheet.

(ii) On the same set of axes, sketch the graph of its derivative, $y = f'(x)$. **3**

(b) Find the equation of the normal to the curve $y = \sqrt{x}$ at the point on the curve where $x = 9$. **4**

(c) For the function $f(x) = x^3 - 3x^2 - 9x + 5$, find the values of x for which:

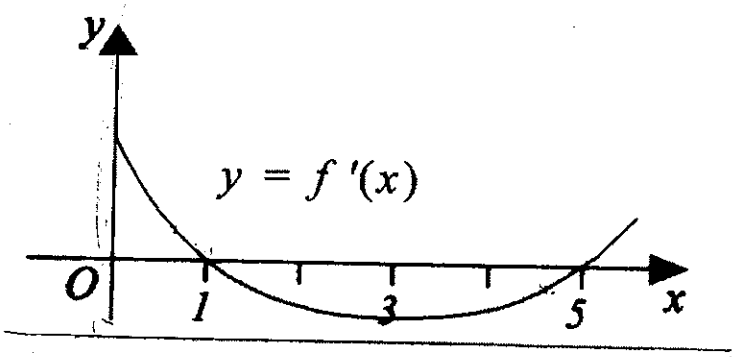
(i) the function $f(x)$ is increasing **3**

(ii) the curve $y = f(x)$ is concave up **2**

Question 3: Start question on a new page.

(a) The curve $y = 3x^2 + \frac{a}{x^2}$ has a turning point at $x = 2$. Find the constant a. **3**

(b) The diagram **shows the** graph of a gradient function of the curve $y = f'(x)$.



For what values of x does $f(x)$ have a local maximum? Justify your answer. **2**

(c) The gradient of a curve is given by $\frac{dy}{dx} = 3x^2 - 6x - 9$. The curve passes through the point (1, -2).

(i) Find the equation of the curve.

(ii) Find the coordinates of the stationary points and determine their nature.

(iii) Find the coordinates of the point of inflexion.

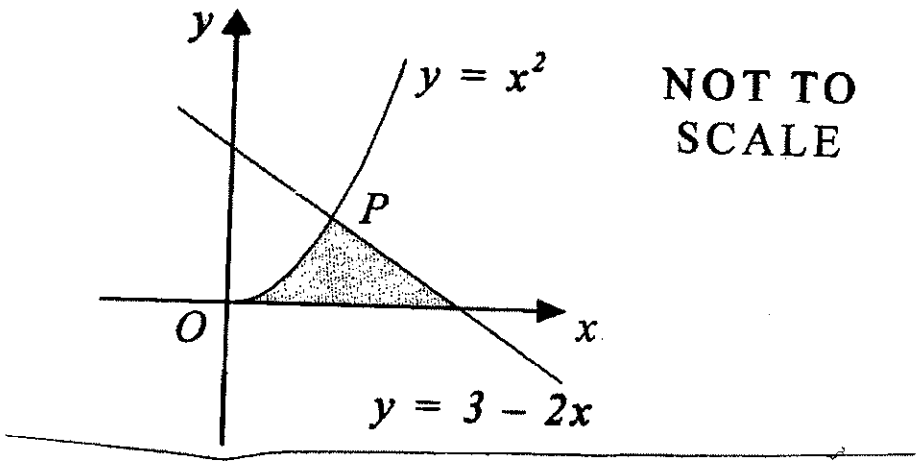
(iv) Sketch the curve in the domain $-2 \leq x \leq 3$ showing its stationary points, point of Inflexion and the y-intercept. **7**

Question 4: Start question on a new page.

(a) (i) Sketch the function $y = x(x^2 - 1)$ on your answer sheet, showing all x-intercepts. **2**

(ii) Find the area bounded by the function $y = x(x^2 - 1)$ and the x-axis. **4**

(b)



(i) Show that the coordinates of the point P are (1, 1). **2**

(ii) Find the volume of the solid formed when the area is rotated about the x-axis. **4**

Question 5: Start question on a new page.

(a) At the point P(-1, 13) on the curve $y = ax^3 + bx^2 + 15x + 26$, there is a point of inflexion. Find the values of a and b. **4**

(b) A closed water tank in the shape of a right cylinder is to be constructed with a surface area of $54\pi\ cm^2$. The height of the cylinder is hcm and the base radiusis $r\ cm$.

(i) Show that the height of the water tank in terms of r is given by $h = \frac{27}{r} - r$. **2**

(ii) Show that the volume V that can be contained in the tank is given by $V = 27\pi r - \pi r^3$. **2**

(iii) Find the radius $r\ cm$ which will give the cylinder its greatest possible volume. Justify your answer. **4**

END OF ASSESSMENT

Solutions.

TRINITY GRAMMAR SCHOOL
MATHEMATICS DEPARTMENT

YEAR 12 2010 ASSESSMENT TASK 2

MATHEMATICS

(2 UNIT/EXTENSION 1)

Time Allowed – *one hour*

WEIGHTING 20% towards final result

Outcomes referred to: P2, P4, P5, P6, P7, P8, H1, H2, H4, H5, H6, H7, H8, H9

Qu 1 - Weremy
Qu 2 - Stathalis
Qu 3 - chandler
Qu 4 - Wardes
Qu 5 - Rogerson

INSTRUCTIONS:

1. Attempt **ALL** questions.
2. Show all necessary working.
3. **Begin** each question on a **new page.**
4. Each question is of equal value. Mark values are shown besides each part.
5. Non-programmable silent Board of Studies approved calculators are permitted.
6. If requested, additional writing sheets may be obtained from the examinations supervisor upon request.
7. A double sided A4 page of notes is permitted to be referred to throughout this task.

Question 1:

(a) (i) $3x + c$ 1

(ii) $\frac{x^4}{4} - \frac{8x^3}{3} + c$ 2

(iii) $\frac{(3x+5)^4}{3 \times 4} + c$ 2

$= \frac{(3x+5)^4}{12} + c$

(iv) $\int \frac{x}{\sqrt{x}} + \frac{1}{\sqrt{x}}\, dx$ 1

$= \int x^{1/2} + x^{-1/2}\, dx$ 1

$= \frac{2x^{3/2}}{3} + 2x^{1/2} + c$ 1

(b) $\int_2^4 \left(x + \frac{1}{x}\right)^2 dx$

$= \int_2^4 \left(x^2 + 2 + x^{-2}\right) dx$ 1

$= \left[\frac{x^3}{3} + 2x + \frac{x^{-1}}{-1}\right]_2^4$ 1

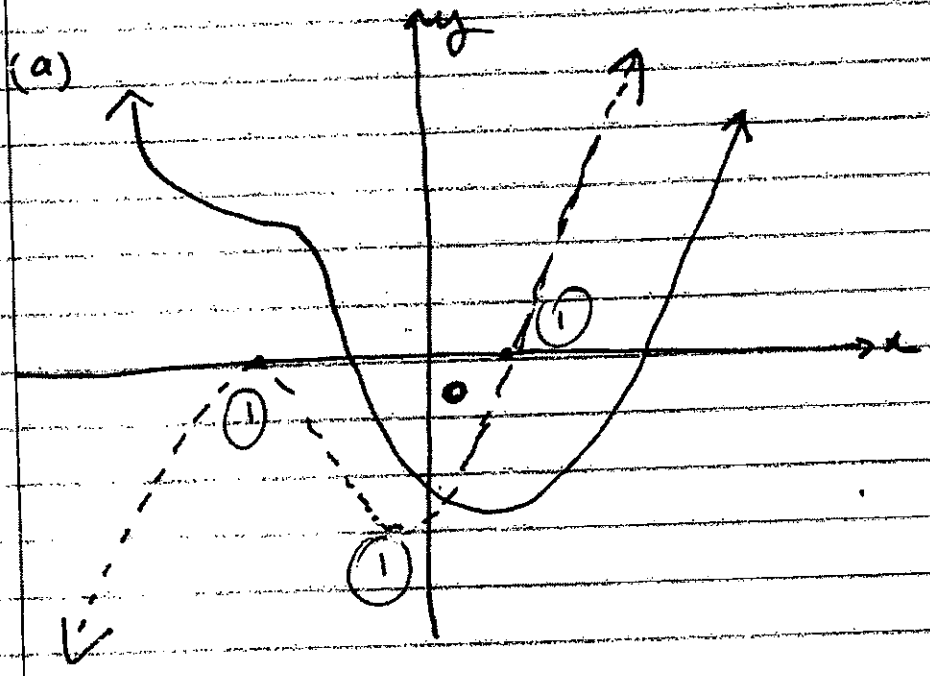
$= \left[\frac{x^3}{3} + 2x - \frac{1}{x}\right]_2^4$ 1

$= \left(\frac{64}{3} + 8 - \frac{1}{4}\right) - \left(\frac{8}{3} + 4 - \frac{1}{2}\right)$

$= \left(\frac{349}{12}\right) - \left(\frac{37}{6}\right) = \frac{275}{12}$

Question 2:

(a)



(b) $y = \sqrt{x}$ $\quad x = 9,\ y = 3$

$y = x^{1/2}$ (1)

$\frac{dy}{dx} = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

$\therefore m = \frac{1}{6}$ (1) $\quad \therefore M_{\perp} = -6$

$y - 3 = -6(x - 9)$ (1)

$y - 3 = -6x + 54$

$y = -6x + 57$ (1)

(c) $f(x) = x^3 - 3x^2 - 9x + 5$.

(i) $f'(x) = 3x^2 - 6x - 9$ (1)

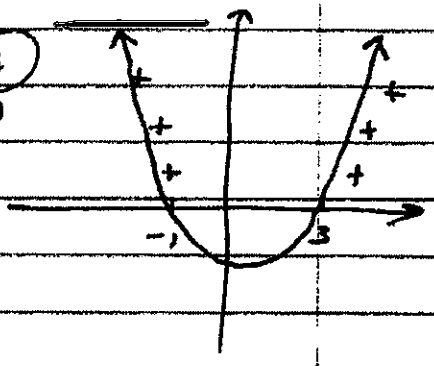
$\therefore f'(x) > 0$

$3(x^2 - 2x - 3) > 0$

$3(x+1)(x-3) > 0$ (1)

$\therefore x > 3$

$x < -1$ (1)



(ii) $f''(x) = 6x - 6$

$\therefore f''(x) > 0$ (1)

$6x - 6 > 0$

$6x > 6$

$x > 1$ (1)

Question 3:

(a) $y' = 6x - 2ax^{-3}$

$y' = 6x - \frac{2a}{x^3}$ (1)

at $x = 2$

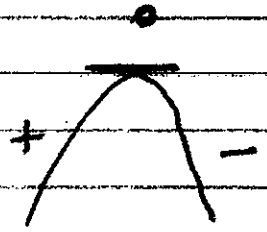
$0 = 6(2) - \frac{2a}{8}$ (1)

$0 = 12 - \frac{2a}{8}$

$\frac{2a}{8} = 12$

$2a = 96$ (1)

$a = 48$

(b) 

at $x = 1$ (1)

$x < 1 \quad \frac{dy}{dx} > 0$

$x > 1 \quad \frac{dy}{dx} < 0$ (1)

(5) (i) $y = x^3 - 3x^2 - 9x + c$

$(1, -2)$

$-2 = 1 - 3 - 9 + c$

$-2 = -11 + c$

$9 = c$ (1)

$\therefore y = x^3 - 3x^2 - 9x + 9$

(ii) $0 = 3(x^2 - 2x - 3)$

$0 = (x-3)(x+1)$

$x = 3, \quad x = -1$ (1)

$y = -18, \quad x = 14$ (1)

$\frac{dy}{dx} = 6x - 6$

$\frac{d^2y}{dx^2} = 6x - 6$

At $x = 3$ $\frac{d^2y}{dx^2} = 6(3) - 6$

$= 12$ ∪ min.

At $x = -1$ $\frac{d^2y}{dx^2} = 6(-1) - 6$

$= -6 - 6$

$= -12$ (1) max.

(iii) $\frac{d^2y}{dx^2} = 0$

$6x - 6 = 0$

$6x = 6$

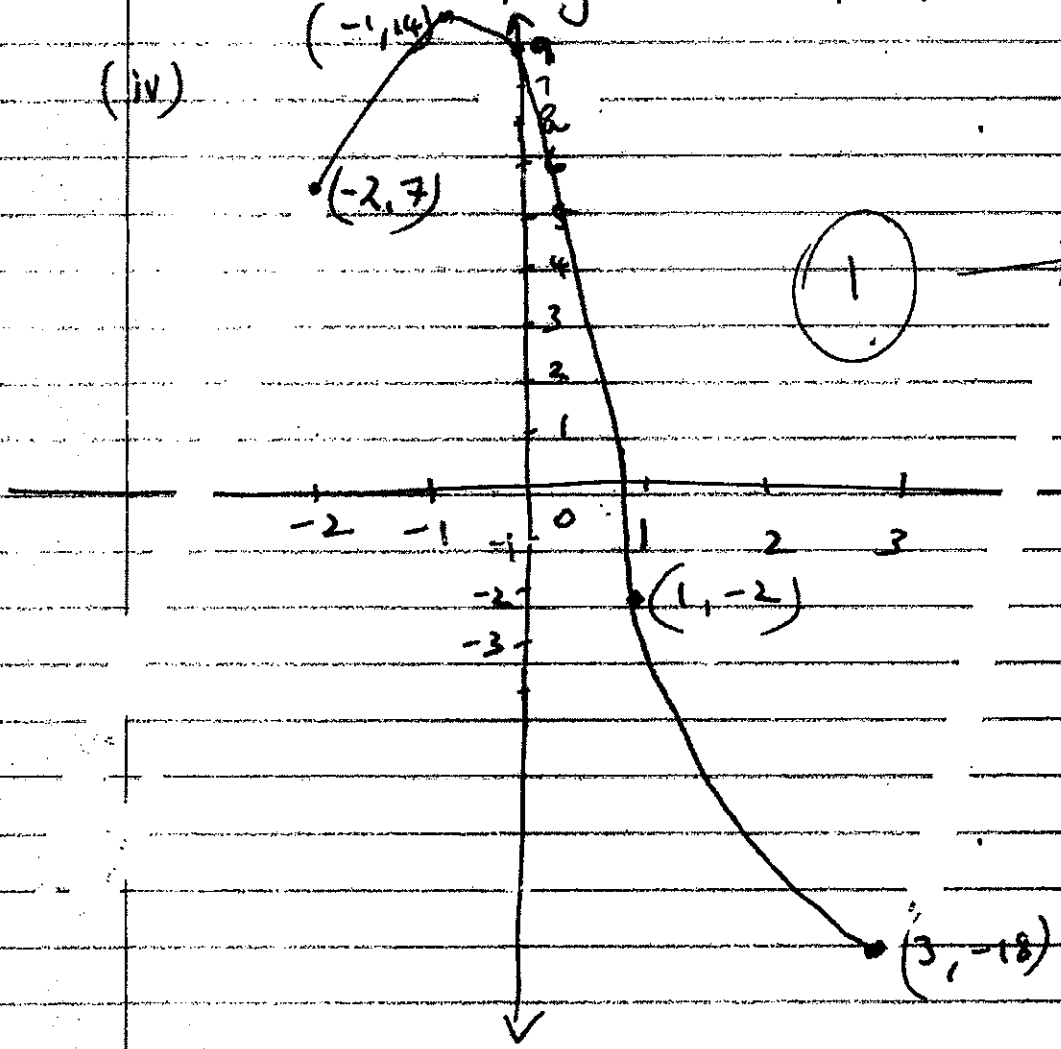
$x = 1$

x	0	1	2
y''	-6	0	6

(1)

$\therefore x = 1,\ y = -2$ pt of inflexion.

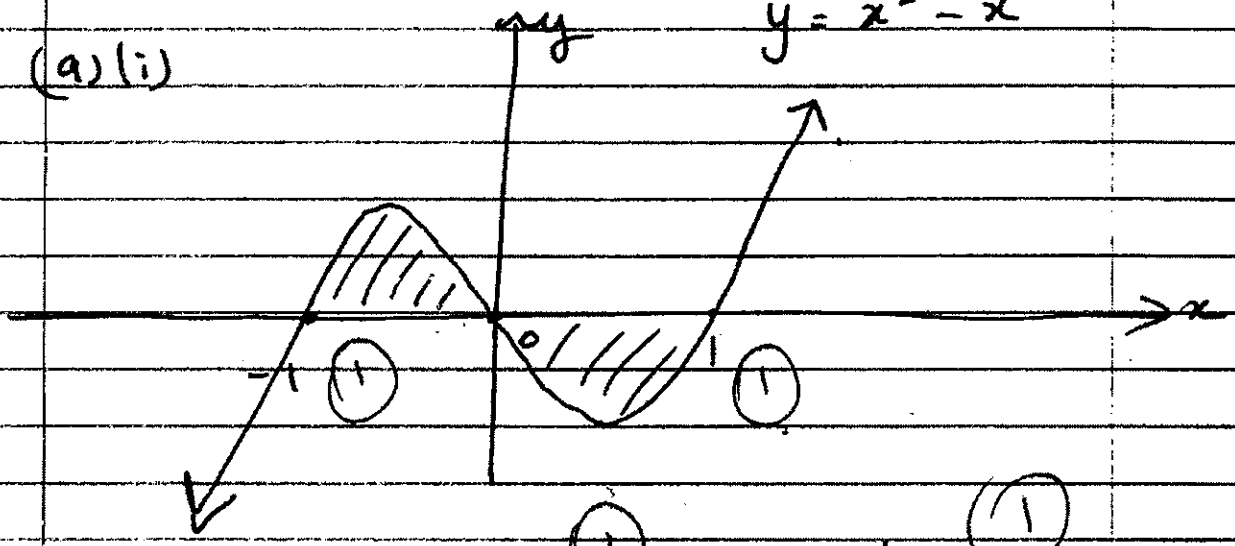
(iv)



(1) → all points shown.

Question 4:

(a)(i)



(1) (1) (1)

(ii) $A = \int_{-1}^{0} x^3 - x \, dx + \left| \int_{0}^{1} x^3 - x \, dx \right|$ (1)

$A = \left[\frac{x^4}{4} - \frac{x^2}{2} \right]_{-1}^{0} + \left| \left[\frac{x^4}{4} - \frac{x^2}{2} \right]_{0}^{1} \right|$ (1)

$A = \left[(0) - \left(\frac{1}{4} - \frac{1}{2} \right) \right] + \left| \left(\frac{1}{4} - \frac{1}{2} \right) - (0) \right|$

$A = \frac{1}{4} + \frac{1}{4}$

$A = \frac{1}{2}$ unit^2 (1)

(b) (i) $x^2 = 3 - 2x$

$x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0$ (1)

$\therefore x = -3, \quad x = 1$

if $x = 1$, $y = 3 - 2(1)$ (1)

$y = 1$ $\quad P(1,1)$

(ii) $V = \pi \int_0^1 (x^2)^2 \, dx + \pi \int_1^{3/2} (3 - 2x)^2 \, dx$

$V = \pi \int_0^1 x^4 \, dx + \pi \int_1^{3/2} 9 - 12x + 4x^2 \, dx$ (1)

$V = \pi \left[\left(\frac{x^5}{5} \right)_0^1 + \left[9x - 6x^2 + \frac{4x^3}{3} \right]_1^{3/2} \right]$ (1)

$V = \pi \left[\frac{1}{5} + \left(\left(9x^{3/2} \right) - 6\left(\frac{9}{4}\right) + \frac{4\left(\frac{27}{8}\right)}{3} \right) - \left(9 - 6 + \frac{4}{3} \right) \right]$ (1)

$V = \pi \left[\frac{1}{5} + \left[4\frac{1}{2} - \left(4\frac{1}{3}\right) \right] \right]$

$V = \pi \left[\frac{1}{5} + \frac{1}{6} \right]$

$V = \frac{11\pi}{30}$ units^3 (1)

Question 5:

(a) $13 = -a + b - 15 + 26$

$13 = -a + b + 11$

$2 = -a + b$ ——— (1) (1)

$\frac{dy}{dx} = 3ax^2 + 2bx + 15$

$\frac{d^2y}{dx^2} = 6ax + 2b$

$0 = -6a + 2b$ ——— (2) (1)

$4 = -2a + 2b$ ——— (1)'

$0 = -6a + 2b$ ——— (2)

∴ (1)' − (2)

$4 = 4a$

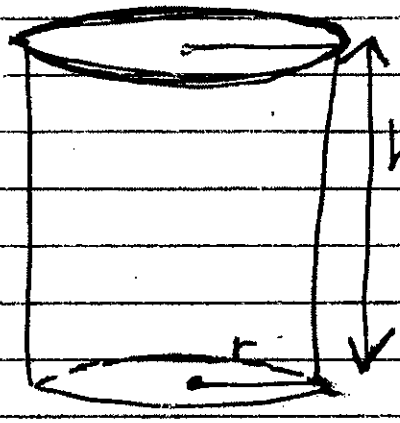
$1 = a$ (1)

sub a into (1)

$2 = -1 + b$

$3 = b$ (1)

(b)



$SA = 54\pi cm^2$

(i) $SA = 2\pi r^2 + 2\pi rh$ (1)

$54\pi = 2\pi r^2 + 2\pi rh$

$\frac{54 - 2r^2}{2r} = h$

$\frac{27}{r} - r = h$ (1)

(ii) $V = \pi r^2 h$

$V = \pi \times r^2 \times \left(\frac{27}{r} - r\right)$ (1)

$V = \frac{27\pi r^2}{r} - \pi r^3$

$V = 27\pi r - \pi r^3$ (1)

(iii) $\frac{dV}{dr} = 27\pi - 3\pi r^2$ (1)

$0 = 27\pi - 3\pi r^2$

$0 = 27 - 3r^2$ (1)

$3r^2 = 27$

$r^2 = \frac{27}{3}$

$r = +\sqrt{\frac{27}{3}}$ (1)

r	0.5	$\sqrt{\frac{27}{3}}$	1
$\frac{dV}{dr}$	+	0	−

(1) Max