TRINITY GRAMMAR SCHOOL
MATHEMATICS DEPARTMENT

YEAR 12 - ASSESSMENT TASK 2

MATHEMATICS

WEIGHTING 15% towards final result

Wednesday 22nd February 2012

OUTCOMES REFERRED TO: P2, P3, P4, P5, P6, P7, P8, H1, H2, H3, H4, H5, H6, H8, H9

General Instructions

- Working time – **45 minutes.**
- Write using blue or black pen.
- Board-approved calculators may be used.
- A double sided A4 page of notes is permitted to be referred to throughout this task
- A table of standard integrals is provided at the back of this paper.
- All necessary working should be shown in every question.
- **Begin** each question on a **new page.**
- Write your **name** and your **teacher's name** on the front of each page.

Total marks – 44

- Attempt Questions 1-4.
- All questions are of equal value.
- Mark values are shown at the side of each question part.

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Marks

Question 1. (Start this question on a new page)

(a) Differentiate with respect to x.

(i) $x^3 + \frac{x}{2}$ **2**

(ii) $\frac{x^2}{x-2}$ **2**

(b) (i) Find $\int \left(\frac{2}{x^2} + x^4 \right) dx$ **2**

(ii) Find $\int x(3x+4)\, dx$ **2**

(iii) Evaluate $\int_1^4 \left(\sqrt{x} + x \right) dx$ **3**

Marks

Question 2. (Start this question on a new page)

(a) Find the equation of the tangent to the curve $y=\sqrt{2x-1}$ at the point where $x=5$. **4**

(b) Given that $\frac{d^2y}{dx^2}=12x^2+12x$ and when $x=-1$, $\frac{dy}{dx}=-9$ and $y=4$.
Find the equation of the curve. **4**

(c) Find the area bounded by the graph of $y=\sqrt{x}+1$, and the y-axis between the lines $y=1$ and $y=4$. **3**

Marks

Question 3. (Start this question on a new page)

(a) Let $f(x) = x^3 - 3x^2 - 9x + 6$.

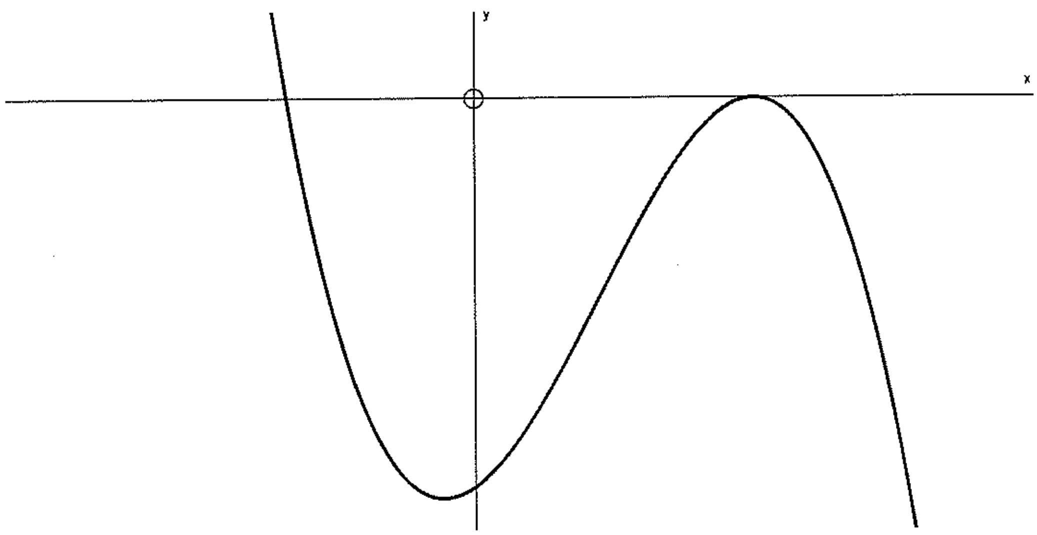
(i) Find the coordinates of any turning points of $y = f(x)$ and determine their nature. **4**

(ii) Find the coordinates of the point of inflexion. **1**

(iii) Sketch the graph of $y = f(x)$ showing the turning points, the y-intercept and any points of inflexion. **3**

(iv) Determine the minimum value of $f(x)$ in the domain $-4 \le x \le 5$. **1**

(b) The graph shows the graph of $y = f(x)$.



(i) Copy this diagram on to your answer sheet.

(ii) On the same set of axes, sketch the graph of its derivative $y = f'(x)$. **2**

Marks

Question 4. (Start this question on a new page)

(a) If $y = x(x+2)^2$, show that $\frac{dy}{dx} = (x+2)(3x+2)$. . **2**

(b) Calculate the area of the region enclosed by the graphs of $f(x) = x^2 - 2x + 1$ and $g(x) = x + 1$. **4**

(c) Given that $\int_1^4 f(x)\ dx = 10$, find the value of $\int_1^4 \left(f(x) + 3\right)\ dx$. **2**

(d) Let $f(x) = x^3 - 3x^2 + kx + 6$ where k is a constant. Find the values of k for which $f(x)$ is an increasing function. **3**

End of Assessment Task

Year 12 Mathematics - AT2 - 2012

Q1.

a) i) $\frac{d}{dx}\left(x^3 + \frac{x}{2}\right)$

$= 3x^2 \; (1) + \frac{1}{2} \; (1)$

ii) $\frac{d}{dx}\left(\frac{x^2}{x-2}\right)$

$= \frac{(x-2)(2x) - (x^2)(1)}{(x-2)^2}$ (M1)

$= \frac{2x^2 - 4x - x^2}{(x-2)^2}$

$= \frac{x^2 - 4x}{(x-2)^2}$ (1)

b) i) $\int (2x^{-2} + x^4)\, dx$

$= -2x^{-1} \; (1) + \frac{x^5}{5} \; (1) + C$

ii) $\int (3x^2 + 4x)\, dx$

$= x^3 \; (1) + 2x^2 \; (1) + C$

iii) $\int_1^4 (x^{\frac{1}{2}} + x)\, dx$

$= \left[\frac{2}{3}x^{\frac{3}{2}} + \frac{x^2}{2}\right]_1^4$ (M1)

$= \left(\frac{2(8)}{3} + 8\right) - \left(\frac{2}{3} + \frac{1}{2}\right)$ (M1)

$= \frac{73}{6}$ OR $12\frac{1}{6}$ (1)

Q2. a) $y = (2x-1)^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2}(2x-1)^{-\frac{1}{2}}(2)$

$= \frac{1}{\sqrt{2x-1}}$ (M1)

when $x = 5$ $\quad m = \frac{1}{3}$

$y = 3$ (M1)

$y - 3 = \frac{1}{3}(x-5)$ (M1)

$3y - 9 = x - 5$

$x - 3y + 4 = 0$ OR $y = \frac{1}{3}x + \frac{4}{3}$ (1)

b) $\frac{d^2y}{dx^2} = 12x^2 + 12x$

$\frac{dy}{dx} = 4x^3 + 6x^2 + C_1$ (M1)

$-9 = -4 + 6 + C_1$ (M1)

$\therefore C_1 = -11$

$\frac{dy}{dx} = 4x^3 + 6x^2 - 11$

$y = x^4 + 2x^3 - 11x + C_2$

$4 = 1 - 2 + 11 + C_2$ (M1)

$\therefore C_2 = -6$

$\therefore y = x^4 + 2x^3 - 11x - 6$ (1)

c) $y - 1 = \sqrt{x}$

$(y-1)^2 = x$ (M1)

$A = \int_1^4 (y-1)^2 \, dy$

$= \left[\frac{(y-1)^3}{3}\right]_1^4$ (M1)

$= \frac{27}{3} - 0$

$= 9u^2$ (1)

Q3. a) $f(x) = x^3 - 3x^2 - 9x + 6$

i) $f'(x) = 3x^2 - 6x - 9$

$3(x^2 - 2x - 3) = 0$ (M1)

$(x-3)(x+1)$

$x = 3 \qquad x = -1$

$y = -21 \qquad y = 11$

$(3, -21)$ (1) $\qquad (-1, 11)$ (1)

MINIMUM $\qquad$ MAXIMUM

$f''(x) = 6x - 6$

$f''(3) = 18 - 6 = 12$

$f''(-1) = -6 - 6 = -12$

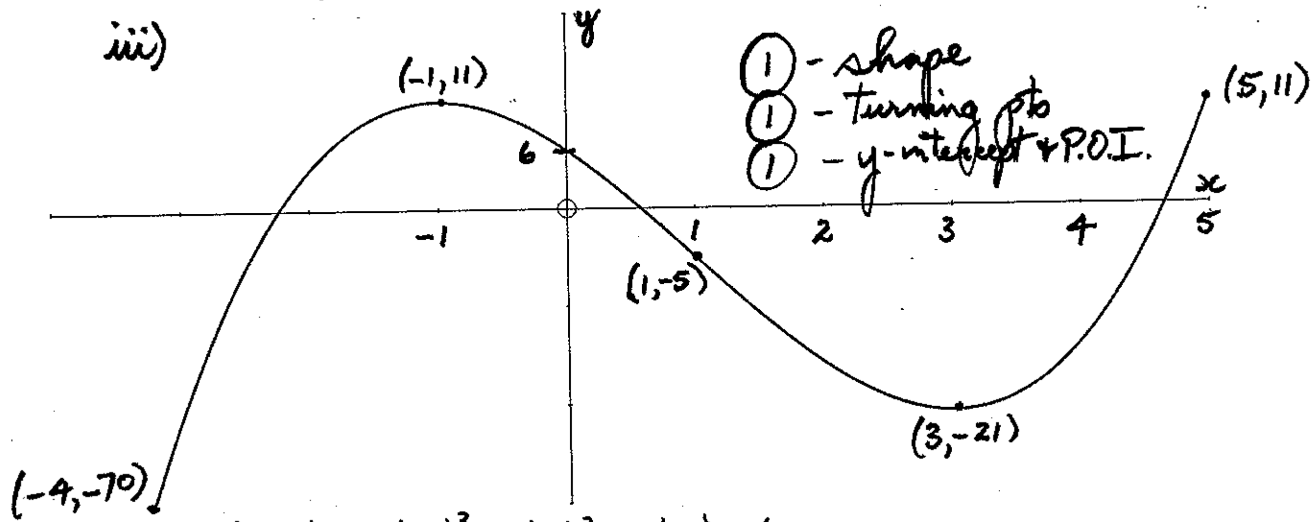
TEST (1)

ii) $6x - 6 = 0$

$x = 1$

$y = -5 \quad (1, -5)$ (1)

iii)

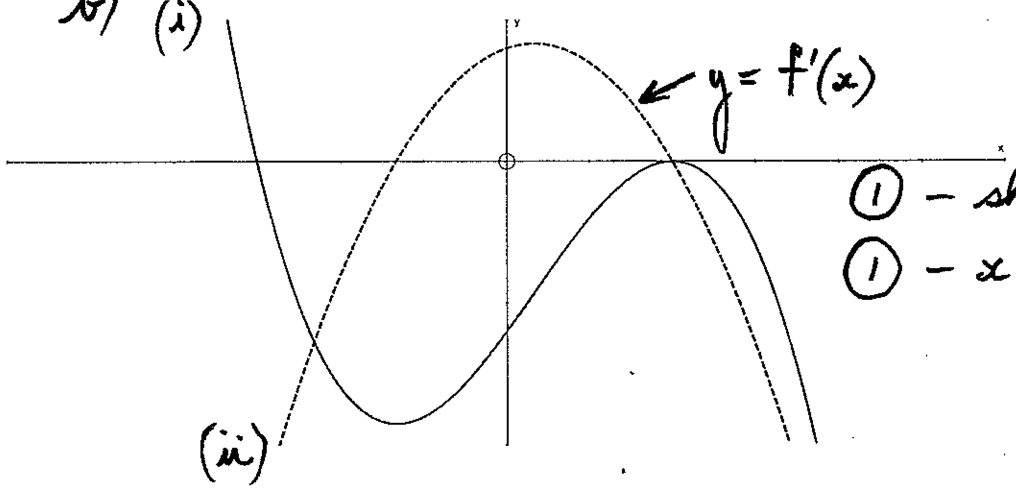


iv) $f(-4) = (-4)^3 - 3(-4)^2 - 9(-4) + 6$

$= -70$

Minimum value $= -70$ (1)

b) (i)



Q4. a) $y = x(x+2)^2$

$\frac{dy}{dx} = (x)2(x+2) + (x+2)^2(1)$ (M1)

$= (x+2)[(2x) + x + 2]$

$= (x+2)(3x+2)$ (1)

b) $x^2 - 2x + 1 = x + 1$

$x^2 - 3x = 0$

$x(x-3) = 0$ (M1)

$x = 0 \quad x = 3$

$A = \int_0^3 [(x+1) - (x^2 - 2x + 1)]\,dx$ (M1)

$= \int_0^3 (3x - x^2)\,dx$

$= \left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_0^3$ (M1)

$= \left(\frac{27}{2} - 9\right) - 0$

$= 4\frac{1}{2}\ u^2$ (1)

c) $\int_1^4 f(x)\,dx + \int_1^4 3\,dx$

$= 10 + \left[3x^1\right]_1^4$ (M1)

$= 10 + (12 - 3)$

$= 19$ (1)

d) $f'(x) = 3x^2 - 6x + k$

$b^2 - 4ac < 0$ since $a > 0$ OR $f'(x) > 0$ (M1)

$\therefore (-6)^2 - 4(3)(k) < 0$ (M1)

$36 - 12k < 0$

$36 < 12k$

$3 < k$

$\therefore k > 3$ (1)