Trinity Grammar School

Mathematics Department

2013

HALF-YEARLY EXAMINATION

HSC ASSESSMENT TASK 3

Year 12

Mathematics

Board of Studies Student Number
(Year 12 only)

Class Teacher:

....

Name:
(Year 11 students)

....

Do NOT write solutions on this question paper. Any working on this question paper will NOT be marked.

General Instructions

- Reading time – 5 minutes
- Writing time – 3 hours
- Write using black or blue pen
- Only approved calculators for this course are allowed in this task
- A table of Standard Integrals (if required) in this course is supplied
- Show all necessary working
- Write your Board of Studies Student Number (Year 12 HSC) or Name (Year 11) and your Class teacher on the question paper and on any answer sheets or booklets used to write your responses to the questions submitted
- If you do not attempt a question you must submit an answer sheet or writing booklet for that question clearly indicating **N/A** and your Student Number or Name.

- **Assessment Weighting:** 30%

Total marks – 100

Section 1

10 marks

- Attemtpt all Questions
- Allow about **15** minutes for this section

Section II

90 marks

- Attempt all Questions
- Allow about **2** hours **45** minutes for this section

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SECTION I Total marks (10)

- Shade the correct response on the answer sheet provided.
- Each question is worth 1 mark.

---

1 What is 4.097 84 correct to three significant figures?

(A) 4.09

(B) 4.10

(C) 4.097

(D) 4.098

2. Let $a = e^x$. Which expression is equal to $\log_e(a^2)$?

(A) e^{2x} (B) e^{x^2} (C) $2x$ (D) x^2

3. A bag contains seven yellow balls and three white balls. Trish selects three balls at random from the bag. What is the probability that all three balls are yellow?

(A) $\frac{1}{180}$ (B) $\frac{7}{24}$ (C) $\frac{343}{1000}$ (D) $\frac{7}{10}$

4. If $f''(x) > 0$ and $f'(x) < 0$ for all x over a given domain, which of the following describes the graph of $y = f(x)$?

(A) Increasing and concave up

(B) Increasing and concave down

(C) Decreasing and concave up

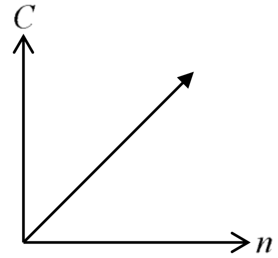
(D) Decreasing and concave down

5. What is the value of $\int_1^4 \frac{1}{3x}\,dx$?

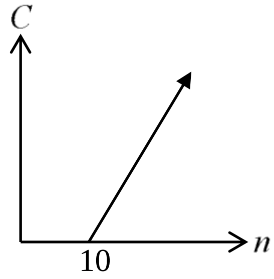
(A) $\frac{1}{3}\ln 3$ (B) $\frac{1}{3}\ln 4$ (C) ln9 (D) ln12

6 The cost, C, of producing n textbooks is given by the formula $C = 20n + 10$.
Which of the graphs below best represents this equation?

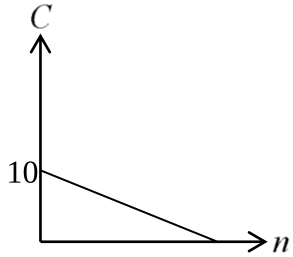
(A)



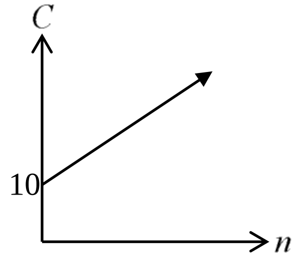
(B)



(C)



(D)



7. If $5 + x + 45 + \ldots$ is a geometric series, find the value(s) of x.

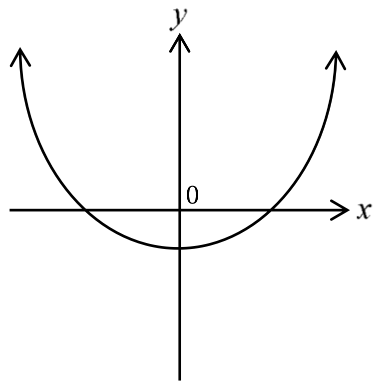
(A) 25 (B) 15 (C) -15 (D) ± 15

8. Differentiate with respect to x :

$$y = \log_e(5+4x).$$

(A) 4 (B) $4\log_e(5+4x)$ (C) $\frac{4}{5+4x}$ (D) $\frac{4}{\log_e(5+4x)}$

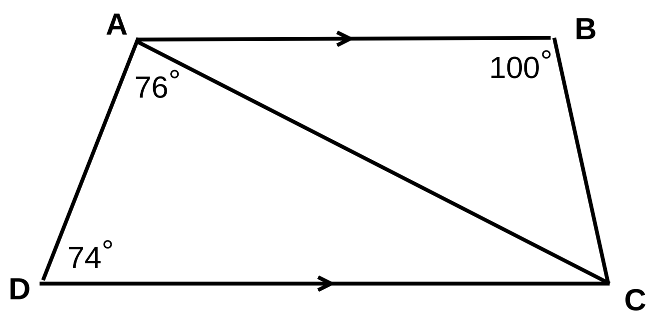
9.



The parabola shown could have the equation.

(A) $y = x^2 - 1$ (B) $y = -x^2 + 1$ (C) $y = -x^2 - 1$ (D) $y = x^2 + 1$

10.



NOT TO SCALE

In the above figure ABCD is a quadrilateral in which AB is parallel to DC.
Angle CAD is 76^0, angle ADC is 74^0 and angle ABC is 100^0.

Find the size of angle BCA.

(A) 76^0 (B) 80^0 (C) 30^0 (D) 50^0

END OF SECTION I
SECTION II COMMENCES ON THE NEXT PAGE

SECTION II Total marks (90)

- Begin each question in a new writing booklet.
- Show all necessary working.
- Each question is worth 15 marks.

Marks

Question 11 (15 marks)

(a) Evaluate e^3 correct to 3 decimal places. **2**

(b) Calculate $\sum_{r=3}^{5} r^2$ **1**

.

(c) Simplify fully $\dfrac{3}{x+3} - \dfrac{1}{x-3}$ **2**

(d) If $\dfrac{14}{3+\sqrt{2}} = a+b\sqrt{2}$, find the values of a and b. **2**

(e) Solve $|4x-1| = 3$ **2**

(f) Factorise fully: $2x^3 - 54y^3$ **2**

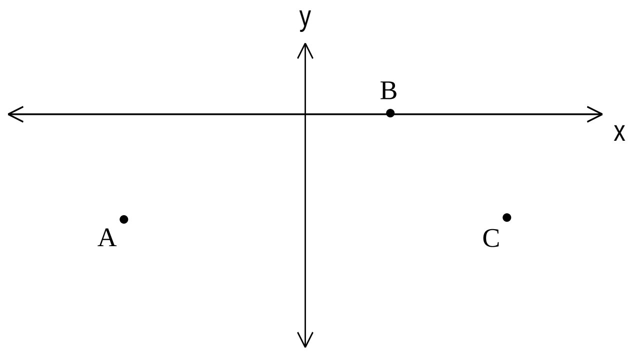
(g) Given $\log_a 3 = 0.6$ and $\log_a 2 = 0.4$, find the value of $\log_a 18$. **2**

(h) Find the sum of the first 18 terms of the arithmetic series $5 + 3 + 1 + \ldots\ldots\ldots$ **2**

End of Question 11

Marks

Question 12 (15 marks) **[START A NEW WRITING BOOKLET]**



The coordinates of the points A, B and C are $(-3,-2)$, $(1, 0)$ and $(5, -2)$ respectively.
Copy this diagram into your writing booklet.

(a) Calculate the exact length of the interval AB. **2**

(b) Find the gradient of the line AB. **2**

(c) Show that the equation of line l, drawn through C parallel to AB is $x - 2y - 9 = 0$. **2**

(d) Find the coordinates of D, the point where l intersects the x-axis. **1**

(e) What is the size of the acute angle (to the nearest degree) made by the line AB with the positive direction of the x-axis? **1**

(f) Hence, determine the size of $\angle ABD$. **1**

(g) Find the perpendicular distance of the point A from the line l. **2**

(h) Find the area of parallelogram ABDC. **2**

(i) Sketch the line l and shade the area satisfied by the following inequalities simultaneously; $x \geq 0$, $y \leq 0$, $x - 2y - 9 \geq 0$ **2**

End of Question 12

Marks

Question 13 (15 marks) **[START A NEW WRITING BOOKLET]**

(a) Differentiate the following;

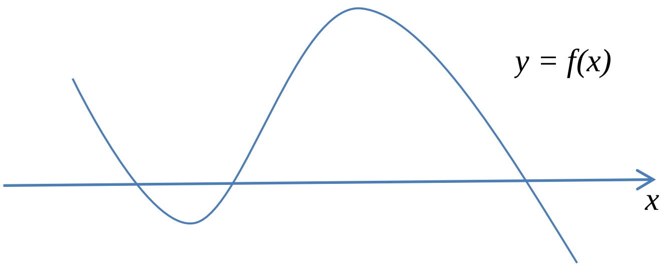
(i) $y = 7x^4 - 3x^3 + x^2 - 8x - 4$ **1**

(ii) $y = x^2 \ln x$ **2**

(iii) $y = \frac{3x-4}{2x+1}$ **2**

(iv) $y = (1 + e^{4x})^5$ **2**

(b) **2**



Copy or trace the function above into your writing booklet.

On the same axes, sketch and label a possible graph of the gradient function $f'(x)$.

(c) Find:

(i) $\int_0^1 3\sqrt{x}\, dx$ **2**

(ii) $\int \frac{8x + 10}{2x^2 + 5x}\, dx$ **2**

(iii) $\int \frac{3x^2 + 2x + 1}{x^2}\, dx$ **2**

End of Question 13

Marks

Question 14 (15 marks) **[START A NEW WRITING BOOKLET]**

(a) Find the equation, in general form, of the normal to the curve $y = 2x^3 + 6$ at the point where $x = -2$. **3**

(b) Consider the function, $y = x^3 - 3x^2 - 9x + 1$.

(i) Find the stationary points and determine their nature. **3**

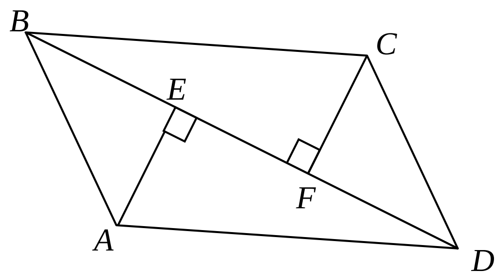
(ii) Find the point of inflexion. **1**

(iii) Sketch the graph of $y = x^3 - 3x^2 - 9x + 1$ in the interval $-3 \leq x \leq 6$. **1**

(iv) Find the maximum value of the function in the domain, $-3 \leq x \leq 6$. **2**

(c) Solve $2\log_4 x = 9$. **2**

(d)



ABCD is a parallelogram with AE and CF drawn perpendicular to the diagonal DB. Prove with reasons that :

(i) $\Delta AEB \equiv \Delta CFD$ **2**

(ii) AE = FC. **1**

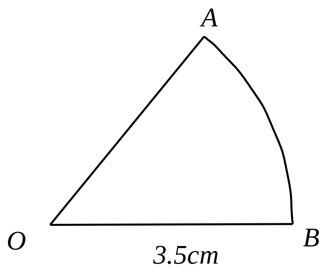
End of Question 14

Marks

Question 15 (15 marks) **[START A NEW WRITING BOOKLET]**

(a) A sector *AOB* of a circle has a radius of *3.5cm* and its perimeter is *9.5cm*.

NOT TO SCALE



(i) Find the length of the arc *AB*. **1**

(ii) Find the size of $\angle AOB$ to the nearest degree. **2**

(iii) Find the area of the sector *AOB*. **2**

(b) The gradient of a curve is given by $\frac{dy}{dx} = 3x^2 + 2x$. **2**

The curve passes through the point (2, 13).

Find the equation of the curve.

(c) Solve $2^{2x} - 9(2^x) + 8 = 0$ **3**

(d) The first four terms of a sequence are 3, 6, 9, 12

(i) Show that 102 is a term of this sequence. **2**

(ii) Hence, or otherwise, find the sum of the terms of this sequence between 100 and 200. **3**

End of Question 15

Marks

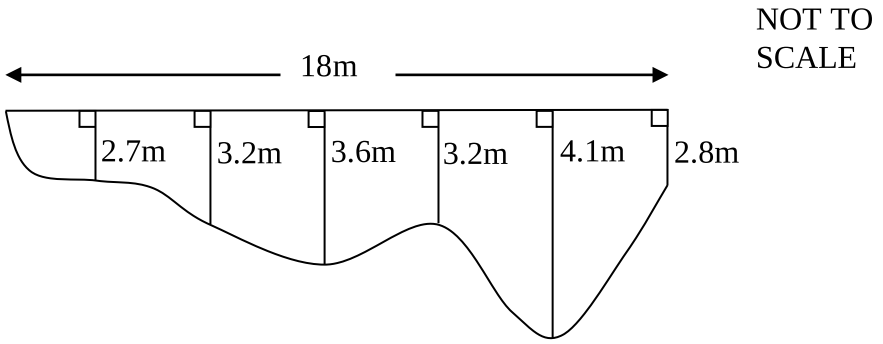
Question 16 (15 marks) **[START A NEW WRITING BOOKLET]**

(a) (i) Show that $y = x^2$ and $y = x + 2$ intersect when $x = 2$ and $x = -1$ **2**

(ii) Find the area of the region bounded by the two curves, $y = x^2$ and $y = x + 2$ **3**

(b) The council wanted to make a rectangular swimming area at the beach using a straight cliff on one side and a length of 300 m of sharkproof netting for the other three sides. Using calculus, find the dimensions of the rectangle that encloses the greatest area? **3**

(c) A surveyor measures the depth of a river at equal intervals across its width. The river is 18m wide and the measurements are as follows:

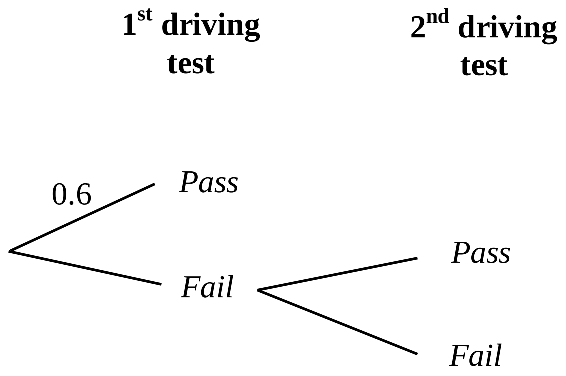


Using the trapezoidal rule with 7 function values, find the approximate cross-sectional area of the river. **3**

Question 16 continues on the next page

(d) The probability that Belinda will pass her driving test first time is 0.6, while the probability that Belinda will pass her driving test on the second attempt is 0.85.
Once she passes a driving test, she will get her P plates.

Part of the tree diagram is shown, indicating possible outcomes.



(i) Copy and complete the tree diagram showing the relevant probabilities **1**

Hence, or otherwise, find the probability that Belinda:

(ii) will need to sit a 3rd driving test to get her P plates **1**

(iii) obtains her P plates without the need for a 3rd driving test **2**

End of Question 16

END OF EXAMINATION

SOLUTIONS

TRINITY GRAMMAR SCHOOL
2013, Year 12 Mathematics
Half Yearly Examination
SECTION I
ANSWER SHEET

Board of Studies Student Number

Class Teacher: ……………………………….

Be sure to write your answers for Section I on this answer sheet. After you have selected an answer, **CIRCLE** the correct answer. To change an answer, erase your previous mark completely, and then record your new answer. Mark only one answer for each question.

Q1.	A	B	C	D
Q2.	A	B	C	D
Q3.	A	B	C	D
Q4.	A	B	C	D
Q5.	A	B	C	D
Q6.	A	B	C	D
Q7.	A	B	C	D
Q8.	A	B	C	D
Q9.	A	B	C	D
Q10.	A	B	C	D

1 each. (10)

SOLUTIONS

Question 11

a) 20.086 1 for incorrect rounding   (2)

b) $3^2 + 4^2 + 5^2 = 50$ ✓   (1)

c)
$$\frac{3(x-3) - 1(x+3)}{(x+3)(x-3)} = \frac{3x - 9 - x - 3}{(x+3)(x-3)} \checkmark$$
$$= \frac{2x - 12}{x^2 - 9} \checkmark \qquad (2)$$

d)
$$\frac{14}{3+\sqrt{2}} \times \frac{3-\sqrt{2}}{3-\sqrt{2}} = \frac{14(3-\sqrt{2})}{9-2} \checkmark$$
$$= \frac{14(3-\sqrt{2})}{7}$$
$$= 2(3-\sqrt{2})$$
$$= 6 - 2\sqrt{2} \checkmark \qquad (2)$$

e)
$4x - 1 = 3$ OR $-(4x-1) = 3$

$4x = 4$ $-4x + 1 = 3$

$x = 1$ ✓ $-4x = 2$

$x = -\frac{1}{2}$ ✓   (2)

f) $2(x^3 - 27y^3) = 2(x-3y)(x^2 + 3xy + 9y^2)$ ✓   (2)

g)
$$\log_a 18 = \log_a(3^2 \times 2)$$
$$= \log_a 3^2 + \log_a 2 \checkmark$$
$$= 2\log_a 3 + \log_a 2$$
$$= 2 \times 0.6 + 0.4$$
$$= 1.6 \checkmark \qquad (2)$$

h) $S_{18} = \frac{18}{2}\left[2 \times 5 + (18-1) \times {}^{-}2\right]$ ✓

$= 9\left[10 - 17 \times 2\right]$ (2)

$= -216$ ✓

Question 12

a) $d = \sqrt{(1--3)^2 + (0--2)^2}$ ✓

$= \sqrt{4^2+2^2}$

$= \sqrt{20}$

$= 2\sqrt{5}$ ✓ (2)

b) $m = \dfrac{0--2}{1--3}$ ✓

$= \dfrac{2}{4}$

$= \dfrac{1}{2}$ ✓ (2)

c) $y--2 = \frac{1}{2}(x-5)$ ✓

$y+2 = \frac{1}{2}x - \frac{5}{2}$

$y = \frac{1}{2}x - \frac{9}{2}$

$x-2y-9 = 0$ ✓ (2)

d) If $y=0$, ✓ $x-0-9=0$

$x = 9$

$\therefore$ D is $(9,0)$ ✓ (1)

) $\tan\theta = \frac{1}{2}$

$\theta = 27^\circ$ ✓ (1)

f) $\angle ABD = 180^\circ - 27^\circ$

$= 153^\circ$ ✓ (1)

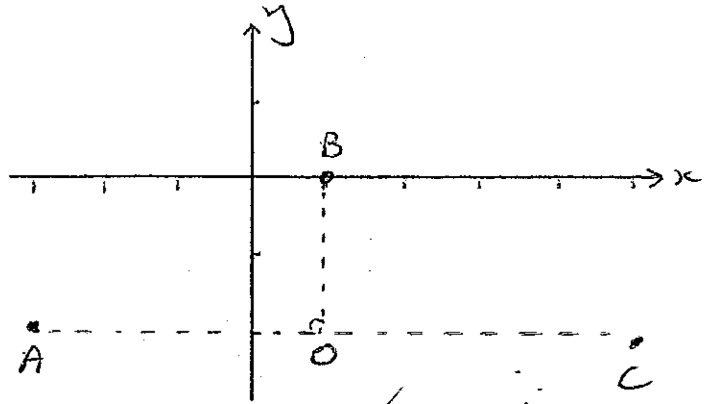
g) $d = \dfrac{|1(-3)-2(-2)-9|}{\sqrt{1^2+(-2)^2}}$ ✓

$= \dfrac{8}{\sqrt{5}}$

$= \dfrac{8}{\sqrt{5}} \times \dfrac{\sqrt{5}}{\sqrt{5}}$

$= \dfrac{8\sqrt{5}}{5}$ ✓ (2)

h)



$A = AC \times AO$ ✓

$= 8 \times 2$

$= 16$ sq units. ✓

or $A = d_{AB} \times pd$

$= 2\sqrt{5} \times \dfrac{8}{\sqrt{5}}$

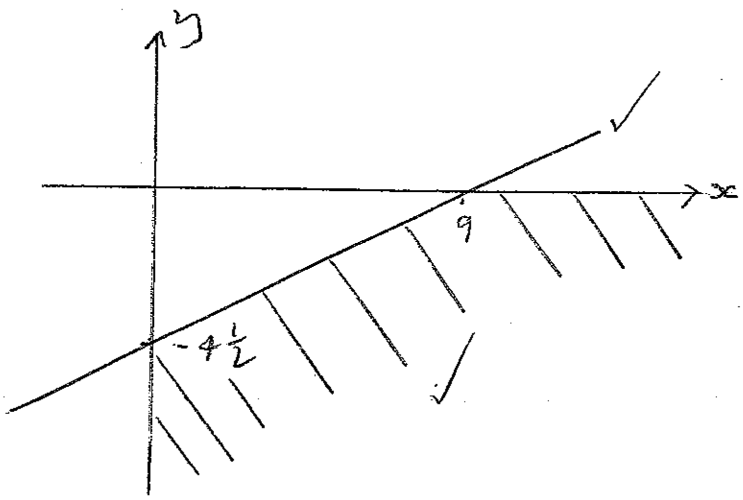
$= 16$ (2)

i) x-intercept $= 9$

y-intercept $(x=0)$: $0 - 2y - 9 = 0$

$-2y = 9$

$y = -4\frac{1}{2}$



(2)

Question 13

a) (i) $\frac{dy}{dx} = 28x^3 - 9x^2 + 2x - 8$ ✓ (1)

(ii) $\frac{dy}{dx} = \ln x \times 2x + x^2 \times \frac{1}{x}$ ✓

$= 2x\ln x + x$ ✓ (2)

(iii) $\frac{dy}{dx} = \frac{(2x+1)\times 3 - 2(3x-4)}{(2x+1)^2}$ ✓

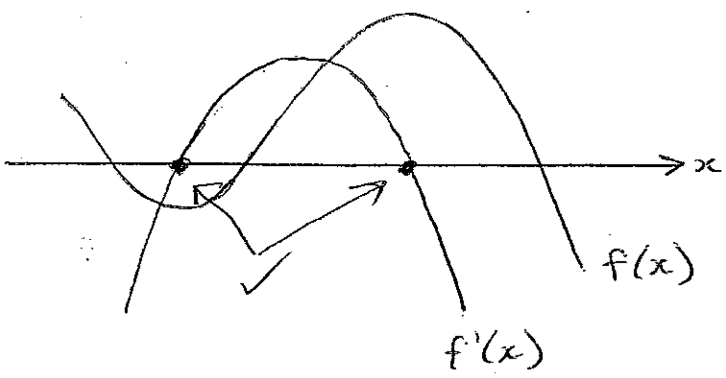
$= \frac{6x+3-6x+8}{(2x+1)^2}$

$= \frac{11}{(2x+1)^2}$ ✓ (2)

(iv) $\frac{dy}{dx} = 5(1+e^{4x})^4 \times 4e^{4x}$ ✓

$= 20e^{4x}(1+e^{4x})^4$ ✓ (2)

b)



(2)

c) (i) $\int_0^1 3x^{\frac{1}{2}}\,dx = \frac{2}{3}\times 3x^{3/2}\Big]_0^1$ ✓

$= 2x^{3/2}\Big]_0^1$

$= 2$ ✓ (2)

(ii) $\int \frac{8x+10}{2x^2+5x}\,dx = \underbrace{2\ln(2x^2+5x)}_{\checkmark} + c$ (2)

(iii) $\int 3 + \frac{2}{x} + \frac{1}{x^2}\,dx = \int (3 + 2x^{-1} + x^{-2})\,dx$ ✓

$= 3x + 2\ln x - \frac{1}{x} + c$ ✓ (2)

Question 14

a) $y = 2x^3 + 6$

$y' = 6x^2 = 6(-2)^2 = 24$ where $x = -2$

$\therefore$ gradient of the normal is $-\frac{1}{24}$ ✓

when $x = -2$, $y = 2(-2)^3 + 6 = -10$

Now, $y + 10 = -\frac{1}{24}(x+2)$ ✓

$-24y - 240 = x + 2$ (3)

$x + 24y + 24\overset{2}{0} = 0$ ✓

b) (i) $y = x^3 - 3x^2 - 9x + 1$

$y' = 3x^2 - 6x - 9 = 0$ for stat. pts.

$3(x^2 - 2x - 3) = 0$

$3(x-3)(x+1) = 0$

$x = 3$ or $x = -1$ ✓

when $x = 3$, $y = 27 - 27 - 27 + 1 = -26$

when $x = -1$, $y = -1 - 3 + 9 + 1 = 6$

Test $(3, -26)$: $y'' = 6x - 6 = 18 - 6 > 0$

$\therefore (3, -26)$ is a min turning pt. ✓

Test $(-1, 6)$: $y'' = 6 \times -1 - 6 = -12 < 0$

$\therefore (-1, 6)$ is a max turning pt. ✓ (3)

(ii) $y'' = 6x - 6 = 0$ for possible pt of inflexion

$6x = 6$

$x = 1$

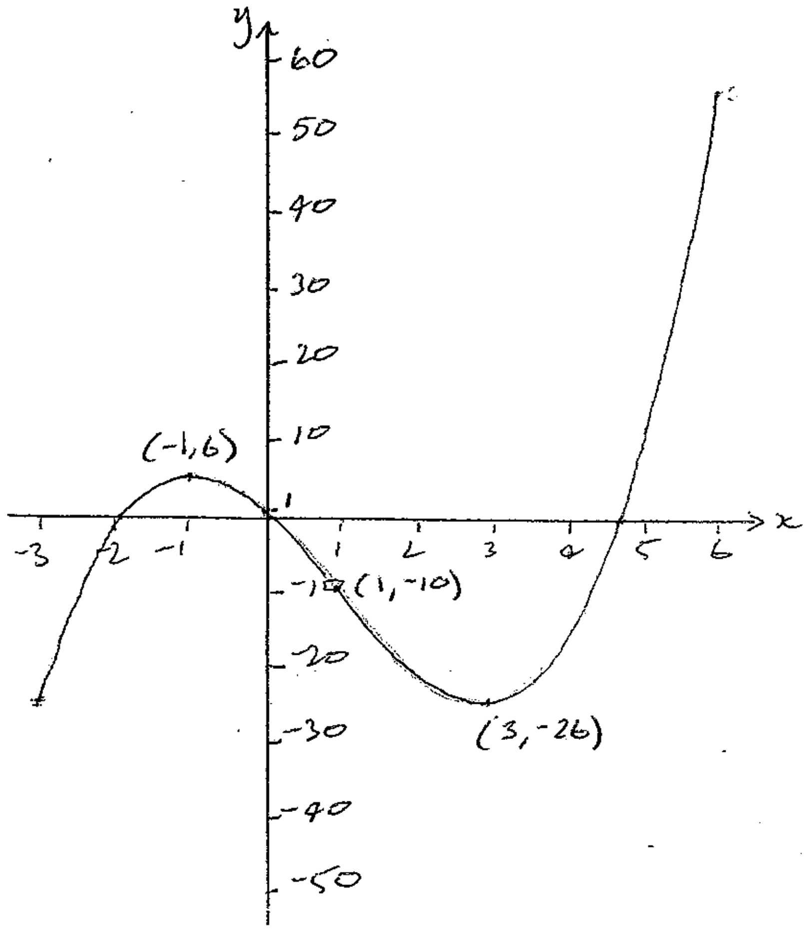
when $x = 1$, $y = 1 - 3 - 9 + 1 = -10$

Test $(1, -10)$:

x	0	1	2
y''	$-$	0	$+$

$\therefore (1, -10)$ is a pt of inflexion ✓ (1)

(iii)



(1)

(iv) When $x=6$, $y = 6^3 - 3(6)^2 - 9(6) + 1$ ✓

$= 55$ ✓ (2)

c) $2\log_4 x = 9$

$\log_4 x = \frac{9}{2}$ ✓

$\therefore x = 4^{\frac{9}{2}}$

$= 512$ ✓ (2)

d) (i) $AB = CD$ (opp sides of parallelogram equal)

$\angle AEB = \angle CFD = 90°$ (Given)

$\angle ABE = \angle CDF$ (alternate angles equal in parallel lines) ✓

$\therefore \triangle AEB \equiv \triangle CFD$ (AAS test) ✓ (2)

(ii) $AE = FC$ (corresponding sides in congruent triangles) ✓ (1)

Question 15

a) (i) $AB = 9.5 - 2 \times 3.5$
$= 2.5 \text{ cm}$ ✓ (1)

(ii) $\angle AOB = \dfrac{2.5}{7\pi} \times 360^\circ$ ✓

$= 41^\circ$ (correct to nearest degree) ✓ (2)

(iii) $A = \dfrac{41}{360} \times \pi \times 3.5^2$ ✓

$= 4.4 \text{ cm}^2$ (correct to 1 dec. place) ✓ (2)

b) $y = \int 3x^2 + 2x \, dx = x^3 + x^2 + c$ ✓

Now, $y = 13$ when $x = 2$

so, $13 = 2^3 + 2^2 + c$

$\therefore c = 1$ (2)

$\therefore y = x^3 + x^2 + 1$ ✓

c) Let $m = 2^x$

$\therefore m^2 - 9m + 8 = 0$

$(m-8)(m-1) = 0$ ✓

$m = 8$ or $m = 1$

$2^x = 8$ $\quad 2^x = 1$ (3)

$\therefore \underline{x = 3}$ ✓ $\quad \underline{x = 0}$ ✓

d) (i) $102 = 3 + (n-1) \times 3$ ✓

$102 = 3 + 3n - 3$

$102 = 3n$ (2)

$n = 34$ ✓

Since $n \in \mathbb{Z}$, 102 is a term of the sequence

(ii) $S_{66} = \dfrac{66}{2}\left[2 \times 3 + (66-1) \times 3\right] = 6633$ ✓

$S_{33} = \dfrac{33}{2}\left[2 \times 3 + (33-1) \times 3\right] = 1683$ ✓

Sum $= 6633 - 1683 = 4950$ ✓ (3)

Question 16

a) (i) $x^2 = x + 2$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

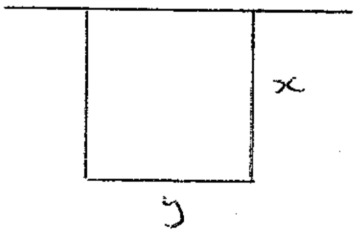
$x = 2$ and $x = -1$ (2)

(ii) $A = \int_{-1}^{2} (x + 2 - x^2)\,dx$

$= \left[\frac{1}{2}x^2 + 2x - \frac{x^3}{3}\right]_{-1}^{2}$

$= 4\frac{1}{2}$ sq units (3)

b)



$P = 2x + y = 300$

$y = 300 - 2x$

$A = xy$

$= x(300 - 2x)$

$= 300x - 2x^2$

$\frac{dA}{dx} = 300 - 4x = 0$ for max/min

$-4x = -300$

$x = 75$

$\frac{d^2A}{dx^2} = -4 < 0$ $\therefore x = 75$ gives a max

When $x = 75$, $y = 300 - 2 \times 75 = 150$

$\therefore$ The dimensions that give a max area are 150m by 75m. (3)

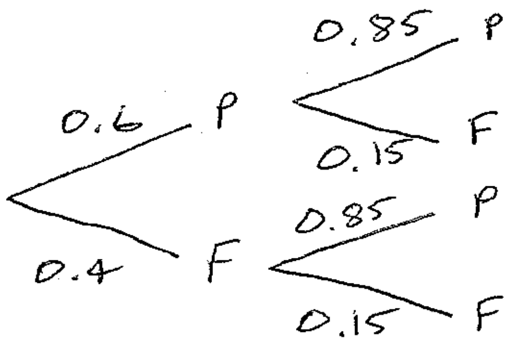
c) $h = \frac{18 - 0}{6} = 3$

$\therefore A \doteqdot \frac{3}{2}\left[0 + 2.8 + 2(2.7 + 3.2 + 3.6 + 3.2 + 4.1)\right]$

$= 54.6 \text{ m}^2$

(3)

d) (i)



(1)

(ii) $P(FF) = 0.4 \times 0.15$
$= 0.06$ (1)

(iii) $P(P) + P(FP) = 0.6 + 0.4 \times 0.85$ (2)
$= 0.94$