Westfields Sports High School

YEAR 12 HALF YEARLY EXAMINATION

2000

MATHEMATICS

2/3 UNIT COMMON

Time allowed - Two and half hours
(Plus 5 minutes reading time)

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- Board-approved calculators may be used.
- Show all necessary working .
- Marks may be deducted for careless or badly arranged work
- All questions are of equal value.
- Start a **NEW PAGE** for each question.

QUESTION 1 (*Start on a new page*)

a) Expand and simplify -5x - 3(2x + 1)

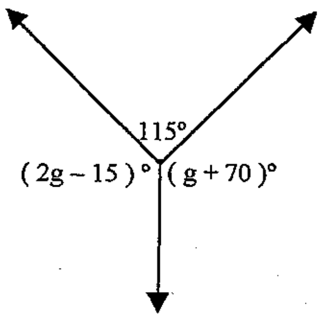
b) Simplify $\dfrac{3x-2y}{6x-4y}$

c) Simplify $2\sqrt{3}-\sqrt{48}+\sqrt{75}$

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$= x^{1} \frac{1}{2} x^{-\frac{1}{2}} x$

d) Find the value of the pronumeral giving reasons.



NOT TO SCALE

$360 - (2g-15) - (g+70)$ ∠ sum at point

$= 360$ ~~sum~~

~~$360 - 2g + 15 - g - 70$~~

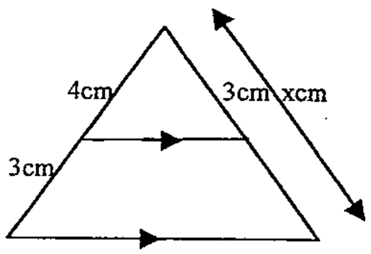
$360 - 115 = (2g-15) + (g+70)$

$245 = 3g + 55$

$3g = 190$

$g =$

e) Find x



NOT TO SCALE

f) If F(x) =3- x^2, find F(2) – F(-1)

QUESTION 2 (*Start on a new page*)

a) Differentiate the following: $\frac{1}{x}$ x

(i) $x\sqrt{x}$

(ii) $x^2+\dfrac{1}{x}$

b) The diagram shows the points A(1, 5), B(8, -2) and the point C(-6, -2)

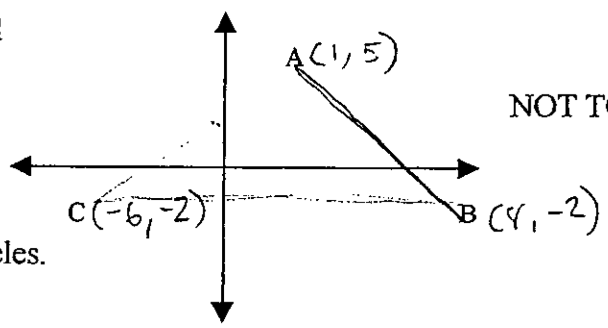
(i) Show that the gradient of AB is -1

(ii) Find the equation of line AB

(iii) Find the exact length of AB

(iv) Show that triangle ABC is isosceles.

(v) Find the mid- point of AC



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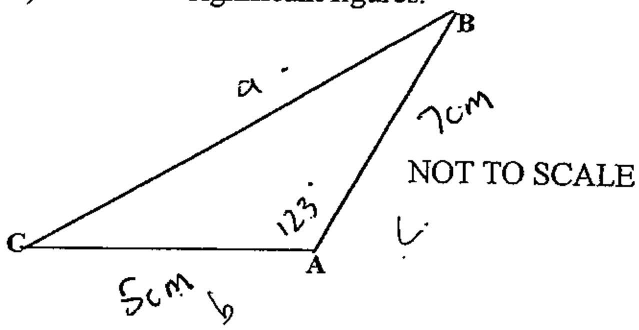
QUESTION 3 (*Start on a new page*)

a) Find the equation of the normal to the curve $y = \sqrt{x+2}$, at the point (7, 3)

b) (i) In triangle ABC , $\angle A = 123°$, b=5cm and c =7cm , find a to 3 significant figures.

(ii) Find the size of $\angle B$ to the nearest minute.

(iii) Find the area of $\triangle$ ABC



c) Calculate $\sqrt{\dfrac{1-0.25^2}{2.7^4}}$ to 3 decimal places

d) The volume of a sphere is given by $V = \frac{1}{3}\pi r^2(2r+h)$. Find the value of V if r = 3cm and h = 7cm to the closest unit.

QUESTION 4 (*Start on a new page*)

a) Solve $2x^2 - 6x + 3 = 0$

b) A curve y = f(x) is defined by its derivative $\frac{\partial y}{\partial x} = x^2 + 2x - 4$ and it is known to pass through the point (1,3). What is the equation of the curve?

c) Find (i) the co-ordinates of the stationary points of $y = x^3 - 3x + 2$ and hence determine the nature of these points.

(ii) the point of inflexion

(iii) neatly sketch the graph of $y = x^3 - 3x + 2$ showing all major features

QUESTION 5 (*Start on a new page*)

a) Solve the following equations simultaneously:

$$3x + 2y = 5$$
$$2x + y = 3$$

b) Solve $|x + 1| = 5$

c) A student lies down on the ground and views the top of a church tower at an angle of elevation of 40°. If the student is 50m from the foot of the tower, which is on the same level with the student, how high is the tower?

d) Prove $(sec\theta + tan\theta)(sec\theta - tan\theta) = 1$

e) If α *and* β are the roots of the equation $3x^2 + 15x + 6 = 0$, without solving, find the values of:

(i) $\alpha + \beta$ (ii) $\alpha\beta$ (iii) $\frac{1}{\alpha} + \frac{1}{\beta}$ (iv) $(\alpha + 2)(\beta + 2)$

QUESTION 6 (*Start on a new page*)

a) Evaluate $\sum_{n=3}^{6} \frac{1}{n}$ leaving your answer in fractional form.

b) If $T_n = 3^{2n-1}$, find the first 4 terms.

c) Solve for x, if $4Sin^2 x = 1$ and $0 \le x \le 360°$

d) State the largest possible domain of the function $f(x) = \frac{x}{x^2 - 1}$

e) For the parabola $y^2 = -8(x-3)$, find:

(i) the focal length

(ii) the co-ordinates of the focus

(iii) and the equation of the directrix

f) Find the values of k for which the equation $x^2 + 6x + k = 0$ has

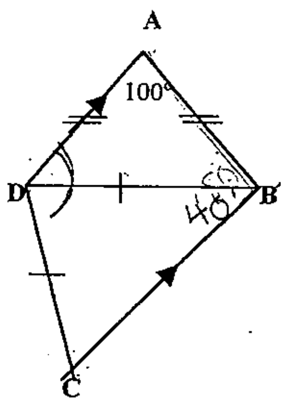
(i) equal roots

(ii) no real roots

QUESTION7 (*Start on a new page*)

a) Solve $9^x - 4(3^x) + 3 = 0$

b) Given AD = AB , DB = DC and AD || BC , find $\angle$BDC



NOT TO SCALE

In $\triangle ABD$
$\angle ADB$ ~~=~~ 40 (angle sum $\triangle$, base $\angle$s um isos $\triangle$)

$\angle DBC = 40'$ (alt $\angle$'s, AD || BC)

$\angle ADC + \angle BCD = 180'$ (co-int, $\angle$'s)

c) Find the co-ordinates of the centre and the length of the radius of the circle whose equation is

$x^2 + y^2 - 4x + 10y + 14 = 0$

$40 + x = \angle ADC$
$180 - 40 - x$

d)If $Sin\theta = \frac{24}{25}$, find the value of:

(i) $\tan\theta$

(ii) $\cot\theta$ in fractional form. $(0 \le \theta \le 90°)$

f) Factorise $16x^2 - 1$

QUESTION 8 (*Start on a new page*)

a) Change to a fraction $0.2\dot{7}$

b) Show that the locus of a point P(x, y) which moves so that its distance from the line x = 8 is twice its distance from the point (2, 0) is $3x^2 + 4y^2 = 48$

c) Solve $-5 < 2x - 3 \leq 7$ for $x \in R$ and graph the solution on the number line.

d) Express $x^2 + 2x - 2$ in the form $Ax(x+1) + Bx^2 + C(x+1)$

e) Solve $\frac{2x+7}{3} = \frac{x}{2} + 5$

g) Rewrite as a fraction 3^{-4}

g) If $Cos\theta = \frac{4^2 + 5^2 - 6^2}{2x4x5}$, find θ to the nearest minute.

h) By rationalizing the denominator express $\frac{3\sqrt{2} + 2\sqrt{3}}{3\sqrt{2} - 2\sqrt{3}}$ in the form $a + b\sqrt{6}$

QUESTION 9 (*Start on a new page*)

a) Evaluate $\lim_{x \to \infty} \frac{x}{x+1}$

b) Differentiate $\frac{3x-7}{4x+5}$

c) If a+3, 2a + 5, and 5a – 7 are in arithmetic progression, find the value of **a** and the common difference.

d) In a arithmetic progression, the first term is 3 and the twentieth term is 81. Find the sum of the first 20 terms.

e) How many terms are there in the following series?

$\frac{1}{4} + \frac{1}{2} + 1 + \ldots\ldots\ldots\ldots + 128$

f) The annual depreciation value of cars is 16%. If a car was bought for $21000 five years a go, how much is it worth now?

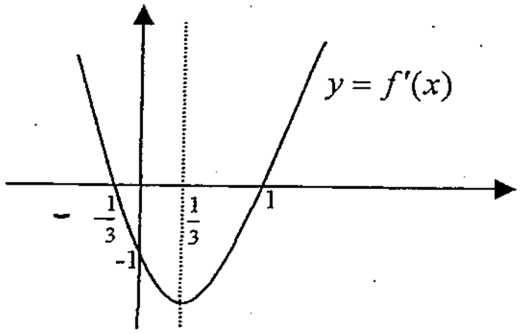
$y = 2x - 3$

y int $= -3$

x int $= \frac{3}{2}$

QUESTON 10 (*Start on a new page*)

a)



The diagram shows the graph of the gradient function of the curve y = f(x). For what value of x does f(x) have a local maximum? Justify your answer.

c) What is the condition if the series $1 - 2x + 4x^2 - 8x^3 + \ldots\ldots\ldots\ldots\ldots\ldots$ is to have a sum to infinity?

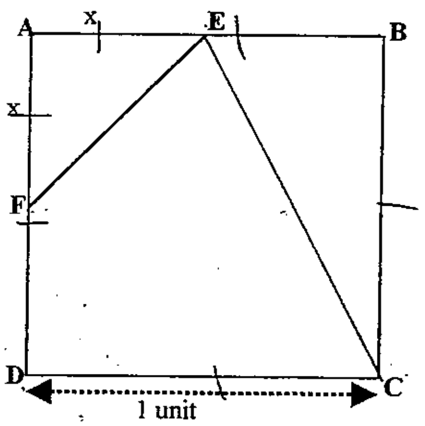
d) A man invests $1000 at the beginning of each year in a superannuation fund. Assuming interest is paid at 8% p.a on the investment, how much will his investment amount to in 30 years?

e) ABCD is a square of unit length and points E and F are taken on the sides AB and AD respectively such that AE = AF = x.

(i) Show that the area y, of the quadrilateral CDFE is given by

$$y = \frac{1}{2}(1 + x - x^2).$$

(ii) What is the greatest area the quadrilateral can have?



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