BAULKHAM HILLS HIGH SCHOOL

2016

YEAR 12 HSC ASSESSMENT TASK 2
HALF YEARLY

# Mathematics Extension 1

**General Instructions**

- Reading time – 5 minutes
- Working time – 120 minutes
- Write using black or blue pen
- Board-approved calculators may be used
- Show all necessary working in Questions 11-14
- Marks may be deducted for careless or badly arranged work

**Total marks – 70**
**Exam consists of 9 pages.**

This paper consists of TWO sections.

**Section 1 – Pages 2-4 (10 marks)**
**Questions 1-10**

- Attempt Questions 1-10
  Allow about 15 minutes for this section.

**Section II – Pages 5-9 (60 marks)**

- Attempt questions 11-14
  Allow about 1 hour and 45 minutes for this section.

**Reference Sheet is provided.**

**Section I - Multiple Choice (10 marks)**
**Allow about 15 minutes for this section.**

**Use the multiple choice page for Question 1-10**

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1 The remainder when $P(x) = 2x^3 - 5x - 3$ is divided by $(2x + 1)$?

(A) $-6$

(B) $-\frac{3}{4}$

(C) $0$

(D) $\frac{3}{4}$

2 What are the solutions to the equation

$$e^{6x} - 7e^{3x} + 6 = 0$$

(A) $x = 1$ and $x = 6$

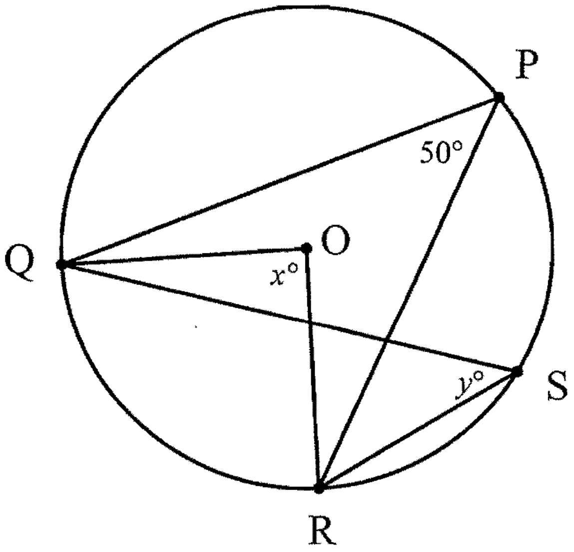
(B) $x = 0$ and $x = \frac{\log_e 6}{2}$

(C) $x = 0$ and $x = \frac{log_e 6}{3}$

(D) $x = 1$ and $x = \frac{log_e 6}{2}$

3 $P, Q, R$ and $S$ are points on a circle with centre $o$.

$\angle QPR = 50°$. Find the values of $x = \angle QOR$ and $y = \angle QSR$



(A) $x = 50°, y = 50°$

(B) $x = 100°, y = 25°$

(C) $x = 100°, y = 50°$

(D) $x = 50°, y = 25°$

4 $\int 6\cos^2 3x\,dx =$

(A) $2\sin^3 3x + c$

(B) $2\cos^3 6x + c$

(C) $3x - \frac{1}{2}\sin 6x + c$

(D) $3x + \frac{1}{2}\sin 6x + c$

5 What is the solution to

$$\frac{2t+1}{t-2} > 1$$

(A) $t > -3$

(B) $t < -3$ or $t > 2$

(C) $-3 < t < 2$

(D) $t < -2$ or $t > 3$

6 The number of ways can a committee of 12 be choosen from 15 men and 6 women so that there are at least 11 men is

(A) ${}^{21}C_{12}$

(B) ${}^{15}C_{11} \times {}^{12}C_1$

(C) ${}^{15}C_{12} + {}^{15}C_{11} \times 6$

(D) ${}^{15}C_{12} + {}^{15}C_{11}$

7 Find the inverse function, $f^{-1}(x)$, in terms of $x$, when the function $f(x)$ is given by

$$y = \frac{2}{x+1}$$

(A) $y = \frac{2-x}{x}$

(B) $y = \frac{x+1}{2}$

(C) $y = \frac{2-x}{2}$

(D) $y = \frac{2+x}{x}$

8 If $x$ is an acute angle, which of the following is equivalent to the expression

$$\sqrt{\frac{4+4\cos 2x}{1-\cos 2x}}$$

(A) $2\cot^2 x$

(B) $2\cot x$

(C) $2\tan x$

(D) $2\tan^2 x$

9 A point $P(x, y)$ moves such that $x = \sin\theta, y = \text{cosec}\theta$.
Which of the following best describes the locus of $P$?

(A) Parabola

(B) Circle

(C) Hyperbola

(D) Straight Line

10 Find $f'(x)$ if $f(x) = \tan^{-1}(x) + x\tan^{-1}(x)$

(A) $\frac{1}{1+x^2}$

(B) $\frac{x+1}{1+x^2}$

(C) $\tan^{-1} x + \frac{1}{1+x^2}$

(D) $\tan^{-1} x + \frac{x+1}{1+x^2}$

**End of Section 1**

**Section II (90 marks)**
**Allow about 2 hours and 45 minutes for this section.**

**Answer each question on the appropriate page in the writing booklet.**

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**Question 11 (15 marks)** **Marks**

a) Show that **1**

$$\lim_{x\to 0}\frac{\sin 5x}{7x}=\frac{5}{7}$$

b) Given $y = 3\cos^{-1} 2x$

(i) State the domain and range **2**

(ii) Hence sketch $y = 3\cos^{-1} 2x$ **2**

c) An object, removed from a freezer at $-5°C$, is placed in a room where the temperature is kept constant at $15°C$. Thereafter, the object's temperature, $T°C$, is changing so that after $t$ minutes,

$$\frac{dT}{dt} = k(15 - T) \qquad \text{where } k \text{ is constant}$$

(i) Verify that the function $T = 15 - Ae^{-kt}$ satisfies this condition **1**

(ii) Find the value of $A$ **1**

(iii) If initially, the temperature was increasing at $5°C$ per minute, find the value of $k$ **1**

(iv) Find to the nearest second, the time taken for the temperature to rise to 0°C **1**

d) Find the acute angle (correct to the nearest minute) between the lines **2**
$2x - 3y - 1 = 0$ and $y = \frac{x}{5} - 7$ .

e) A curve is defined by the parametric equations $x = t - 3$ and $y = t^2 - 9$.

(i) Show that $\frac{dy}{dx} = 2x + 6$. **2**

(ii) Find the equation of the normal to the curve at $t = -3$. **2**

**End of Question 11**

**Question 12 (15 marks)**

a) Given $y = 1 + \sin 4t + \sqrt{3}\cos 4t$:

(i) Express $\sin 4t + \sqrt{3}\cos 4t$ in the form $R\sin(4t + \alpha)$. **2**

(ii) Hence or otherwise, solve $1 + \sin 4t + \sqrt{3}\cos 4t = 0$ for the domain $0 \le t \le \frac{\pi}{2}$. **2**

b) A polynomial $P(x)$ is monic and of degree 5.
It has a single zero at $x = -2$ and a double zero at $x = 1$. **3**
The other two zeroes are equal in magnitude and opposite in sign.

Given $P(0) = -18$, write $P(x)$ in fully factorised form.

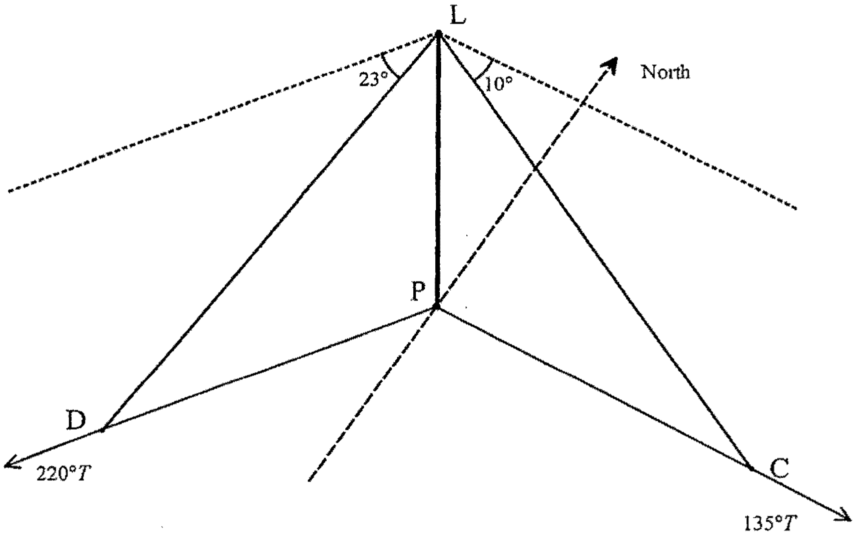
c) Evaluate **2**

$$\int_1^{\sqrt{3}} \frac{dx}{\sqrt{4 - x^2}}$$

d) Solve for $x$ **3**

$$x + 4 \ge \frac{2}{x + 3}$$

e) From the base of a lighthouse (P), a yacht (C) is on a bearing of 135°T and another vessel (D) is on a bearing of 220°T. From the top of the lighthouse (L), 115 metres above sea-level, Yacht C is at an angle of depression of 10° while vessel D is at an angle of depression of 23°. **3**



Calculate the distance between the two vessels at that moment, correct to the nearest metre.

**End of Question 12**

**Question 13 (15 marks)** **Marks**

a) (i) Show that $\sin(A+B)+\sin(A-B)=2\sin A\cos B$ **1**

(ii) Hence or otherwise, find the exact value of **2**

$$\int_0^{\frac{\pi}{6}} \sin 4x \cos 2x \, dx$$

b) Use the method of mathematical induction to prove that $4^n + 14$ is a multiple of 6 for all integers $n \geq 1$. **3**

c) The equation $x^3 + 2x^2 + 3x + 6 = 0$ has roots $\alpha, \beta$ and $\gamma$. **2**

Find the value of $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}$

d) Consider the word ***M A T H E M A T I C S***

(i) Find how many distinct ways the letters can be arranged **1**

(ii) How many arrangements are the 4 vowels together? **2**

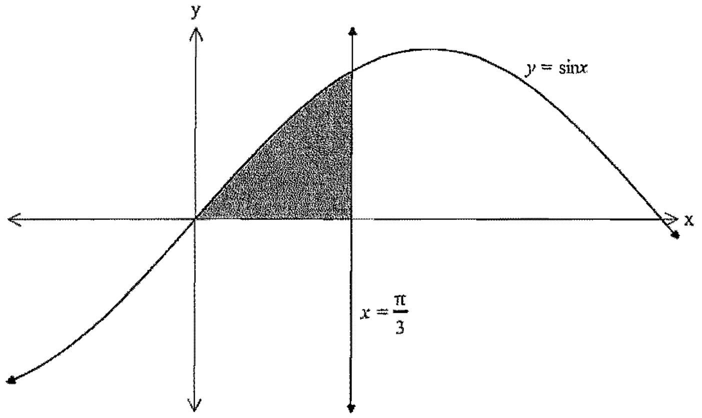
(iii) How many arrangements begin and end with the letter ***M***? **2**

e) Show that $\sin\left(2\cos^{-1}\left(\frac{p-q}{p+q}\right)\right)=\frac{4\sqrt{pq}(p-q)}{(p+q)^2}$ **2**

**End of Question 13**

## Question 14 (15 marks)

a) Find the volume formed when the area bounded by the curve $y = \sin x$, the line $x = \frac{\pi}{3}$ and the $x$-axis is rotated about the $x$-axis. **3**



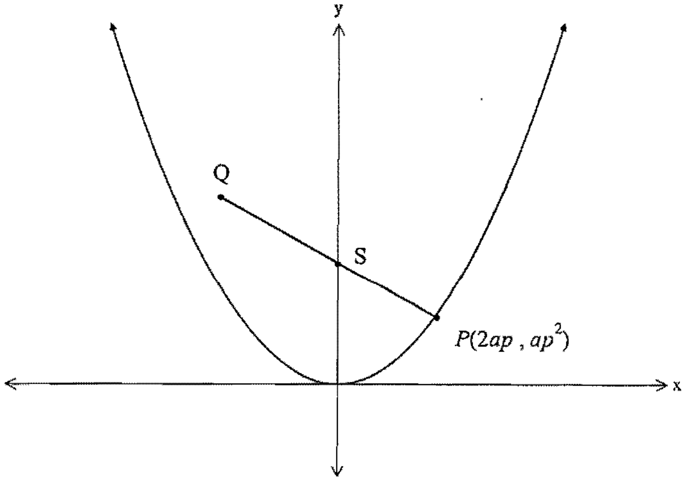
b) After $t$ months, the number of goats on an island is given by $N = 500 - 400e^{-0.1t}$.

(i) Sketch the graph of $N$ as a function of $t$, showing clearly the initial population size. **2**

(ii) Show that the rate of growth $\frac{dN}{dt}$ is given by $\frac{dN}{dt} = 0.1(500 - N)$. **1**

(iii) Find the population size for which the rate of growth is half the initial rate of growth. **1**

c) $P(2ap, ap^2)$ is a point on the parabola $x^2 = 4ay$ with focus, $S(0, a)$. The point $Q$ lies on $PS$ produced and $Q$ divides $PS$ externally in the ratio 4: 3.

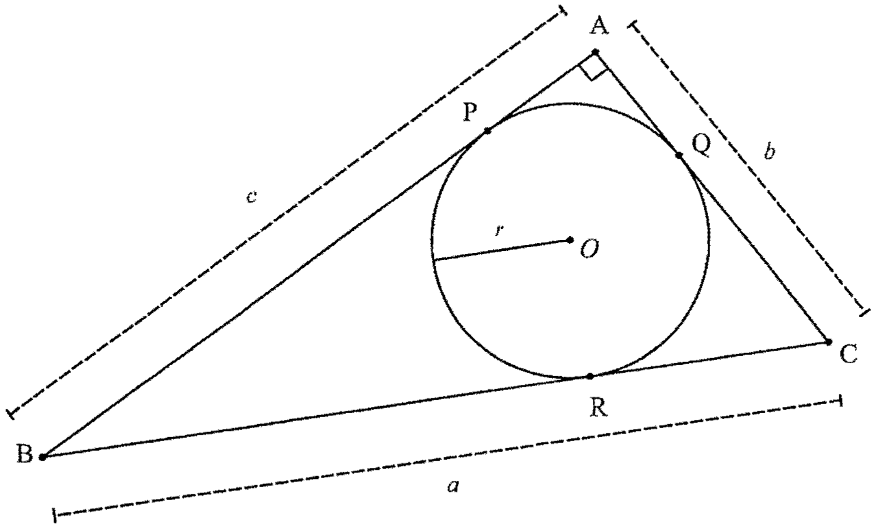


i) Show that $Q$ has co-ordinates $\left(-6ap, a(4 - 3p^2)\right)$ **2**

ii) Show that the locus of $Q$, as $P$ varies, is a new parabola and find its focal point. **2**

**Question 14 continued on the next page**

d) A triangle $ABC$, is right angled at $A$ and has sides of length $a, b$ and $c$. A circle of radius $r$, centre at $O$, is drawn so that the sides of the triangle are tangents to the circle. $P, Q$ and $R$ are the points where the tangents $AB, AC$ and $BC$ meet the circles respectively.



Copy the diagram into your answer booklet

i) Prove that $AQOP$ is a square **2**

ii) Prove that $PA$ is $\frac{c+b-a}{2}$ **2**

**End of Exam**

# Mathematics Extension 1

**Angle sum identities**

$$\sin(\theta+\phi)=\sin\theta\cos\phi+\cos\theta\sin\phi$$

$$\cos(\theta+\phi)=\cos\theta\cos\phi-\sin\theta\sin\phi$$

$$\tan(\theta+\phi)=\frac{\tan\theta+\tan\phi}{1-\tan\theta\tan\phi}$$

**$t$ formulae**

If $t=\tan\frac{\theta}{2}$, then

$$\sin\theta=\frac{2t}{1+t^2}$$

$$\cos\theta=\frac{1-t^2}{1+t^2}$$

$$\tan\theta=\frac{2t}{1-t^2}$$

**General solution of trigonometric equations**

$\sin\theta=a,\qquad \theta=n\pi+(-1)^n\sin^{-1}a$

$\cos\theta=a,\qquad \theta=2n\pi\pm\cos^{-1}a$

$\tan\theta=a,\qquad \theta=n\pi+\tan^{-1}a$

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**Division of an interval in a given ratio**

$$\left(\frac{mx_2+nx_1}{m+n},\ \frac{my_2+ny_1}{m+n}\right)$$

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**Parametric representation of a parabola**

For $x^2=4ay$,

$$x=2at,\quad y=at^2$$

At $\left(2at,\ at^2\right)$,

tangent: $y=tx-at^2$

normal: $x+ty=at^3+2at$

At $\left(x_1,\ y_1\right)$,

tangent: $xx_1=2a\left(y+y_1\right)$

normal: $y-y_1=-\frac{2a}{x_1}\left(x-x_1\right)$

Chord of contact from $\left(x_0,\ y_0\right)$: $xx_0=2a\left(y+y_0\right)$

**Acceleration**

$$\frac{d^2x}{dt^2}=\frac{dv}{dt}=v\frac{dv}{dx}=\frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

**Simple harmonic motion**

$$x=b+a\cos(nt+\alpha)$$

$$\ddot{x}=-n^2(x-b)$$

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**Further integrals**

$$\int\frac{1}{\sqrt{a^2-x^2}}\,dx=\sin^{-1}\frac{x}{a}+C$$

$$\int\frac{1}{a^2+x^2}\,dx=\frac{1}{a}\tan^{-1}\frac{x}{a}+C$$

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**Sum and product of roots of a cubic equation**

$$\alpha+\beta+\gamma=-\frac{b}{a}$$

$$\alpha\beta+\alpha\gamma+\beta\gamma=\frac{c}{a}$$

$$\alpha\beta\gamma=-\frac{d}{a}$$

**Estimation of roots of a polynomial equation**

Newton's method

$$x_2=x_1-\frac{f(x_1)}{f'(x_1)}$$

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**Binomial theorem**

$$(a+b)^n=\sum_{k=0}^{n}\binom{n}{k}a^kb^{n-k}=\sum_{k=0}^{n}\binom{n}{k}a^{n-k}b^k$$

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BOARD OF STUDIES TEACHING & EDUCATIONAL STANDARDS NSW

**2016** HIGHER SCHOOL CERTIFICATE EXAMINATION

# REFERENCE SHEET

– Mathematics –

– Mathematics Extension 1 –

– Mathematics Extension 2 –

# Mathematics

### Factorisation

$$a^2-b^2=(a+b)(a-b)$$

$$a^3+b^3=(a+b)(a^2-ab+b^2)$$

$$a^3-b^3=(a-b)(a^2+ab+b^2)$$

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### Angle sum of a polygon

$$S=(n-2)\times 180^\circ$$

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### Equation of a circle

$$(x-h)^2+(y-k)^2=r^2$$

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### Trigonometric ratios and identities

$$\sin\theta=\frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\cos\theta=\frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\tan\theta=\frac{\text{opposite side}}{\text{adjacent side}}$$

$$\operatorname{cosec}\theta=\frac{1}{\sin\theta}$$

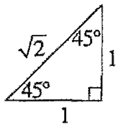
$$\sec\theta=\frac{1}{\cos\theta}$$

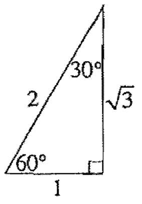
$$\tan\theta=\frac{\sin\theta}{\cos\theta}$$

$$\cot\theta=\frac{\cos\theta}{\sin\theta}$$

$$\sin^2\theta+\cos^2\theta=1$$

### Exact ratios





### Sine rule

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

### Cosine rule

$$c^2=a^2+b^2-2ab\cos C$$

### Area of a triangle

$$\text{Area}=\frac{1}{2}ab\sin C$$

### Distance between two points

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

### Perpendicular distance of a point from a line

$$d=\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$$

### Slope (gradient) of a line

$$m=\frac{y_2-y_1}{x_2-x_1}$$

### Point-gradient form of the equation of a line

$$y-y_1=m(x-x_1)$$

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### *n*th term of an arithmetic series

$$T_n=a+(n-1)d$$

### Sum to *n* terms of an arithmetic series

$$S_n=\frac{n}{2}[2a+(n-1)d] \quad \text{or} \quad S_n=\frac{n}{2}(a+l)$$

### *n*th term of a geometric series

$$T_n=ar^{n-1}$$

### Sum to *n* terms of a geometric series

$$S_n=\frac{a(r^n-1)}{r-1} \quad \text{or} \quad S_n=\frac{a(1-r^n)}{1-r}$$

### Limiting sum of a geometric series

$$S=\frac{a}{1-r}$$

### Compound interest

$$A_n=P\left(1+\frac{r}{100}\right)^n$$

# Mathematics (continued)

### Differentiation from first principles

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

### Derivatives

If $y=x^n$, then $\dfrac{dy}{dx}=nx^{n-1}$

If $y=uv$, then $\dfrac{dy}{dx}=u\dfrac{dv}{dx}+v\dfrac{du}{dx}$

If $y=\dfrac{u}{v}$, then $\dfrac{dy}{dx}=\dfrac{v\dfrac{du}{dx}-u\dfrac{dv}{dx}}{v^2}$

If $y=F(u)$, then $\dfrac{dy}{dx}=F'(u)\dfrac{du}{dx}$

If $y=e^{f(x)}$, then $\dfrac{dy}{dx}=f'(x)e^{f(x)}$

If $y=\log_e f(x)=\ln f(x)$, then $\dfrac{dy}{dx}=\dfrac{f'(x)}{f(x)}$

If $y=\sin f(x)$, then $\dfrac{dy}{dx}=f'(x)\cos f(x)$

If $y=\cos f(x)$, then $\dfrac{dy}{dx}=-f'(x)\sin f(x)$

If $y=\tan f(x)$, then $\dfrac{dy}{dx}=f'(x)\sec^2 f(x)$

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### Solution of a quadratic equation

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

### Sum and product of roots of a quadratic equation

$$\alpha+\beta=-\frac{b}{a} \qquad \alpha\beta=\frac{c}{a}$$

### Equation of a parabola

$$(x-h)^2=\pm 4a(y-k)$$

### Integrals

$$\int(ax+b)^n\,dx=\frac{(ax+b)^{n+1}}{a(n+1)}+C$$

$$\int e^{ax+b}\,dx=\frac{1}{a}e^{ax+b}+C$$

$$\int\frac{f'(x)}{f(x)}\,dx=\ln|f(x)|+C$$

$$\int\sin(ax+b)\,dx=-\frac{1}{a}\cos(ax+b)+C$$

$$\int\cos(ax+b)\,dx=\frac{1}{a}\sin(ax+b)+C$$

$$\int\sec^2(ax+b)\,dx=\frac{1}{a}\tan(ax+b)+C$$

### Trapezoidal rule (one application)

$$\int_a^b f(x)\,dx\approx\frac{b-a}{2}\big[f(a)+f(b)\big]$$

### Simpson's rule (one application)

$$\int_a^b f(x)\,dx\approx\frac{b-a}{6}\left[f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right]$$

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### Logarithms – change of base

$$\log_a x=\frac{\log_b x}{\log_b a}$$

---

### Angle measure

$180^\circ=\pi$ radians

### Length of an arc

$$l=r\theta$$

### Area of a sector

$$\text{Area}=\frac{1}{2}r^2\theta$$

X1 - Half Yearly Yr12 - 2016 soln.

## Multiple Choice

Q1 $P(-\frac{1}{2}) = 2(-\frac{1}{2})^3 - 5(-\frac{1}{2}) - 3.$

$= -0.75$ . ∴ Ⓑ

Q2. $(e^{3x}-6)(e^{3x}-1) = 0$

$e^{3x} = 6$ or $e^{3x} = 1$

$x = \frac{\ln 6}{3}$ or $x = 0$. ∴ Ⓒ

Q3. $x = 100$ (∠ at centre = 2× ∠ at circumf.)

$y = 50$ (∠ standing on same arc) ∴ Ⓒ.

Q4. $\int 6 \times \frac{1}{2}(\cos 6x + 1)\,dx$.

$= \int 3\cos 6x + 3\,dx$.

$= \frac{1}{2}\sin 6x + 3x + c$ Ⓓ

Q5. $t \neq 2$.

boundary: $2t + 1 = t - 2$.

$t = -3$. Ⓑ.

test $t = 0$ $\frac{1}{-2} \not> 1$

Q6. ${}^{15}C_{12}$ if all men.

${}^{15}C_{11} \times 6$ for 11 men and 1 women. Ⓒ

Q7. inverse : $x = \frac{2}{y+1}$.

$y + 1 = \frac{2}{x}$.

$y = \frac{2}{x} - 1$ Ⓐ

Q8 $\sqrt{\frac{4(1+\cos 2x)}{1-\cos 2x}}$.

$= 2 \times \sqrt{\frac{1 + \frac{1-t^2}{1+t^2}}{1 - \frac{1-t^2}{1+t^2}}}$ where $t = \tan x$

$= 2 \times \sqrt{\frac{1+t^2+1-t^2}{1+t^2-1+t^2}}$

$= 2 \times \sqrt{\frac{\cancel{2}}{2t^2}} = 2\sqrt{\frac{1}{t^2}} = 2\frac{1}{t}$

$= 2\cot x$ Ⓑ

Q9. $x = \sin\theta$, $y = \frac{1}{\sin\theta}$

$\therefore y = \frac{1}{x}$. Ⓒ.

Q10 $f(x) = \tan^{-1}x + x\tan^{-1}x$.

$f'(x) = \frac{1}{x^2+1} + \tan^{-1}x + \frac{1}{x^2+1} \times x$

$= \tan^{-1}x + \frac{x+1}{x^2+1}$. Ⓓ.

## Question 11.

marks.

a) $\lim_{x \to 0} \frac{5}{5} \times \frac{\sin 5x}{7x} = \lim_{x \to 0} \frac{5}{7}\ \frac{\sin 5x}{5x}$

$= \frac{5}{7}$

1mk - correct soln.

b) $y = 3\cos^{-1}2x$

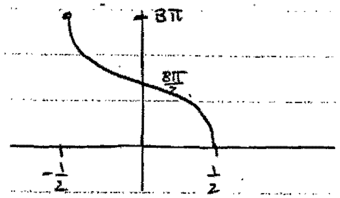
(i) Domain: $-\frac{1}{2} \leq x \leq \frac{1}{2}$

Range: $0 \leq y \leq 3\pi$

2mk - correct answer

1mk - either correct domain OR range.

(ii)



2mk - correct graph and relevant values.

1mk either correct graph or correct values

c)(i) $T = 15 - Ae^{-kt}$. $\rightarrow -Ae^{-kt} = T - 15$

$\frac{dT}{dt} = -k \times -Ae^{-kt}$

$= -k(T - 15)$

$= k(15 - T)$

1mk - correct soln

(ii) when $t = 0$, $T = -5°C$

$-5 = 15 - Ae^{0}$

$A = 20$.

1mk - correct answer

(iii) when $t = 0$ $\frac{dT}{dt} = 5$.

$5 = k(15 - -5)$.

$k = 0.25$.

1mk - correct answer.

(iv) $0 = 15 - 20e^{-0.25t}$.

$\frac{15}{20} = e^{-0.25t}$.

$-0.25t = \ln(\frac{15}{20})$

$t = 1.15$

ie 1min and 9 sec.

1mk correct answer

Marks.

Q11. d) $3y = 2x - 1$

$y = \frac{2}{3}x - \frac{1}{3}$

$m_1 = \frac{2}{3}$

$y = \frac{x}{5} - 1$

$m_2 = \frac{1}{5}$

$\tan\theta = \left| \frac{\frac{2}{3} - \frac{1}{5}}{1 + \frac{2}{3} \times \frac{1}{5}} \right|$

$= \left| \frac{7}{17} \right|$

$\theta = 22°23'$ (nearest min)

OR $\theta = 22.38°$

2mks – correct soln

1mk
- correct use of formula.
- finds both gradients.

e) (i) $x = t - 3$ $\quad y = t^2 - 9$

$t = x + 3$

$y = (x+3)^2 - 9$

$y = x^2 + 6x$.

$\frac{dy}{dx} = 2x + 6$

OR (method 2)

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$\frac{dx}{dt} = 1 \quad \frac{dy}{dt} = 2t$

$\therefore \frac{dy}{dx} = 2t$

$\frac{dy}{dx} = 2(x+3)$.

2mk - correct soln.

1mk
- eliminating t
- correct derivative

(ii) at $t = -3$

$x = -6, \; y = 0$.

$m_T = -12 + 6$

$= -6$.

$m_N = \frac{1}{6}$

qn of normal:

$y - 0 = \frac{1}{6}(x + 6)$

$x - 6y + 6 = 0$.

2mk – correct soln.

1mk – finding gradient of namal.
- finding x & y values and substituting into eqn of the line.

marks.

## Question 12

a) $y = 1 + \sin 4t + \sqrt{3} \cos 4t$

(i) $\sin 4t + \sqrt{3}\cos 4t = R\sin(4t + \alpha)$

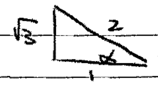
$= R\sin 4t \cos\alpha + R\cos 4t \sin\alpha$

$R\cos\alpha = 1$

$R\sin\alpha = \sqrt{3}$.

$\tan\alpha = \sqrt{3}$.

$\alpha = \frac{\pi}{3}$.



$\therefore R = 2$.

$\therefore \sin 4t + \sqrt{3}\cos 4t = 2\sin\left(4t + \frac{\pi}{3}\right)$

2mk – correct R and α

1mk
- finds R
- finds α

(ii) $1 + \underbrace{\sin 4t + \sqrt{3}\cos 4t}_{} = 0. \quad 0 \leq t \leq \frac{\pi}{2}$.

$\cancel{2}\sin\left(4t + \frac{\pi}{3}\right) = \frac{-1}{2} \qquad \frac{\pi}{3} \leq 4t + \frac{\pi}{3} \leq 2\pi + \frac{\pi}{3}$.

$\therefore 4t + \frac{\pi}{3} = \frac{7\pi}{6}, \frac{11\pi}{6}$

$4t = \frac{5\pi}{6}, \frac{3\pi}{2}$

$t = \frac{5\pi}{24}, \frac{3\pi}{8}$

2mk - correct soln

1mk
- Finds correct angle for $\left(4t + \frac{\pi}{3}\right)$

b) Ⓐ $P(x)$ monic, deg of 5 $\rightarrow x^5$

Ⓑ zero at $x = -2$ $\rightarrow (x+2)$

Ⓒ double zero at $x = 1$ $\rightarrow (x-1)^2$

Ⓓ equal in mag and opp in sign $\rightarrow (x-\alpha)(x+\alpha)$.

$P(x) = (x+2)(x-1)^2(x-\alpha)(x+\alpha)$

$P(0) = -18$.

$-18 = (2)(-1)^2(0 - \alpha^2)$.

$+9 = +\alpha^2$

$\alpha = \pm 3$.

$\therefore P(x) = (x+2)(x-1)^2(x-3)(x+3)$

3mk – correct soln

2mk satisfies any 3 of ABCD

1mk satisfies any 2 of ABCD

c) $\int_1^{\sqrt{3}} \frac{dx}{\sqrt{4-x^2}} = \left[\sin^{-1}\frac{x}{2}\right]_1^{\sqrt{3}}$

$= \sin^{-1}\frac{\sqrt{3}}{2} - \sin^{-1}\frac{1}{2}$

$= \frac{\pi}{2} - \frac{\pi}{} = \frac{\pi}{}$

2mk correct soln.

1mk
- correct integration
- correct evaluation

mark.

Q12 d) $x+4 \geq \frac{2}{x+3}$

multiply both sides by $(x+3)^2$.

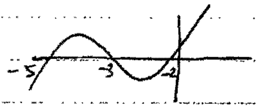
$(x+4)(x+3)^2 \geq 2(x+3)$

$(x+4)(x+3)^2 - 2(x+3) \geq 0$.

$(x+3)[(x+4)(x+3)-2] \geq 0$

$(x+3)(x^2+7x+10) \geq 0$

$(x+3)(x+2)(x+5) \geq 0$



$-5 \leq x < -3$ or $x \geq -2$

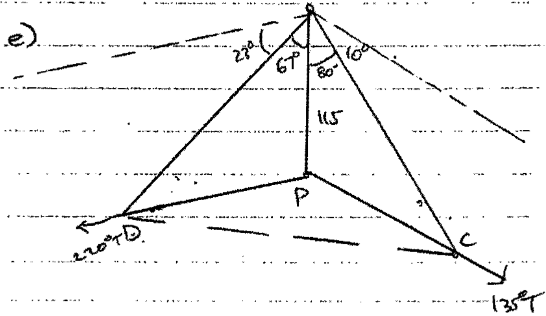
<u>3mk</u> correct soln

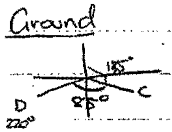
<u>2mk</u>
$-5 \leq x \leq -3$
or $x \geq -2$

<u>1mk</u> working correctly towards soln.

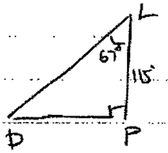
---

e)





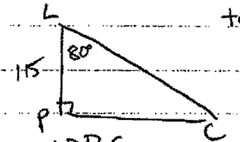
In △LPD



$\tan 67° = \frac{DP}{115°}$

$\therefore DP = 115 \tan 67$

In △ LPC



$\tan 80 = \frac{PC}{115}$

$\therefore PC = 115 \tan 80°$

In △DPC

$x^2 = DP^2 + PC^2 - 2 \times DP \times PC \cos P$

$x^2 = (115\tan 67)^2 + (115\tan 80)^2 - 2(115\tan 67)(115\tan 80)\cos 85°$

$x = 684.076\ldots$

<u>3mk</u> correct soln

<u>2mk</u>
- finds a correct expression for $DC^2$

<u>1mk</u>
- finds DP and PC
- finds $DC^2$ with incorrect value/s.

---

Question 13.

a)(i) $LHS = \sin A \cos B + \cos A \sin B$.
$+ \sin A \cos B - \cos A \sin B$
$= 2\sin A \cos B$
$= RHS$.

<u>1mk</u> correct soln

(ii) $\int_0^{\frac{\pi}{6}} \overset{A}{\sin 4x} \overset{B}{\cos 2x}\, dx$.

$= \int_0^{\frac{\pi}{6}} \frac{1}{2}(\sin 6x + \sin 2x)\, dx$

$= \frac{1}{2}\left[-\frac{1}{6}\cos 6x - \frac{1}{2}\cos 2x\right]_0^{\frac{\pi}{6}}$

$= \frac{1}{2}\left[\left(-\frac{1}{6}\cos\pi - \frac{1}{2}\cos\frac{\pi}{3}\right) - \left(-\frac{1}{6}\cos 0 - \frac{1}{2}\cos 0\right)\right]$

$= \frac{7}{24}$

<u>2mk</u> - correct soln

<u>1mk</u> using (i).

b) $4^n + 14$ is multiple of 6

Ⓐ <u>Show n=1 works</u>

$4^1 + 14 = 18 = 6 \times 3$

$\therefore$ true for $n=1$

Ⓑ <u>Assume true for n=k</u>.

ie $4^k + 14 = 6Q$ where Q is a positive integer.

ie $4^k = 6Q - 14$

Ⓒ <u>Test n=k+1</u>

Aim: $4^{k+1} + 14 = 6P$ where P is a positive integer

$LHS = 4 \times 4^k + 14$
$= 4(6Q-14) + 14$ (if assumption is true)

3mk - correct soln

<u>2mk</u>
- Proves C and, two of ABD.

<u>1mk</u>
- Proves C and one of ABD
- Shows ABD

marks

Q13 b) cont :

$= 24Q - 42$

$= 6(4Q-7)$

$= 6P$ where $P = 4Q-7$.

$\therefore$ divisible by 6

Ⓓ $\therefore$ If it is true for $n=k$, it is now proved for $n=k+1$

Since it is true for $n=1$, then it is true $n=2,3...$ ie for all positive integers of $n \geq 1$.

c) $x^3 + 2x^2 + 3x + 6 = 0$.

$$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma}$$

$$= \frac{3}{-6}$$

$$= -\frac{1}{2}.$$

2mks - correct Soln.

1mk
- finds $\beta\gamma + \alpha\gamma + \alpha\beta$
- Finds $\alpha\beta\gamma$

d) MATHEMATICS

(i) $\frac{11!}{2!2!2!} = 4989600$

1mk - correct answer

(ii) vowels (AAEI) be 1entity.

$$\frac{8!}{2!\,2!} \times \frac{4!}{2!}$$

← number of entities (8!)

← arrangement of vowels (4!)

2m's, 2T's (2! 2!); 2A's (2!).

$\therefore$ total = 120960

2mk correct Soln.

1mk
- groups vowels as one 1entity

(iii) Place M's at beginning and end

$\frac{9!}{2!\,2!}$ ← placing other letters

2T's, 2A's.

$\therefore$ total = 90720

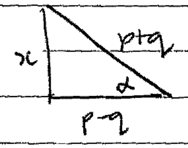
2mks correct Soln.

1mk.
- placing M at beginning and end and arranging 9 other letters

marks

Q13 e)

Let $\cos\alpha = \frac{p-q}{p+q}$.



$x^2 = (p+q)^2 - (p-q)^2$

$x^2 = 4pq$

$x = 2\sqrt{pq}$

$\sin(2\alpha) = 2\sin\alpha\cos\alpha$

$= 2 \times \frac{2\sqrt{pq}}{p+q} \times \frac{p-q}{p+q}$.

$= \frac{4\sqrt{pq}\,(p-q)}{(p+q)^2}$

2mks - correct soln

1mk
- expands $\sin 2\alpha$
- uses triangle to find opposite side ($x$).

marks

## Question 14.

a) $V = \pi \int y^2 \, dx$.

Ⓐ→ $V = \pi \int_0^{\frac{\pi}{3}} \sin^2 x \, dx$.

$V = \pi \int_0^{\frac{\pi}{3}} \frac{1}{2} \times (1-\cos 2x) \, dx$.

Ⓑ→ $V = \frac{\pi}{2} \left[ x - \frac{1}{2}\sin 2x \right]_0^{\frac{\pi}{3}}$

$V = \frac{\pi}{2}\left[ \left(\frac{\pi}{3} - \frac{1}{2}\sin\frac{2\pi}{3}\right) - \left(0 - \frac{1}{2}\sin 2\times 0\right)\right]$

$V = \frac{\pi}{2}\left[\frac{\pi}{3} - \frac{1}{2} \times \frac{\sqrt{3}}{2}\right]$

$= \frac{\pi}{2}\left[\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right]$

3marks – correct soln

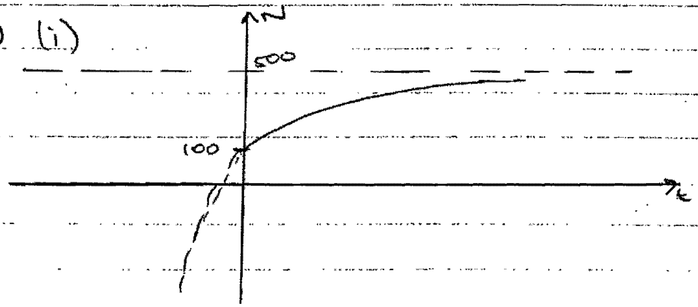
2mks

• successfully integrates $\sin^2 x$.

1mk.

• Correct application of volume formula ie Ⓐ.

b) (i)



2mks

Ⓐ correct curve

Ⓑ correct asymptote

Ⓒ labels intercept

1mk

• shows 2 out of 3 of ABC.

(ii) $N = 500 - 400(e^{-0.1t})$ then $400e^{-0.1t} = 500 - N$

$\frac{dN}{dt} = 0.1 \times 400 \, e^{-0.1t}$

$= 0.1(500 - N)$

1mk – correct soln

(iii) rate of growth $= \frac{1}{2} \times$(initial rate of growth)

initial rate of growth.

$= 0.1(500 - 100) = 0.1 \times 400$

population for $\frac{1}{2}$ rate of growth.

$= 0.1(500 - N) = (0.1 \times 400) \times \frac{1}{2}$

$N = 300$.

1mk – correct answer

---

marks.

Q14c) (i) PQ : QS = −4 : 3

$Q = \left( \frac{-4\times 0 + 3\times 2ap}{-4+3}, \frac{-4\times a + 3\times ap^2}{-4+3} \right)$

$= \left(-6ap, \; 4a - 3ap^2\right)$

$= \left(-6ap, \; -a(4 - 3p^2)\right)$

2mks – correct soln

1mk

• uses internal formula.

• doesnt show subst. of values into the formula.

(ii) $x = -6ap \qquad y = a(4 - 3p^2)$

$p = \frac{x}{-6a}$

Sub p into y.

$y = a\left(4 - 3 \times \left(\frac{-x}{6a}\right)^2\right)$

$y = a\left(4 - 3 \times \frac{x^2}{36a^2}\right)$

$y = 4a - \frac{x^2}{12a^2}$.

$x^2 = -12a(y - 4a)$

4 × focal length = 12a.

focal length = 3a.

vertex (0, 4a).

∴ focus = (0, a).

2mks – correct soln

1mk

• Finds correct cartesian eqn.

• From incorrect cartesian finds the focus.

marks

$OP = OQ$ (radii)

Q14 d) (i) $AP = AQ$ (tangents from an external pt are =) ] or

2mks - correct s

$\angle OQA = 90^\circ$ (tangent ⊥ radius).

$\angle OPA = 90^\circ$ (tangent ⊥ radius)

1mk - shows significant progre towards soln

$\angle POQ + \angle OQA + \angle OPA + \angle PAQ = 360^\circ$ ($\angle$ sum of quad)

$\therefore \angle POQ = 90^\circ$

$\therefore$ AQOP is a square (all angles $90^\circ$ and one pair of adjacent sides =).

---

(ii) Prove: $PA = \frac{1}{2}(c+b-a)$ ①.

2mks - correct Soln

Proof! $PA = AQ = PO$ (sides of a square).

$= $ radius of circle. $r$.

1mk shows significant progres towards soln .

$BP = c - r$

$QC = b - r$

$BR = BP = c - r$ (tangents from external pts are =)

$RC = QC = b - r$ (tangents from external pt are =)

$BC = c - r + b - r = a$ —— ②

① $\therefore$ LHS $= PA = r$

RHS $= \frac{1}{2}(c+b-a)$

$= \frac{1}{2}(c+b-[c-r+b-r]) \rightarrow$ using ②.

$= \frac{1}{2}(2r)$

$= r$

$= PA$.

$=$ LHS .

$\therefore$ LHS = RHS.