2009

SEMESTER 1
EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total Marks – 84

- Attempt questions 1-7
- All questions are of equal value

Question 1 (12 marks) Start a new writing booklet.

Marks

(a) Write an expression for $\sin 2\theta$ in terms for $\sin\theta$ — 1

(b) Calculate $\sum_{n=4}^{7}(2n+3)$ — 1

(c) Solve $\frac{4-2x}{x+5} \leq 2$ — 3

(d) Determine coordinates of the point P, which divides the interval A(-1,6) and B(4,-6) in the ratio of 2:3. — 2

(e) Find, in degrees and minutes, the acute angle between the lines — 2

$$y = 2x + 3 \text{ and } x - y = 1$$

(f) Calculate x to 3 decimal places, using logs, for the following equation — 2

$$3^x = 2$$

(g) Write the primitive for $2xe^{x^2}$. — 1

End of Question 1

Question 2 (12 marks) Start a new writing booklet.

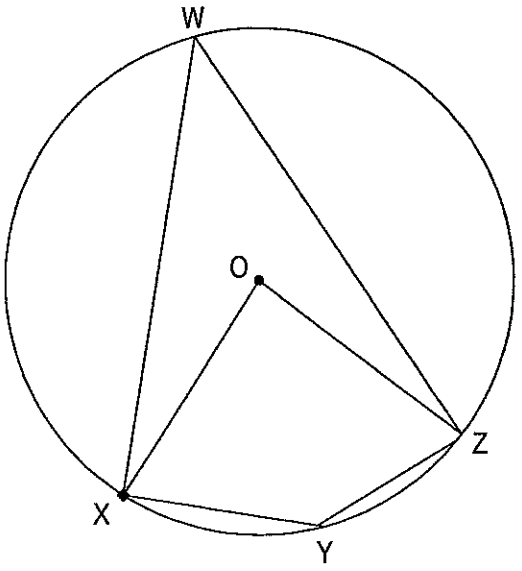
Marks

(a) Using the remainder theorem, or otherwise, fully factorise $6x^3 + 17x^2 - 4x - 3$. — 3

(b) Show that $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} sin^2 3x.\,dx = \frac{1}{2}\left(\frac{\pi}{12} - \frac{1}{6}\right)$ — 3

(c) Prove that $\frac{\sin 2\theta}{\sqrt{4-4\sin^2\theta}} = \sin\theta$. — 3

(d)



Not to Scale

WXYZ is a quadrilateral inscribed in a circle with centre O.
$\angle XWZ = 32^\circ$.
Find, giving reasons, the size of:

(i) $\angle XOZ$ **1**

(ii) $\angle XYZ$ **2**

End of Question 2

Question 3 (12 marks) Start a new writing booklet. Marks

(a) Solve $\cos\theta + \frac{1}{2} = 0, \quad 0 \le \theta \le 2\pi$ **2**

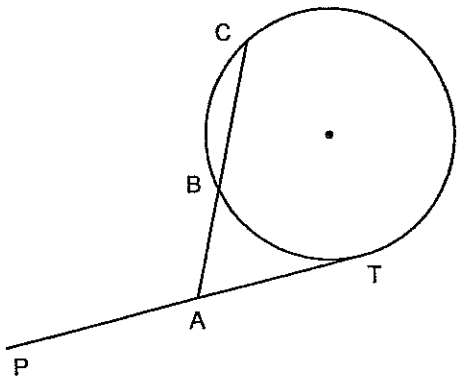
(b) Two of the roots $x^3 + ax^2 + b = 0$ are reciprocals of each other, a, b are real numbers.

(i) Show that the third root is equal to $-b$. **1**

(ii) Show that $a = b - \frac{1}{b}$ **3**

(c) TA is a tangent which touches a circle at T. TA is extended to P. C is a point on the circumference on the circle such that CA cuts the circle at B. CB =BT.

(i) Copy the diagram and add on all of the above information of the above information. **1**



(ii) Prove $\angle BAP = 3 \times \angle BTA$ giving reasons **2**

(d) **(i)** Write the expansion of $\sin(\alpha + \beta)$ **1**

(ii) Hence, find the exact value of $\sin 105^0$ **2**

End of Question 3

Question 4 (12 marks) **Start a new writing booklet.**

Marks

(a) **(i)** Show that there is a root to the equation $sinx = x - \frac{1}{2}$ between $x = 0.5$ and $x = 1.8$. **1**

(ii) Taking $x = 1.2$ as a first approximation to this solution, apply Newton's method once to find a closer approximation to the solution. Give your answer correct to two decimal places. **2**

(b) Write $(1+\sqrt{7})^3$ in the form $a + b\sqrt{7}$ **2**

(c) Using the difference of two cubes, show that

$\frac{sin^3\theta - cos^3\theta}{sin\theta - cos\theta} = 1 + \frac{1}{2}sin2\theta$, where $0 < \theta < \frac{\pi}{2}$. **3**

(d) Calculate the volume formed when $f(x) = \frac{2}{\sqrt{2x-1}}$ is rotated around the x axis between x=1 and x=5. Give an exact answer. **3**

(e) Use the factor theorem to show that $(x+4)$ is a factor of

$$x^4 + 2x^3 - 13x^2 - 14x + 24$$ **1**

End of Question 4

Question 5 (12 marks) Start a new writing booklet.

Marks

(a) Let $f(x) = \frac{x}{x^2-1}$

(i) For what values of x is $f(x)$ undefined. 1

(ii) Show that $y = f(x)$ is odd. 1

(iii) Show that as x is increasing $y = f(x)$ is decreasing for all values of x for which the function is defined. 2

(iv) Hence sketch $y = f(x)$. 2

(b) Use the method of mathematical induction to prove that $2^{2n} + 8$ is divisible by 6. 4

(c) Use $t = tan\frac{\theta}{2}$ to prove that $\frac{sin\theta}{1-cos\theta} = cot\frac{\theta}{2}$ 2

End of Question 5

Question 6 (12 marks) Start a new writing booklet.

Marks

(a) (i) Express $\cos x + \sqrt{3}\sin x$ in the form $R\sin(x + \alpha)$ for $R > 0$ 2

(ii) Hence or otherwise state the least value of $\cos x + \sqrt{3}\sin x$ 1

(b) **(i)** Given the parametric equations $x = 4t$ and $y = 2t^2$ show that the Cartesian equation is $x^2 = 8y$. **1**

(ii) Using the parametric equations show that the gradient to the tangent at $P(4t, 2t^2)$ is equal to *t*. **2**

(iii) Hence, or otherwise prove that the normal to the parabola $x^2 = 8y$ at the point $P(4t, 2t^2)$ has equation $x + ty = 2t^3 + 4t$. **2**

(iv) The point $M(3,1)$ is an external point to the parabola $x^2 = 8y$. Find the equation of the chord of contact and hence by solving two equations simultaneously, show that the coordinates of the points of intersection between the chord of contact and the parabola are $(4, 2)$ and $\left(2, \frac{1}{2}\right)$. **3**

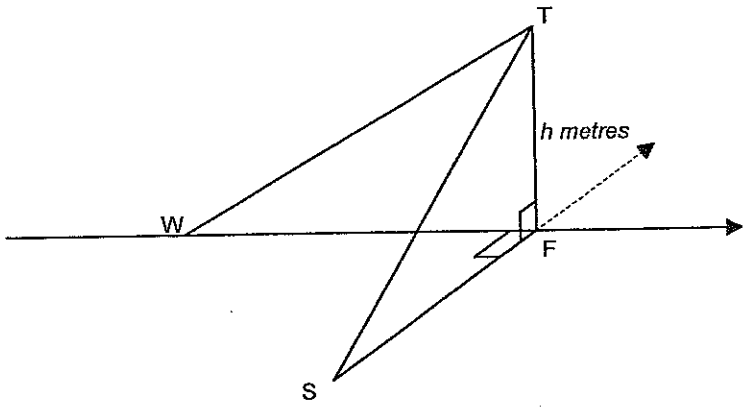
(v) Find the value of the parameter ***t,*** at the two points found in (iv). **1**

End of Question 6

Marks

Question 7 (12 marks) **Start a new writing booklet.**

(a)

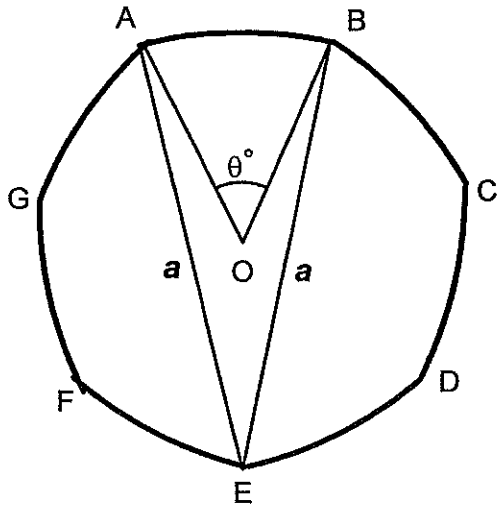


A surveyor estimates the height *h* m of a mesa in the desert by taking two readings. Standing due South of the foot *F* of the mesa at point *S*, the angle of elevation to the top of the mesa is 9°, whereas from point *W*, due west of the foot of the mesa, the angle of elevation is 15° to the top of the mesa. The surveyor knows that the distance from *S* to *W* is 1200 m.
Copy the diagram.

(i) Show that $h^2(cot^2\ 9^0 + cot^2\ 15^0) = 1200^2$ **2**

(ii) Hence find the height of the mesa correct to two significant figures. **2**

(b)



The above diagram shows a British 50 pence coin. The seven circular arcs AB, BC, ..., GA are of equal length and their centres are E, F, ..., D respectively. Each arc has a radius ***a*** units.

(i) If O is the centre of the coin, explain why θ is $\frac{2\pi}{7}$ radians. **1**

(ii) Hence or otherwise show that $\angle AEB = \frac{\pi}{7}$. **1**

(iii) Hence show that area of sector AEB is $\frac{1}{14}\pi a^2$. **1**

(iv) Hence, or otherwise, show that the face of the coin has an area **2**

$$\frac{1}{2}a^2\left(\pi - 7\tan\frac{\pi}{14}\right)$$

(c) Find $\lim\limits_{x \to 0} \frac{\sin \pi x}{x}$ **1**

(d) Find $\frac{d^2}{dx^2} \ln(e^x + 1)$ **2**

End of Question 7

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\quad n\neq -1;\ x\neq 0,\ \text{if } n<0$$

$$\int \frac{1}{x}\,dx = \ln x,\quad x>0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\quad a\neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\quad a\neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\quad a\neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\quad a\neq 0$$

$$\int \sec ax\tan ax\,dx = \frac{1}{a}\sec ax,\quad a\neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\quad a\neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a},\quad a>0,\ -a<x<a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right),\quad x>a>0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x,\quad x>0$