2010
Semester 1
HIGHER SCHOOL CERTIFICATE EXAMINATION

Student Number

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total Marks – 84

- Attempt questions 1-7
- All questions are of equal value

Question 1 (12 marks) **Start a new sheet of writing paper.** **Marks**

(a) Find the perpendicular distance from the point (1, 4) to the line **2**

$$4y = 3x - 2$$

(b) Differentiate with respect to x **1**

$$y = \sin 2x$$

(c) Differentiate with respect to x **2**

$$y = \ln\sqrt{\frac{2x-1}{3x+2}}$$

(d) Prove **3**

$$\frac{tan2\theta + cot2\theta}{tan2\theta - tan\theta} = \cot^2\theta$$

(e) Using the following function

$$y = \frac{\ln x}{x}$$

(i) Show that **1**

$$\frac{dy}{dx} = \frac{1 - \ln x}{x^2}$$

(ii) Evaluate **3**

$$\int_e^{e^2} \frac{1 - \ln x}{x \ln x}.dx = \ln(2 - 1)$$

End of Question 1

Question 2 (12 marks) **Start a new sheet of writing paper.** **Marks**

(a) Find **1**

$$\lim_{x \to 0} \frac{4x}{sin5x}$$

(b) Solve for x **2**

$$\left|\frac{2x+5}{3}\right| < 2$$

(c) Consider the polynomial $P(x) = x^3 + Ax^2 - 2008$ **2**

If $(x-2)$ is a factor of $P(x)$ find A.

(d) The point P divides the line AB externally in the ratio $3:2$. **2**
Find P if A is (2, -5) and B is (6, 1).

(e) P is the point, other than the origin, where $y = ax^2$ meets the line $y = x$

(i) Find the coordinates of P. **1**

(ii) Find, to the nearest minute, the size of the acute angle formed by the line $y = x$ and the tangent to $y = ax^2$ at P. **2**

(f) Two dice are rolled, find the probability that the sum of the two dice is = 5 **1**

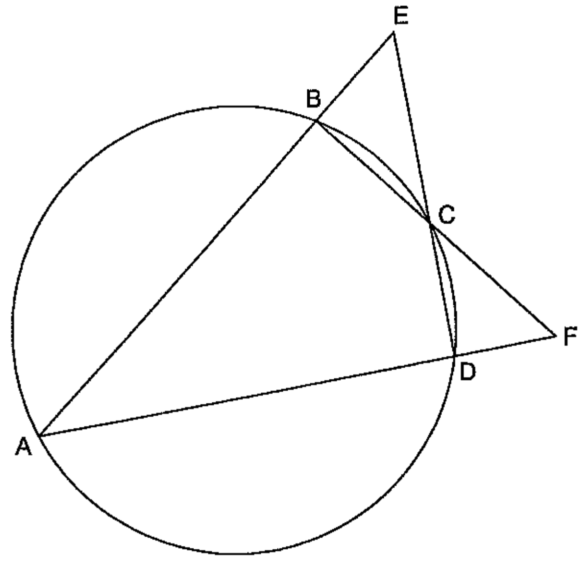
(g) Calculate the value of x to 3 dp in the following equation **1**

$$5^x = 40$$

End of Question 2

Question 3 (12 marks) Start a new sheet of writing paper. Marks

(a)



In the diagram above
DBE, ADF, DCE and BCF are straight lines.
$\angle AED = \angle BFA$.

(i) Copy the diagram into your answer booklets and add in the relevant information. **3**
Prove $\angle ABC = \angle ADC$

(ii) Hence of otherwise prove AC is a diameter. **2**

(b) (i) Find **2**

$$\frac{d}{dx}(\cos^3 x)$$

(ii) Hence or otherwise evaluate **3**

$$\int_0^{\frac{\pi}{3}} 3\sin x \cos^2 x \,.\, dx$$

(c) If $t = \tan\frac{x}{2}$, **2**

express $\frac{1-\cos x}{1+\cos x}$ in terms of t, in simplest form.

End of Question 3

Question 4 (12 marks) **Start a new sheet of writing paper.**

Marks

(a) Use Mathematical Induction to show that if x is a positive integer then $(1+x)^n - 1$ is divisible by x for all integers n such that $n \geq 1$. (Hint in step 2 let $(1+x)^n - 1 = x \times P(x)$) **4**

(b) **(i)** Prove that **2**

$$\int_0^{\frac{\pi}{4}} \sin^2 x \, . \, dx = \frac{\pi}{8} - \frac{1}{4}$$

(ii) Prove that **2**

$$\frac{d}{dx}(x \sin^2 x) - \sin^2 x = x \sin 2x$$

(iii) Hence, or otherwise, prove **2**

$$\int_0^{\frac{\pi}{4}} x \sin 2x \, . \, dx = \frac{1}{4}$$

(c) The probability of a cure with drug A is 0.6 and the probability of a cure with drug B is 0.8. If drug A is administered to one patient and drug B to another patient, what is the probability that neither patient will be cured? **2**

End of Question 4

Question 5 (12 marks) **Start a new sheet of writing paper.** **Marks**

(a) Four cards marked with the numbers 1, 2, 3 and 4 are placed in a box. Two cards are selected at random, one after the other without replacement, to form a two-digit number.

(i) Draw a tree diagram to show the possible outcomes. **1**

(ii) How many different two-digit numbers can be formed? **1**

(iii) What is the probability that the number formed is less than 34? **1**

(iv) What is the probability that the number formed is divisible by 3? **1**

(b) Graph $y = \sin 3\theta$ showing all relevant points for $-\pi \geq \theta \geq \pi$. **3**

(c) Find **2**

$$\frac{d}{dx}\frac{\sin 2x}{\cos 4x}$$

(d) Solve $\sqrt{3}\cos 2\theta - \sin 2\theta = 1$ where $0 \leq \theta \leq 2\pi$ **3**

End of Question 5

Question 6 (12 marks) **Start a new sheet of writing paper.** **Marks**

(a) **(i)** State the domain of the function 1

$$\ln(2x+3)+\ln(x-2)=2\ln(x+4)$$

(ii) Find all of the real numbers such that 2

$$\ln(2x+3)+\ln(x-2)=2\ln(x+4)$$

(b) For the function

$$f(x)=\ln\left(\frac{x^2}{x-1}\right)$$

(i) State the domain of $f(x)$ 1

(ii) Find the coordinates and nature of the stationary point on the curve $y=f(x)$ 3

(iii) Find the coordinates of the point of inflexion. 3

(iv) Sketch the graph of showing the coordinates of the stationary point and the equations of any asymptotes. 2

End of Question 6

Question 7 (12 marks) **Start a new sheet of writing paper.**

Marks

(a) Show that **3**

$$\frac{\tan 2\theta - \tan\theta}{\tan 2\theta + \cot\theta} = \tan^2\theta$$

(b) Use log laws to find **1**

$$\frac{d}{dx}(\ln\sqrt{e^x})$$

(hint $\sqrt{e^x} = e^{\frac{1}{2}x}$)

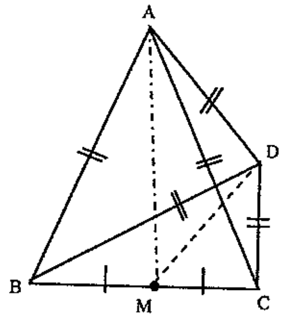
(c) **2**

$$\int \frac{e^{3x} + e^x - 5}{e^{2x}}.dx$$

(d) If α, β, γ are the roots of the equation $2x^3 - 14x - 1 = 0$ **2**
Find the value of

$$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$$

(e) The figure below ABCD is a regular tetrahedron, with



AB = AC =BC = BD = DC = AD =20*cm*.
(2M)

(i) Draw the triangle BCD in your writing booklet and mark 'M' the midpoint of BC. Prove that $DM = 10\sqrt{3}$ **2**

(ii) Determine the size of $\angle AMD$ **2**

End of Question 7

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1;\ x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0,\ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$