FINAL MARK

GIRRAWEEN HIGH SCHOOL

Mathematics Extension 1

HSC ASSESSMENT

Midyear Examination 2013

ANSWERS COVER SHEET

Name:______________________________Teacher:__________.

QUESTION	MARK	HE2	HE3	HE4	HE5	HE6	HE7
1 -5	/5						√
	/5						
6	/20						√
	/20						
7	/14						√
	/14						
8a	/4	√					√
bc	/9		√				√
	/13						
9	/18						√
	/18						
10	/14						√
	/14						
TOTAL	**/84**	/4	/9				**/84**

HSC Outcomes **Mathematics Extension 1**

HE1 appreciates interrelationships between ideas drawn from different areas of mathematics.

HE2 uses inductive reasoning in the construction of proofs.

HE3 uses a variety of strategies to investigate mathematical models of situations involving binomial probability, projectiles, simple harmonic motion and exponential growth and decay.

HE4 uses the relationship between functions, inverse functions and their derivatives

HE5 applies the chain rule to problems including those involving velocity and acceleration as functions of displacement.

HE6 determines integrals by reduction to a standard form through a given substitution.

HE7 evaluates mathematical solutions to problems and communicates them in an appropriate form.

GIRRAWEEN HIGH SCHOOL

MATHEMATICS

Midyear Examination

Year 12 Extension 1 **March 2013**

Time allowed: *120 minutes (Plus 5 minutes reading time)*

INSTRUCTIONS:

- Attempt ALL questions.
- All necessary working should be shown in every question. Marks will be deducted for careless or badly arranged work.
- Board-approved calculators may be used.
- Each question attempted is to be returned on a *separate* piece of paper labelled clearly Question 6, Question 7 etc. Each sheet of paper should clearly show your name.
- For multiple choice: write the letter corresponding to the correct answer on your answer paper.

For Questions 1 – 5 write the letter corresponding to the correct answer on your answer sheets.

(1) If α, β and γ are the roots of $2x^3 - 3x^2 + x - 5 = 0$ $\alpha^2 + \beta^2 + \gamma^2 =$

(A) $\frac{1}{2}$ (B) $\frac{5}{4}$ (C) $\frac{3}{2}$ (D) $\frac{9}{4}$

(2) The number of different ways of arranging the letters of the word MOOLOOLABA is

(A) 37 800 (B) 75 600 (C) 151 200 (D) 1 814 400

(3) The domain of $y = \ln(x^2)$ is

(A) $x > 0$ (B) $x < 0$ (C) All reals except $x = 0$ (D) All reals

(4) If $y = \ln\left(\frac{x^2}{x^2+3}\right), \frac{dy}{dx} =$

(A) $\ln\left(\frac{6x}{(x^2+3)^2}\right)$ (B) $\frac{x^2+3}{x^2}$ (C) $\frac{3-3x^2}{x(x^2+3)}$ (D) $\frac{6}{x(x^2+3)}$

(5) $\int \sqrt[3]{e^x}\,.dx =$

(A) $\frac{1}{3}\sqrt[3]{e^x} + C$ (B) $\frac{1}{3\sqrt[3]{e^x}} + C$ (C) $\sqrt[3]{e^x} + C$ (D) $3\sqrt[3]{e^x} + C$

Question 6 ***(20 Marks)Show all necessary working on a new sheet of paper.*** **Marks**

(a) Solve $\frac{2x}{x-3} \leq 8$ showing all workings. 5

(b) Divide the interval between $(4,3)$ and $(-1,13)$ externally in the ratio $7:2$. 2

(c) The angle between the lines $x - 2y = 1$and $y = mx$ is 45^o. Find both possible values of m. 5

(d) (i) Express $\sqrt{2}\mathbf{cos\theta} + \sqrt{3}\mathbf{sin\theta}$ in the form $\boldsymbol{R sin(\theta + \alpha)}$. 5

(ii) Hence or otherwise solve the equation $\sqrt{2}\cos(\boldsymbol{\theta}) + \sqrt{3}\sin\boldsymbol{\theta} = 2$ for $0 \leq \boldsymbol{\theta} \leq \mathbf{360^o}$. 3

Examination continues on the next page

Question 7 ***(14 Marks)Show all necessary working on a new sheet of paper.*** **Marks**

(a) (i) Differentiate $y = xe^x$. **1**

(ii) Hence find $\int xe^x \, .dx$ **2**

(b) Find the equation of the tangent to $y = \frac{lnx}{x^2}$ at the point where $x = e$. Leave your answer in exact form. **5**

(c) Find the volume of the solid of revolution formed when $y = xe^{2x^3}$ is rotated about the x axis between $x = 1$ and $x = 2$. Answer correct to four significant figures. **4**

(d) Find $\int \frac{\sqrt{x}}{x\sqrt{x}+1} \, .dx$ **2**

Question 8 ***(13 Marks)Show all necessary working on a new sheet of paper.***

(a) Prove using mathematical induction: **4**

$$7 + \frac{14}{5} + \frac{28}{25} + \cdots + 7 \times \left(\frac{2}{5}\right)^{n-1} = \frac{7(5^n - 2^n)}{3 \times 5^{n-1}}$$

(b) The probability that a lightbulb manufactured at a certain factory will be defective is 0.015. Out of a sample of 100 lightbulbs what is the probability that

(i) 3 will be defective. **2**

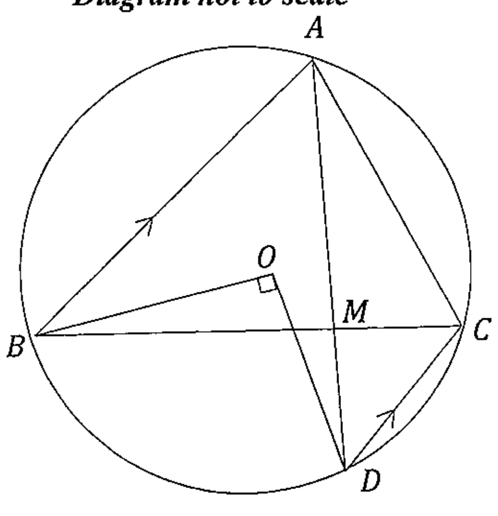
(ii) At least 99 of the lightbulbs will work. **2**

(c) Find the greatest term in the expansion of $\left(2x^2 + \frac{7}{x}\right)^{23}$ if $x = \frac{1}{2}$. **5**

Question 9 ***(18 Marks)Show all necessary working on a new sheet of paper.***

(a) In the diagram below, O is the centre of circle $ABCD$. $BA \parallel CD$ and $BO \perp OD$. AD and BC meet at M.

Diagram not to scale



(i) State why $\angle BAD = 45^o$. **1**

(ii) Prove $BC \perp AD$. **3**

(iii) Show that $BOMD$ is a cyclic quadrilateral. **1**

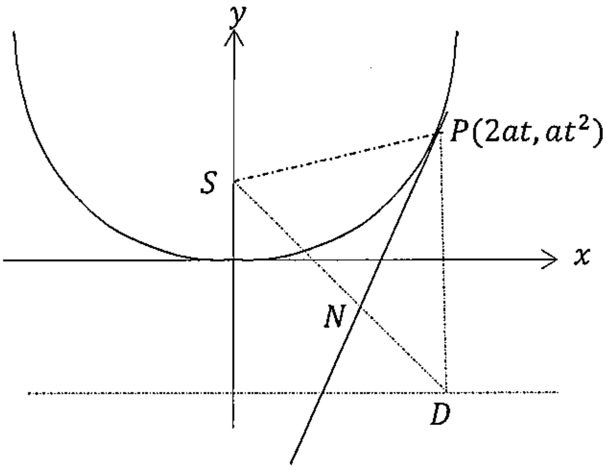
Question 9 continues on the next page.

Question 9 ***(continued)*** **Marks**

(b)Prove that $\frac{\sin\theta}{1-\cos\theta} = \cot(\frac{\theta}{2})$ *(Hint: Use t formulae)* **3**

(c) $P\ (2at, at^2)$ is a point on the parabola $x^2 = 4ay$. S is the focus of the parabola and D is the point on the directrix of the parabola directly below P. N is the point of intersection of SD and the tangent to the parabola at P. *(see diagram)*

Diagram not to scale



(i) Show that the equation of the tangent to the parabola at P is $y = tx - at^2$. **4**

(ii) Show that $SD \perp PN$. **2**

(iii) Show that N is the midpoint of SD. **2**

(iv) Prove that $\angle SPN = \angle DPN$. **2**

Question 10 ***(14 Marks)***

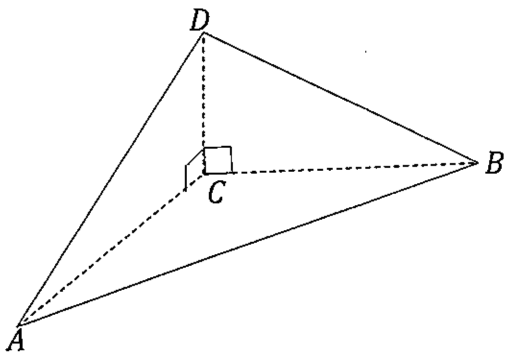
(a) Solve the polynomial equation $4x^3 + 24x^2 + 29x - 21 = 0$ given that one of its roots is equal to the sum of the other two. **6**

Question 10 continues on the next page

Question 10 ***(continued)*** **Marks**

(b) An explorer at A sees a mountain peak on a bearing of 010^o and at an angle of elevation of 2^o. The explorer then walks 263km on a bearing of 060^o to B. The explorer then sights the mountain peak on a bearing of 250^o and at an angle of elevation of 2.5^o. *(See diagram)* If C is the base of the mountain, D is the peak and h is the height of the mountain in metres:

Diagram not to scale



(i) Show that $AC = h\ tan88^o$ and find a similar expression for BC. **2**

(ii) Show that $\angle ACB = 120^o$. **2**

(iii) Show that

$$h^2(tan^2 88^o + tan^2 87.5^o - 2tan88^o tan87.5^o cos120^o) = 263000^2.$$ **2**

(iv) Find the height of the mountain. **2**

HERE ENDETH THE EXAMINATION!!!

Year 12 Ext 1 Midyear 2013 p.1

Solutions: Multiple Choice:

Q.(1) $\alpha^2+\beta^2+y^2$

$= (\alpha+\beta+y)^2 - 2(\alpha\beta+\alpha y+\beta y)$

$= \left(\frac{3}{2}\right)^2 - 2\times\frac{1}{2}$ (B)

$= \frac{5}{4}$

(2) $= \frac{10!}{4!\,2!\,2!}$ (A)

$= 37\,800$

(3) (C)

(4) $y = 2\ln x - \ln(x^2+3)$

$\frac{dy}{dx} = \frac{2}{x} - \frac{2x}{x^2+3}$ (D)

$= \frac{2x^2+6-2x^2}{x(x^2+3)}$

$= \frac{6}{x(x^2+3)}$

(5) $\int \sqrt[3]{e^x}\,dx$ (D)

$= \int e^{\frac{1}{3}x}\,dx$

$= 3e^{\frac{1}{3}x} + C$

$= 3\sqrt[3]{e^x} + C$

p-2

Q.(6)(a) $\frac{2x}{x-3} \leq 8$

By critical points:

Equality:

$\frac{2x}{x-3} = 8$ 1

$2x = 8x - 24.$

$24 = 6x$

$4 = x. [x=4].$ 1

Discontinuity:

$x \neq 3.$ 1

✓✓✓✓✓ ✓ | × × × × • ✓✓✓

3 4

Test: $x < 3$. $[x=0]$ $\frac{2\times 0}{0-3} = 0 \leq 8.$ 1

Test: $3 < x < 4$ $[x = \frac{7}{2}]$ $\frac{2\times\frac{7}{2}}{\frac{7}{2}-3} = 14 \nleq 8$

Test: $x > 4$ $[x=5]$ $\frac{2\times 5}{5-3} = 10$ $\nleq 8$

$\therefore x < 3$ and $x \geq 4.$ 1 $\underline{5}$

OR

By Times Square [New York] method.

$\frac{2x}{x-3} \leq 8$

$2x(x-3) \leq 8(x-3)^2.$ 1

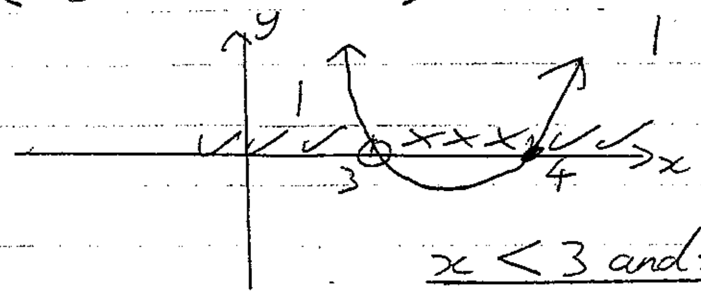
$0 \leq 8(x-3)^2 - 2x(x-3)$

$0 \leq 2(x-3)[4x - 12 - \cancel{x}]$

$0 \leq 2(x-3)(3x-12)$

$0 \leq 8.(x-3)(x-4)$ 1 $\underline{5}$

1



1

$x < 3$ and $x \geq 4.$

(6)(b)

x co-ordinate	y co-ordinate:
$4 \quad -1$	$3 \quad 13$
$7 \quad -2$	$7 \quad -2$
$= \dfrac{4\times -2 - 1\times 7}{7-2}$	$= \dfrac{3\times -2 + 13\times 7}{7-2}$
$= -3$ 1	$= 17$ 1

2

Point $= (-3, 17)$

Or using formula:

$$\text{Point} = \left(\frac{nx_1 + mx_2}{m+n}, \frac{ny_1 + my_2}{m+n}\right)$$

$$= \left(\frac{-2\times 4 + 7\times -1}{7-2}, \frac{-2\times 3 + 7\times 13}{7-2}\right)$$

$$= (-3, 17).$$

(c) $\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$

Note: $x - 2y = 1$.

$y = \frac{1}{2}x - \frac{1}{2}$.

$m_1 = \frac{1}{2}$. $m_2 = m$.

$\tan 45^{\circ} = \left|\dfrac{\frac{1}{2} - m}{1 + \frac{1}{2}m}\right|$ 1

5

$1 = \left|\dfrac{\frac{1}{2} - m}{1 + \frac{1}{2}m}\right|$

Either $\dfrac{\frac{1}{2} - m}{1 + \frac{1}{2}m} = 1$ 1 or $\dfrac{\frac{1}{2} - m}{1 + \frac{1}{2}m} = -1.$ 1

$\dfrac{1 - 2m}{2 + m} = 1$

$1 - 2m = 2 + m$

$-1 = 3m$

$-\frac{1}{3} = m.$ 1

$\dfrac{1 - 2m}{2 + m} = -1$

$1 - 2m = -2 - m$

$3 = m$ 1

$m = 3$ or $m = -\frac{1}{3}.$

(d) (i) $\sqrt{2}\cos\theta + \sqrt{3}\sin\theta \equiv R\sin(\theta+\alpha)$

$\sqrt{2}\cos\theta + \sqrt{3}\sin\theta \equiv R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$ |

Equating parts

$\sqrt{2}\cos\theta = R\cos\theta\sin\alpha$	$\sqrt{3}\sin\theta = R\sin\theta\cos\alpha$
$\sqrt{2} = R\sin\alpha \ (1)$ $_1$	$\sqrt{3} = R\cos\alpha \ (2)$ |

$(1)^2 + (2)^2$: $R^2(\sin^2\alpha + \cos^2\alpha) = (\sqrt{2})^2 + (\sqrt{3})^2$

$R = \sqrt{5}$. 1

We know α is in Q_1 ; $(1) \div (2)$

$$\frac{R\sin\alpha}{R\cos\alpha} = \frac{\sqrt{2}}{\sqrt{3}}$$

$$\tan\alpha = \frac{\sqrt{2}}{\sqrt{3}}$$ 5

$\alpha = 39°14'$. 1

$\therefore \sqrt{2}\cos\theta + \sqrt{3}\sin\theta \equiv \sqrt{5}\sin(\theta + 39°14')$

(ii) $\sqrt{2}\cos\theta + \sqrt{3}\sin\theta = 2$

$\sqrt{5}\sin(\theta + 39°14') = 2$ 1

$\sin(\theta + 39°14') = \frac{2}{\sqrt{5}}$. 3

$\theta + 39°14' = 63°26'$ or $116°34'$. 1

$\theta = 24°12'$ or $77°20'$. 1

Q.(7)(a)(i) $y = xe^x$ $\quad u = x \quad v = e^x$

$u' = 1 \quad v' = e^x$

$$y' = u'v + v'u$$
$$= 1e^x + xe^x \qquad \underline{1}$$

$$\therefore \frac{dy}{dx} = (x+1)e^x \text{ or } xe^x + e^x.$$

(ii) As $\int xe^x + e^x \, dx = xe^x + c$ [from (i)].

$$\int xe^x \, dx + \int e^x \, dx = xe^x + c \qquad \underline{2}$$

$$\int xe^x \, dx = xe^x - \int e^x \, dx + c \qquad 1$$

$$\int xe^x \, dx = xe^x - e^x + c$$
$$= (x-1)e^x + c \qquad 1$$

(b) $y = \dfrac{\ln x}{x^2}$ $\quad u = \ln x \quad v = x^2$

$u' = \dfrac{1}{x} \quad v' = 2x$

$$y' = \frac{u'v - v'u}{v^2}$$
$$= \frac{\frac{1}{x} \times x^2 - 2x\ln x}{(x^2)^2} \qquad 1$$
$$= \frac{x - 2x\ln x}{x^4}$$
$$y' = \frac{1 - 2\ln x}{x^3} \qquad 1$$

Where $x = e$, $y' = \dfrac{1 - 2\ln e}{e^3}$

$$= \frac{-1}{e^3} \qquad 1$$

Where $x = e$, $y = \dfrac{\ln e}{e^2} = \dfrac{1}{e^2}$ $\quad 1$

By $y - y_1 = m(x - x_1)$

$$y - \frac{1}{e^2} = \frac{-1}{e^3}(x - e)$$
$$e^3 y - e = -x + e$$
$$x + e^3 y - 2e = 0. \qquad 1$$

$\underline{5}$

Q.(7)(c) $V = \pi \int y^2 \, dx$

$= \pi \int_1^2 \left(xe^{2x^3}\right)^2 dx$ 1

$= \pi \int_1^2 x^2 e^{4x^3} dx$

$= \frac{\pi}{12} \int_1^2 12x^2 e^{4x^3} dx$ 1

$= \frac{\pi}{12} \left[e^{4x^3} \right]_1^2$ 1 [by $\int f'(x) \times e^{f(x)} dx = e^{f(x)}$

$= \frac{\pi}{12} \left[e^{4\times 2^3} - e^{4\times 1^3} \right]$ $\underline{4}$

$= \frac{\pi}{12} \left[e^{32} - e^{4} \right]$ 1

$= \underline{2.067 \times 10^{13} \text{ cubic units.}}$

(d) $\int \frac{\sqrt{x}}{x\sqrt{x}+1} dx$

Note: $\frac{d}{dx}\left(x\sqrt{x}+1\right) = \frac{3}{2}\sqrt{x}$.

$= \frac{2}{3} \int \frac{\frac{3}{2}\sqrt{x}}{x\sqrt{x}+1} dx$ 1 $\underline{2}$

$= \underline{\frac{2}{3} \ln\left(x\sqrt{x}+1\right) + C}$ by 1 $\int \frac{f'(x)}{f(x)} dx = \ln[f(x)] + C$.

Q.(8)(a) Step 1: Show true for $n=1$.

LHS $=7$

RHS $= 7\left(\dfrac{5^1-2^1}{3\times 5^0}\right)$

$= \dfrac{7\times 3}{3}$

$= 7$

LHS = RHS

$\therefore$ True for $n=1$.

Step 2: Assume true for $n=k$

i.e. $7+\dfrac{14}{5}+\ldots+7\times\left(\dfrac{2}{5}\right)^{k-1} = \dfrac{7(5^k-2^k)}{3\times 5^{k-1}}$ 1

$\underline{4}$

Step 3: Prove true for $n=k+1$

i.e. $7+\dfrac{14}{5}+\ldots+7\times\left(\dfrac{2}{5}\right)^{k-1}+7\times\left(\dfrac{2}{5}\right)^{k} = \dfrac{7(5^{k+1}-2^{k+1})}{3\times 5^{k}}$ 1

LHS:

$7+\dfrac{14}{5}+\ldots+7\times\left(\dfrac{2}{5}\right)^{k-1}+7\times\left(\dfrac{2}{5}\right)^{k}$

$= \dfrac{7(5^k-2^k)}{3\times 5^{k-1}} + \dfrac{7\times 2^k}{5^k}$ 1 [using Step 2]

$= \dfrac{5\times 7(5^k-2^k)+3\times 7\times 2^k}{3\times 5^k}$

$= \dfrac{7\times 5^{k+1}-5\times 7\times 2^k+3\times 7\times 2^k}{3\times 5^k}$

$= \dfrac{7\times 5^{k+1}-7\times 2\times 2^k}{3\times 5^k}$

$= \dfrac{7\times 5^{k+1}-7\times 2^{k+1}}{3\times 5^k}$ 1

$=$ RHS.

$\therefore$ If it is true for $n=k$ it will be true for $n=k+1$. Hence as it is true for $n=1$ it will be true for $n=1+1=2$ and so on for all positive integers n by the principle of mathematical induction.

NO MARKS FOR CONCLUSION!

Ext 1 Solutions: p. 8

Q. (8)(b)(i) Pr [3 defective]

$= {}^{100}C_3 \times (0.015)^3 \times (0.985)^{97}$ 1

$= 0.1260$ [4DP]. 1 2

(ii) At least 99 work

= Pr [1 defective] + Pr [none defective]

$= {}^{100}C_1 \times 0.015^1 \times 0.985^{99} + {}^{100}C_0 \times 0.985^{100}$ 1

$= 0.5566$ [4DP]. 1 2

(c)
$$\frac{T_{k+1}}{T_k} = \frac{{}^{23}C_k \times \left[2\times\left(\tfrac{1}{2}\right)^2\right]^{23-k} \times \left(\frac{7}{\left(\frac{1}{2}\right)}\right)^k}{{}^{23}C_{k-1} \times \left[2\times\left(\tfrac{1}{2}\right)^2\right]^{24-k} \times \left(\frac{7}{\frac{1}{2}}\right)^{k-1}}$$ 1 1

$$= \frac{23! \times \left(\frac{1}{2}\right)^{23-k} \times 14^k}{k!(23-k)!} \div \frac{23! \times \left(\frac{1}{2}\right)^{24-k} \times 14^{k-1}}{(k-1)!(24-k)!}$$

$$= \frac{\cancel{23!} \times \left(\frac{1}{2}\right)^{\cancel{23-k}} \times \cancel{14^k}^{\,14}}{\cancel{k!}_{\,k}(\cancel{23-k})!} \times \frac{\cancel{(k-1)!}\,\cancel{(24-k)!}^{\,(24-k)}}{\cancel{23!} \times \left(\frac{1}{2}\right)^{24-k} \times \cancel{14^{k-1}}}$$ 5

$$= \frac{14 \times (24-k)}{k \quad \frac{1}{2}} \begin{matrix}\times 2 \\ \times 2\end{matrix}$$

$$= \frac{28(24-k)}{k}$$ 1

Finding where $\frac{T_{k+1}}{T_k} > 1$

$$\frac{28(24-k)}{k} > 1$$

$672 - 28k > k$

$672 > 29k$

$23\frac{5}{29} > k$

$\therefore k < 23\frac{5}{29}$ 1

$\therefore$ If $k = 23$, $\frac{T_{k+1}}{T_k} > 1$.

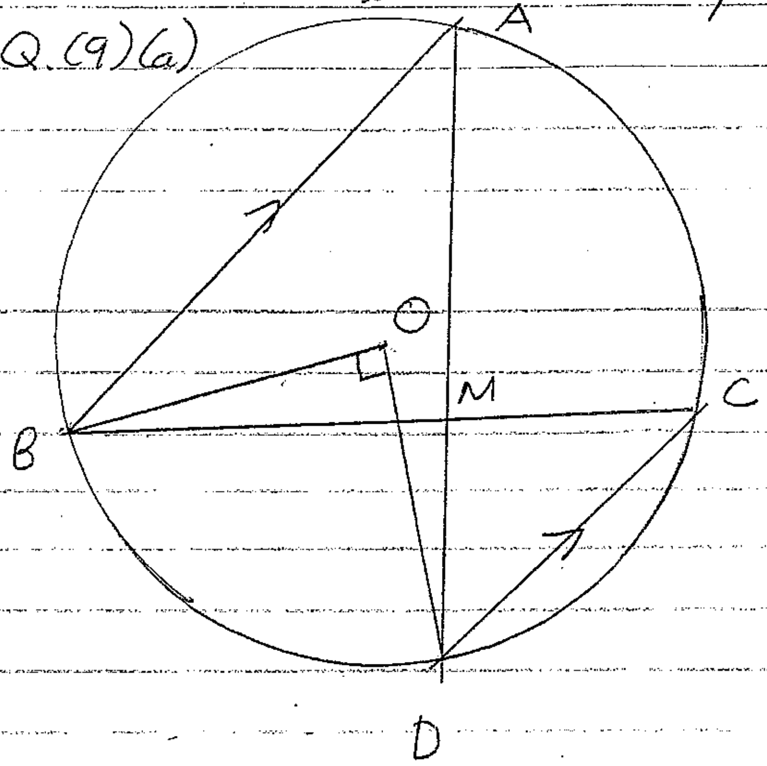
T_{24} is biggest

$= {}^{23}C_{23} \times \left(\frac{1}{2}\right)^0 \times 14^{23}$

$= 2.2959 \times 10^{26}$ [4DP] 1

Ext1 Solutions p. 9

Q.(9)(a)



(i) $\angle BAD = 45^\circ$ [$\angle$ at circumference $= \frac{1}{2} \angle$ at centre on same arc]. 1

(ii) $\angle ADC = 45^\circ$ [alternate $\angle$'s, $BA \parallel DC$]. 1

$\angle BCD = 45^\circ$ [$\angle$'s at circumference on same arc =] 1 — 3

$\angle DMC = 90^\circ$ [$\angle$ sum $\triangle DMC$]. 1

Hence $BC \perp AD$.

(iii) As $\angle BOD = \angle BMD = 90^\circ$

O and M must be on the same arc BD of circle BOMD

[$\angle$'s on same arc =] 1

Hence BOMD is a cyclic quadrilateral.

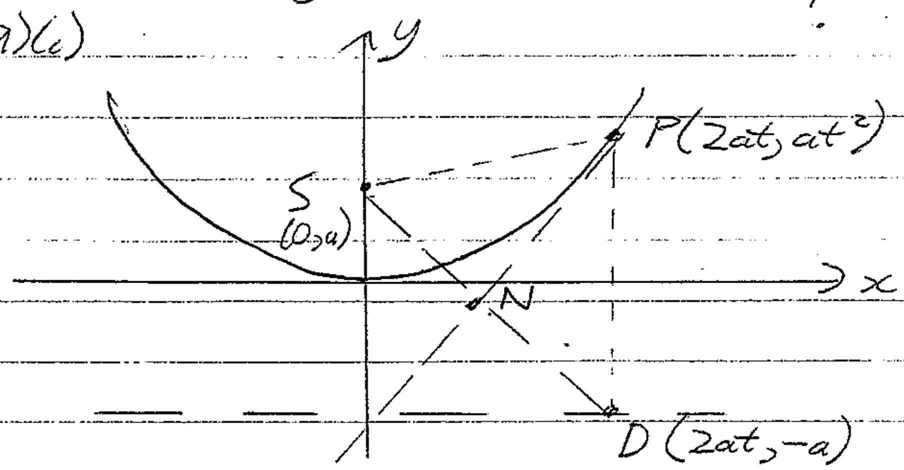
(b)
$$\frac{\sin\theta}{1-\cos\theta}$$
$$= \frac{\frac{2t}{1+t^2}}{1-\frac{1-t^2}{1+t^2}} \quad 1$$
[using $t = \tan\frac{\theta}{2}$].
$$= \frac{2t}{1+t^2-(1-t^2)}$$
$$= \frac{2t}{2t^2}$$
$$= \frac{1}{t} \quad 1$$
$$= \frac{1}{\tan(\frac{\theta}{2})} = \cot\left(\frac{\theta}{2}\right) \quad 1 \quad = RHS$$

3

OR

$$\frac{\sin\theta}{1-\cos\theta}$$
$$= \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{1-(1-2\sin^2\frac{\theta}{2})} \quad 1$$
$$= \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}}$$
$$= \frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}} \quad 1$$
$$= \cot\frac{\theta}{2} \quad 1$$
$$= RHS$$

Q.(9)(i)



(i) $x^2 = 4ay$

$y = \frac{1}{4a}x^2$

$\frac{dy}{dx} = \frac{x}{2a}$ 1

At $(2at, at^2)$ $\frac{dy}{dx} = \frac{2at}{2a}$

$= t.$ 1

By $y - y_1 = m(x - x_1)$

$y - at^2 = t(x - 2at)$ 1

$y - at^2 = tx - 2at^2$

$y = tx - at^2.$ 1 $\underline{4}$

(ii) $mSD = \frac{-a-a}{2at-0}$

$= -\frac{1}{t}$ 1

$\therefore mSD \times mPN = -\frac{1}{t} \times t$

$= -1$ 1 $\underline{2}$

$SD \perp PN.$

(iii) Midpoint of SD

$= \left(\frac{0+2at}{2}, \frac{a-a}{2}\right)$

$= (at, 0)$ 1

Seeing if this is on PN [tangent at P]

$y = tx - at^2$

$0 = t \times at - at^2$

$0 = 0$ 1 $\underline{2}$

$\therefore$ N is midpoint of SD.

(iv) $SN = DN$ [as N is midpoint SD]

PN common

$\angle SNP = \angle DNP = 90°$ [as $SD \perp PN$]

$\therefore \triangle SNP \equiv \triangle DNP$ [SAS] 1

$\therefore \angle SPN = \angle DPN$ [matching $\angle$'s, 1

$\triangle SNP \equiv \triangle DNP$]. $\underline{2}$

OR

$SP = DP$ [definition of parabola]

PN common

$SN = DN$ [as N is midpoint of SD]

$\triangle SNP \equiv \triangle DNP$ [SSS].

$\angle SPN = \angle DPN$ [matching $\angle$'s,

$\triangle SNP \equiv \triangle DNP$].

Note: Could also do RHS with

$SP = PD$, $SN = PN$, $\angle SNP = 90°$.

Q.(10)(a) $4x^3+24x^2+29x-21=0$

Roots $= \alpha, \beta, \alpha+\beta$: 1

Sum of roots: $\alpha+\beta+(\alpha+\beta) = -\frac{24}{4}$

$$2(\alpha+\beta) = -6$$

$$\alpha+\beta = -3. \quad 1 \quad (1)$$

Sum 2 at a time: $\alpha\beta+\alpha(\alpha+\beta)+\beta(\alpha+\beta) = \frac{29}{4}$

$$\alpha\beta+(\alpha+\beta)^2 = \frac{29}{4}$$

$$\alpha\beta+(-3)^2 = \frac{29}{4} \text{ from (1)}$$

$$\alpha\beta = -\frac{7}{4} \quad (2) \quad 1$$

Sub. (1) in (2): $\alpha(-3-\alpha) = -\frac{7}{4}$. 1

$$4\alpha(3+\alpha) = 7$$

$$4\alpha^2+12\alpha-7=0$$

$$(2\alpha-1)(2\alpha+7)=0$$

$$\alpha=\frac{1}{2} \text{ or } \alpha=-\frac{7}{2}. \quad 1$$

$\underline{6}$

If $\alpha=\frac{1}{2}$, $\beta=-3-\frac{1}{2}=-\frac{7}{2}$

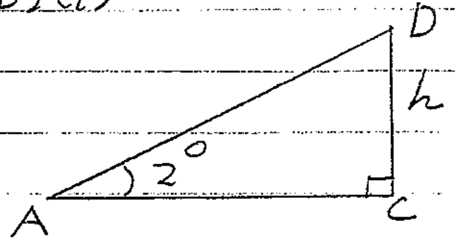
If $\alpha=-\frac{7}{2}$, $\beta=-3--\frac{7}{2}=\frac{1}{2}$.

Also $\alpha+\beta=-3$

$\therefore$ Roots of $4x^3+24x^2+29x-21=0$

are $\frac{1}{2}, -\frac{7}{2}, -3$. 1

Q.(10)(b)(i)



$\angle ADC = 88°$ [$\angle$ sum $\triangle ADC$]

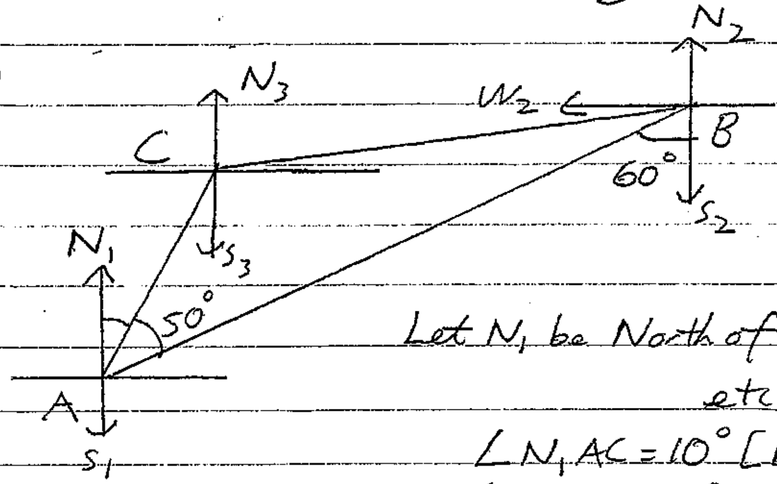
By $\tan\theta = \frac{O}{A}$

$\tan 88° = \frac{AC}{h}$

$h \tan 88° = AC$ 1 $\underline{2}$

& $h\tan 87.5° = BC$ [using $\triangle BDC$]. 1

(ii)



Let N_1 be North of A, N_2 North of B, N_3 North of C etc.

$\angle N_1AC = 10°$ [bearing of C from A] $\underline{2}$

$\angle N_1AB = 60°$ [" B "]

$\angle CAB = 50°$ [adjacent $\angle$'s]. 1

$\angle S_2BA = 60°$ [alternate $\angle$'s, $N_1A \parallel BS_2$ OR bearing of A from B]

$\angle CBS_2 = 70°$ [bearing of C from B] 1

$\therefore \angle CBA = 10°$ [adjacent $\angle$'s]

$\therefore \angle ACB = 120°$ [$\angle$ sum $\triangle ACB$].

PTO →

Q. (10)(b)(iii)

Using the cosine rule on $\triangle ACB$

$$(AB)^2 = (AC)^2 + (BC)^2 - 2 \times AC \times BC \times \cos\angle ACB$$

$$263\,000^2 = h^2\tan^2 88^\circ + h^2\tan^2 87{\cdot}5^\circ - 2h^2\tan 88^\circ \tan 87{\cdot}5^\circ \cos 120^\circ \quad 1$$

$$\therefore 263\,000^2 = h^2\left[\tan^2 88^\circ + \tan^2 87{\cdot}5^\circ - 2\tan 88^\circ \tan 87{\cdot}5^\circ \cos 120^\circ\right] \quad 1 \quad \underline{2}$$

(iv) $$h^2 = \frac{263\,000^2}{\tan^2 88^\circ + \tan^2 87{\cdot}5^\circ - 2\tan 88^\circ \tan 87{\cdot}5^\circ \cos 120^\circ} \quad 1$$

$$h^2 = 34575933{\cdot}64\ldots$$

$$h = 5880{\cdot}13041 \quad 1 \quad \underline{2}$$

$\therefore$ The height of the mountain is 5880 m [nearest metre].