**GOSFORD HIGH SCHOOL**
**YEAR 12 - 2000**

**HALF YEARLY EXAMINATION**

*MATHEMATICS*

*3 Unit (Additional)*
*&*
*3/4 Unit (Common)*

Time Allowed - 2 hours
(plus 5 mins .reading time)

**Directions to Students:**

* Attempt <u>ALL</u> questions

* All questions are of equal value

* All necessary working is to be shown, in every question. (Marks may be deducted for careless or badly arranged work)

* Board approved calculators may be used

* Standard integral sheets are supplied

* Start each question on a **SEPARATE** page

## Question 1

a) Differentiate $2x\tan^{-1}x$

b) Find the exact value of $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec^2 x\,dx$

c) Find the acute angle between the lines $3y = 2x + 8$ and $y = 5x - 9$.

d) If $\alpha, \beta, \gamma$ are the roots of the polynomial $2x^3 - 14x - 1 = 0$, find $\alpha\beta\gamma$

e) Prove the identity $\dfrac{2\tan A}{1+\tan^2 A} = \sin 2A$

f) Evaluate $\int_2^{10} \dfrac{x}{\sqrt{x-1}}dx$ using the substitution $x = t^2 + 1$

## Question 2

a) i) By considering the sum of the terms of an arithmetic series show that

$$(1+2+3+\ldots\ldots+n)^2 = \frac{1}{4}n^2(n+1)^2$$

ii) By using the Principle of Mathematical induction prove that $1^3 + 2^3 + \ldots\ldots + n^3 = (1+2+\ldots\ldots+n)^2$ for all $n \geq 1$.

b) i) The polynomial equation $P(x) = 0$ has a double root at $x = a$. By writing $P(x) = (x-a)^2\ Q(x)$, where $Q(x)$ is a polynomial, show that $P^1(a) = 0$

ii) Hence or otherwise, find the values of a and b if $x = 1$ is a double root of $x^4 + ax^3 + bx^2 - 5x + 1 = 0$

## Question 3

a) Sketch a curve which has <u>all</u> of the following properties:

| | | | |
|---|---|---|---|
| $f(-4)=0$ | $f(8)=0$ | $f^1(-5) > 0$ | $f^1(-3)\rangle 0$ |
| $f^1(5)=0$ | $f^{11}(-6)\langle 0$ | $f^{11}(-1)\rangle 0$ | $f^{11}(0)=0$ |
| and | $f^{11}(5)\langle 0$ | | |

b) Solve $\dfrac{1}{1-x^2} \leq 4$ and graph the solution set on the number line.

c) i) Find the co ordinates of the point of intersection of the curves $y = \sin^{-1} x$ and $y = \cos^{-1} x$.

ii) Find the angle between the tangents to these curves at their point of intersection.

## Question 4

a) Find the co-ordinates of the point which divides the line joining the points (-1,3) and (5, -7) externally in the ratio 4:3.

b) Sketch the graph $|x| + |2y| = 8$

c) $P$ and $Q$ are points on the parabola $x^2 = 4ay$ with parameters $p$ and $q$. $PQ$ subtends a right angle at the origin.

i) Show that $pq + 4 = 0$.

ii) Find the co-ordinates of $M$, the mid-point of $PQ$.

iii) Find the locus of $M$.

## Question 5

a) Sketch the function $\sin^{-1} 2|x|$ for $-\dfrac{1}{2} \leq x \leq \dfrac{1}{2}$

b) i) Write down the range of the function $y = \dfrac{-1}{2(1+x^2)}$

ii) Write down the range of $\sin^{-1}\left[\dfrac{-1}{2(1+x^2)}\right]$

c) $P(2ap, ap^2)$ lies on the parabola $x^2 = 4ay$. the line L is a tangent at P. S is the focus. Show that L is equally inclined to SP and the axis of the parabola.

d) Use mathematical induction to prove that, for every positive integer n, $13 \times 6^n + 2$ is divisible by 5.

**Question 6**

a) Find $\displaystyle\int \frac{dx}{\sqrt{1-4x^2}}$ , using the substitution $x = \frac{1}{2}\sin\Theta$

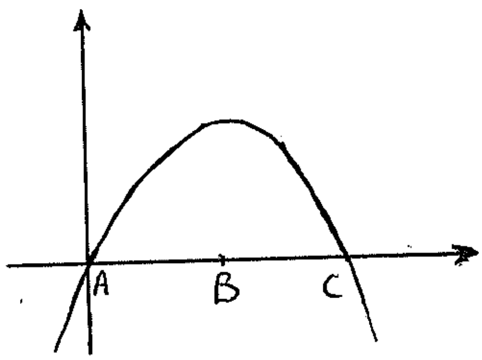
b) A piece of wire 24 cm long is cut into two pieces. The first is bent to form a square of side $x$ cm and the second is bent to form a circle.

i) Write down an expression for the sum of their areas.

ii) Find the greatest value for the sum of the areas.

c) Solve $\dfrac{1}{|3-x|} \geq \dfrac{1}{2}$

d) The sketch shows $y = f^1(x)$




Sketch i) $y = f(x)$

ii) $y = f^1(x)$

and iii) $y = f''(x)$

<u>down</u> the page, showing clearly the shape and position of each graph at the points which correspond with A, B and C in the graph of $y = f'(x)$ shown.

## Question 7

a) The roots of $x^2 + mx + n = 0$ are $\propto, \beta$
Find the values, in terms of $m$ and $n$ of

i) $\propto + \beta$

ii) $\propto \beta$

iii) $\propto^2 + \beta^2$

iv) $3 \propto^2 \beta + 3 \propto \beta^2$

b) Ascertain whether the following statements are TRUE or FALSE. Use diagrams and words of the English language to support your answers.

i) $\int_{-1}^{1} x^2 \cos x dx = 2\int_{0}^{1} x^2 \cos x dx$

ii) $\int_{-3}^{3} (x \cos x + \sin x) = 0$

c) Differentiate $2x \tan^{-1} x - \ell n(1+x^2)$ and hence find
$\int_0^1 \tan^{-1} x \ dx$

d) Consider the circle $x^2 + y^2 - 2x - 14y + 25 = 0$
24

i) Show that if the line $y = mx$ intersects the circle in two distinct points, then $(1+7m)^2 - 25(1+m^2) \rangle 0$.

ii) For what values of $m$ is a line $y = mx$ a tangent to the circle?

The following list of standard integrals may be used:

$$\int x^n dx = \frac{1}{n+1}x^{n+1}, \quad n \neq 1; \ x \neq 0, \ \text{if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\, dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\, dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\, dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\, dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right).$$

Note: $\ln x = \log_e x, \quad x > 0$

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1,2 Rick Q3,4 Tony Q5,6 Chris Q7 Allan

# Year 12 Half Yearly – 3 Unit Maths [Solutions]

## Question 1 [12 mks]

(2) (a) $\frac{d}{dx}(2x \tan^{-1}x)$

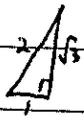
$= \tan^{-1}x\,(2) + 2x \cdot \frac{1}{1+x^2}$

$= 2\tan^{-1}x + \frac{2x}{1+x^2}$

(2) (b) $\int_{\frac{\pi}{6}}^{\pi/3} \sec^2 x \, dx$

$= \left[\tan x\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$

$= \tan\frac{\pi}{3} - \tan\frac{\pi}{6}$



$= \sqrt{3} - \frac{1}{\sqrt{3}}$

(2) (c) $3y = 2x + 8 \longrightarrow y = \frac{2}{3}x + \frac{8}{3}$

$m_1 = \frac{2}{3}$

$y = 5x - 9 \longrightarrow m_2 = 5$

$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$= \left|\frac{5 - 2/3}{1 + 5 \times \frac{2}{3}}\right| = \frac{4\frac{1}{3}}{4\frac{1}{3}}$

$= 1$

$\therefore$ Acute angle between lines $= 45^\circ$

(1) (d) $2x^3 - 14x - 1 = 0$

$\alpha\beta\gamma = \frac{-d}{a} = -\frac{-1}{2}$

$= \frac{1}{2}$

(2) (e) Prove that $\frac{2\tan A}{1+\tan^2 A} = \sin 2A$

LHS $= \frac{2\tan A}{\sec^2 A} = \frac{2\sin A}{\cos A} \times \frac{1}{\sec^2 A}$

$= \frac{2\sin A}{\cos A} \cdot \cos^2 A$

$= 2\sin A\cos A$

$= \sin 2A$

(3) (f) $\int_2^{10} \frac{x}{\sqrt{x-1}}\, dx = I$

Put $x = t^2 + 1$, $\frac{dx}{dt} = 2t$ When $x = 2$, $t = 1$

$I = \int_1^3 \frac{t^2+1}{\sqrt{t^2+1-1}} \frac{dx}{dt}\, dt$ When $x = 10$, $t = 3$

$= \int_1^3 \frac{t^2+1}{\cancel{t}} \cdot 2\cancel{t}\, dt$

$= \int_1^3 (2t^2 + 2)\, dt$

$= \left[\frac{2t^3}{3} + 2t\right]_1^3$

$= F(3) - F(1)$

$= (2 \times 9 + 6) - \left(\frac{2}{3} + 2\right)$

$= 24 - 2\frac{2}{3}$

$= 21\frac{1}{3}$

## Question 2 [12 mks]

(6) 5 (a) i) $(1 + 2 + 3 + \ldots + n)^2$

$= (\text{Sum of an A.P.})^2$

$= \left[\frac{n}{2}(a + l)\right]^2$

$= \left[\frac{n}{2}(1 + n)\right]^2$

$= \frac{n^2}{4}(1+n)^2$

(ii) Let $S_n = 1^3 + 2^3 + \ldots + n^3 = (1 + 2 + \ldots + n)^2$

Step 1: Show that $S_n$ is true for $n = 1$

LHS $= 1^3 = 1$ RHS $= (1)^2 = 1$

$S_n$ is true for $n = 1$

Step 2: Assume $S_n$ true for $n = k$

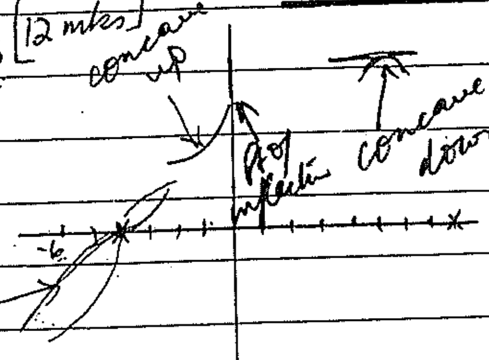
ie that

$1^3 + 2^3 + \ldots + k^3 = (1 + 2 + \ldots + k)^2$

## Question 3 [12 mks]

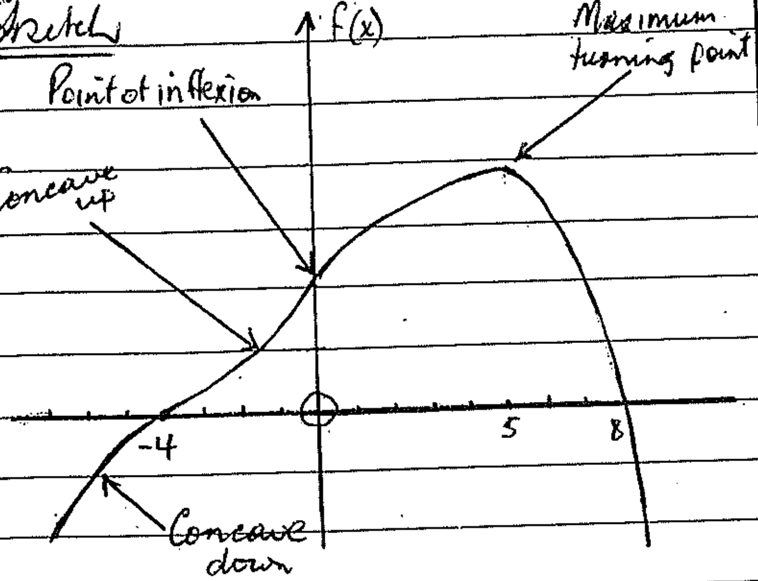
a)

Thinking!




(4) (a)

Sketch




(4) (b)

$$\frac{1}{1-x^2} \leq 4$$

$$1-x^2 \geq \frac{1}{4}$$

Solve $1-x^2 = \frac{1}{4}$

$$x^2 = 3/4$$

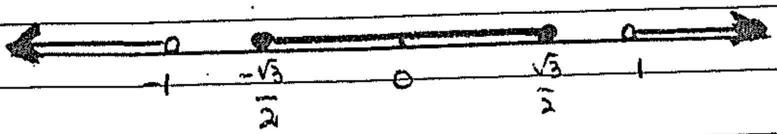
$$x = \pm \frac{\sqrt{3}}{2}$$

Also $1-x^2 < 0$ (cannot equal 0)

ie $x^2 > 1$

$x > 1$ and $x < -1$

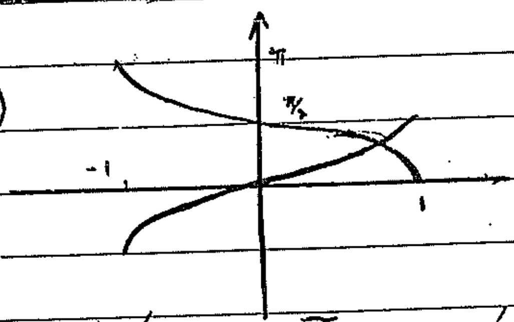
[Plot the four points on the number line and test regions]




Solution is

$x < -1,\ -\frac{\sqrt{3}}{2} \leq x \leq \frac{\sqrt{3}}{2},\ x > 1$

(c) i) (4) 2 2




$\frac{\pi}{4} = \sin^{-1}\frac{1}{\sqrt{2}}$ $\qquad \frac{\pi}{4} = \cos^{-1}\frac{1}{\sqrt{2}}$

Point of intersection is $\left(\frac{1}{\sqrt{2}}, \frac{\pi}{4}\right)$

[OR solve $x = \sin y$ $x = \cos y$ simultaneously.

ii) $y = \sin^{-1} x$ $\qquad y = \cos^{-1} x$

$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$ $\qquad \frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}$

$$= \frac{1}{\sqrt{1-\frac{1}{2}}} \text{ at } x = \frac{1}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{\frac{1}{2}}} = \frac{1}{\frac{1}{\sqrt{2}}}$$

$$= \sqrt{2}$$

$\therefore m_1 = \sqrt{2}$ $\qquad m_2 = -\sqrt{2}$

Angle between the tangents to the curves at $x = \frac{1}{\sqrt{2}}$ given by

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$= \left|\frac{\sqrt{2} - -\sqrt{2}}{1-2}\right|$$

$$= \left|\frac{\sqrt{2}+\sqrt{2}}{-1}\right|$$

$$= 2\sqrt{2}$$

$$\theta = \tan^{-1} 2\sqrt{2}$$

$$= \underline{\underline{70^\circ 32'}}$$

Step 3 Prove $S_n$ true for $n=k+1$

ie that

$$1^3+2^3+\ldots+k^3+(k+1)^3 = [1+2+\ldots+k+(k+1)]^2$$

$$LHS = (1+2+3+\ldots+k)^2+(k+1)^3$$

using assumption Step 2

$$= \frac{1}{4}k^2(k+1)^2 + (k+1)^3$$

using part (i)

$$= \frac{1}{4}(k+1)^2[k^2+4(k+1)]$$

$$= \frac{1}{4}(k+1)^2[k^2+4k+4]$$

$$= \frac{1}{4}(k+1)^2(k+2)^2$$

$$RHS = [1+2+\ldots+k+(k+1)]^2$$

$$= \frac{(k+1)^2(k+1+1)^2}{4}$$

using part (i)

$$= \frac{(k+1)^2(k+2)^2}{4}$$

$\therefore$ $S_n$ is true for $n=k+1$

Step 4 Since $S_n$ is true for $n=1$ (Step 1)

then from steps 2 and 3 it is also true for $n=1+1$

ie $n=2$ and so on for all $n \geq 1$

(b) i) $P(x) = (x-a)^2 Q(x)$

6 2

$$P'(x) = vu' + uv'$$

$$= Q(x)\,2(x-a)^1 \cdot 1 + (x-a)^2 Q'(x)$$

$$= (x-a)[2Q(x)+(x-a)Q'(x)]$$

$$\therefore P'(a) = (a-a)(\quad\sim\quad) = 0$$

ie. $P'(a) = 0$

(ii) $x^4+ax^3+bx^2-5x+1=0$

$$P(x) = 0$$

$$P'(x) = 4x^3+3ax^2+2bx-5$$

If $x=1$ is a double root

then $P'(1) = 0$

ie $4+3a+2b-5=0$

$$3a+2b = 1 \quad (1)$$

Also, since $x=1$ is a root

$$P(1) = 0$$

ie $1+a+b-5+1=0$

$$a+b = 3 \quad (2)$$

Solving simultaneously

$$3a+2b = 1 \quad (1)$$

$$3a+3b = 9 \quad (3)$$

(3)−(1); $b = 8$

and $a = 3-b$

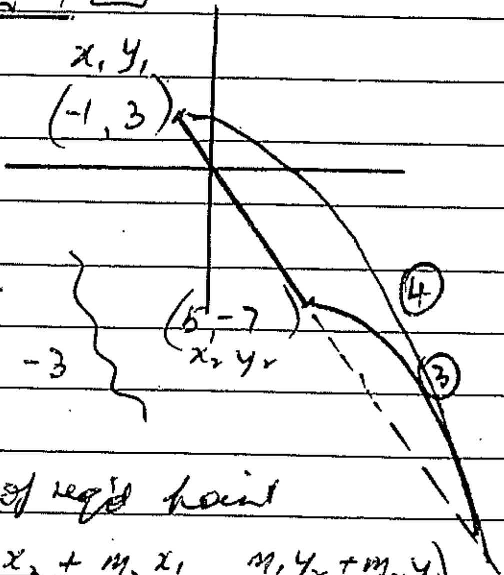
$$= 3-8$$

$$= -5$$

ie $a=-5,\ b=8$

## Question 4 [12]

(a) (3)



$m_1 : m_2 = 4 : -3$

Coords of req'd point are $\left(\dfrac{m_1x_2 + m_2x_1}{m_1+m_2}, \dfrac{m_1y_2+m_2y_1}{m_1+m_2}\right)$

$= \left(\dfrac{4\times 5 + -3\times -1}{4+-3}, \dfrac{4\times -7 + -3\times 3}{4+-3}\right)$

$= \underline{\underline{(23, -37)}}$

(b) (3) $|x| + |2y| = 8$

When $x = 0$ $\quad |2y| = 8$

$2y = \pm 8$

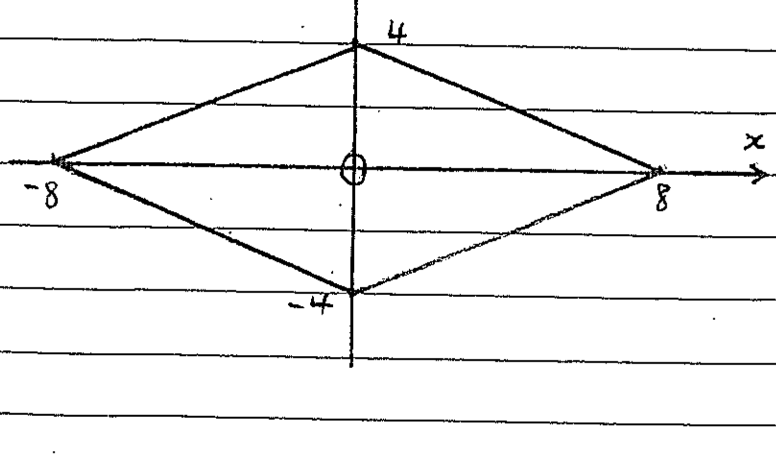
$y = \pm 4$

When $y = 0$ $\quad |x| = 8$

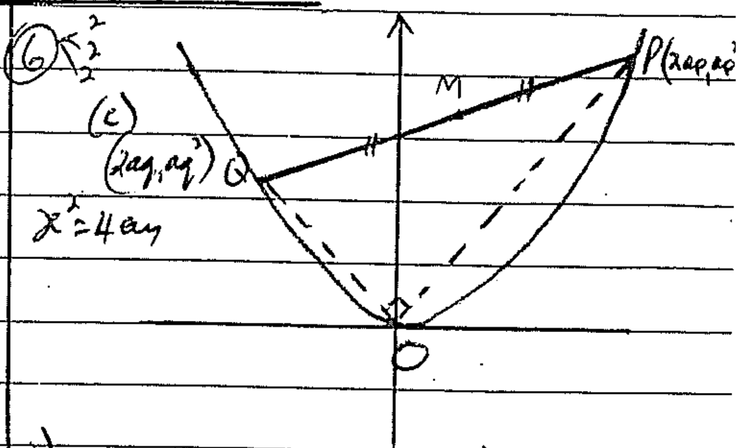
$x = \pm 8$

Because of absolute values

$-8 \le x \le 8$ and $-4 \le y \le 4$



(c) (6)



$x^2 = 4ay$

i) Gradient of $OP = \dfrac{ap^2}{2ap} = \dfrac{p}{2}$

" " $OQ = q/2$

If $OP$ is perp to $OQ$

$m_1 m_2 = -1$ (2)

$\dfrac{p}{2} \times \dfrac{q}{2} = -1$

$pq = -4$

ie $\underline{pq + 4 = 0}$

ii) Coordinates of $M$, mid-point of $PQ$ are $\dfrac{2ap+2aq}{2}, \dfrac{ap^2+aq^2}{2}$

$= \underline{\underline{\left[a(p+q), \frac{a}{2}(p^2+q^2)\right]}}$ (2)

iii) Locus of $M$ given by

$X = a(p+q)$ $\quad \therefore p+q = \dfrac{X}{a}$

$Y = \frac{a}{2}(p^2+q^2)$

$= \frac{a}{2}\left[(p+q)^2 - 2pq\right]$

$= \frac{a}{2}\left[\left(\frac{X}{a}\right)^2 - 2(-4)\right]$ using $pq = -4$

$= \frac{a}{2}\left[\frac{X^2}{a^2} + 8\right]$

$= \dfrac{X^2}{2a} + 4a$ (2)

$\therefore$ Locus of $M$ is

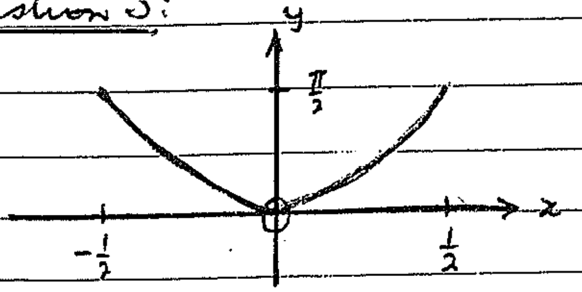
$2aY = X^2 + 8a^2$

$\underline{\underline{x^2 = 2ay - 8a^2}}$

(12) Question 5:

a)

(2)

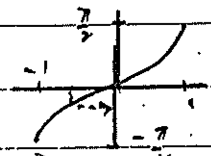



b) i) Range of $y = \frac{-1}{2(1+x^2)}$

(3) is $\underline{-\frac{1}{2} \le y < 0}$

ii) $\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

$\sin^{-1}(0) = 0$



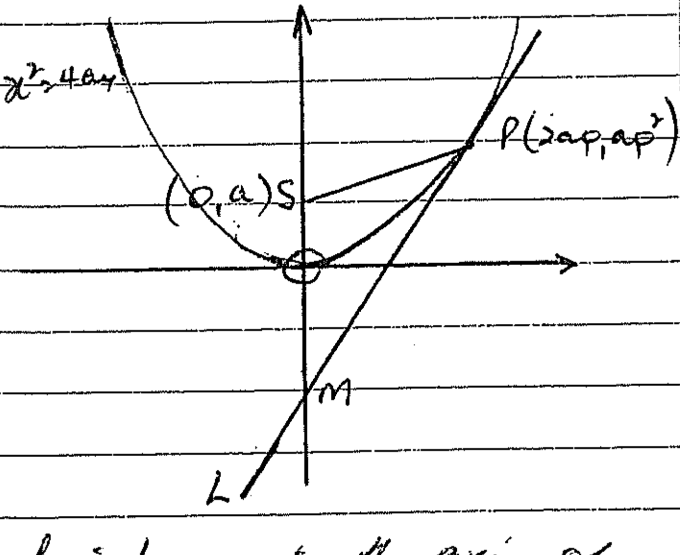

Range of $\sin^{-1}\left[\frac{-1}{2(1+x^2)}\right]$

is

$\underline{-\frac{\pi}{6} \le y < 0}$

c)

(3)




If line L meets the axis of the parabola at M, we have to prove that $\angle SPM = \angle SMP$

Proof: The tangent at P has eqn. $y = px - ap^2$

$\therefore$ Co-ords of M (where $x=0$) are $(0, -ap^2)$

$\therefore$ Length $SM = a + ap^2$

Length $SP = \sqrt{(ap^2-a)^2 + (2ap-0)^2}$

$= \sqrt{a^2p^4 + a^2 - 2a^2p^2 + 4a^2p^2}$

$= \sqrt{a^2p^4 + 2a^2p^2 + a^2}$

$= \sqrt{a^2(p^4 + 2p^2 + 1)}$

$= \sqrt{a^2(p^2+1)^2} = a(p^2+1)$

$= ap^2 + a$

$\therefore SP = SM$

ie $\triangle SPM$ is isosceles

ie L is equally inclined to SP and the axis of the parabola

(d) $S_n \equiv 13 \times 6^n + 2 = 5M$

(4) where M is integral

Assume $S_k$ ie

$13 \times 6^k + 2 = 5M$

Prove $S_{k+1}$ ie

that $13 \times 6^{k+1} + 2 = 5N$

(where N is integer)

$13 \times 6^{k+1} + 2 = 13 \times 6^k \cdot 6 + 2$

$= 6(13 \times 6^k) + 2$

$= 6(13 \times 6^k + 2) + 2 - 12$

$= 6(5M) - 10$

$= 5(6M - 2)$

$= 5N$

where N is integral $(= 6M - 2)$

$S_1 = 13 \times 6^1 + 2 = 80$

which is a multiple of 5

Since $S_1$ is true then from step 1 and 2 above $S_{1+1}$ is true ie $S_2$ & so on for all positive integers $(n \ge 1)$.

# 3U Maths Solutions (Page 6)

## Question 6 [12]

(3) a) $\int \frac{dx}{\sqrt{1-4x^2}}$ If $x = \frac{1}{2}\sin\theta$, $\frac{dx}{d\theta} = \frac{1}{2}\cos\theta$

$= \int \frac{dx}{\sqrt{(1-4\frac{\sin^2\theta}{4})}}$

$= \int \frac{1}{\sqrt{(1-\sin^2\theta)}} \frac{dx}{d\theta}\, d\theta$

$= \int \frac{1}{\cos\theta} \frac{1}{2}\cos\theta\, d\theta$

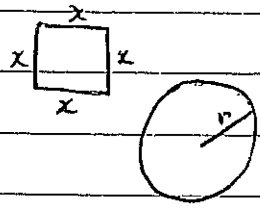
$= \int \frac{1}{2}\, d\theta = \frac{1}{2}\theta + c$

If $x = \frac{1}{2}\sin\theta$, $2x = \sin\theta$

$\theta = \sin^{-1} 2x$

∴ Integral $= \underline{\underline{\frac{1}{2}\sin^{-1} 2x + c}}$

b) (4) 24 cm



i) $4x + 2\pi r = 24$

$2x + \pi r = 12$

$r = \frac{12-2x}{\pi}$

Sum of areas

$A = x^2 + \pi r^2$

$= x^2 + \pi\left(\frac{12-2x}{\pi}\right)^2$

$= \underline{\underline{x^2 + \frac{1}{\pi}(12-2x)^2}}$

ii) $A$ is a maximum when $\frac{dA}{dx} = 0$ and $\frac{d^2A}{dx^2} < 0$

$\frac{dA}{dx} = 2x + \frac{1}{\pi} 2(12-2x)(-2)$

$= 2x - \frac{4}{\pi}(12-2x)$

$= 0$

when $2x = \frac{4}{\pi}(12-2x)$

$2\pi x = 48 - 8x$

$(2\pi + 8)x = 48$

$x = \frac{48}{8+2\pi} = \frac{24}{4+\pi}$ ($\doteqdot 3.36$ cm)

$\frac{d^2A}{dx^2} = 2 - \frac{4}{\pi}(-2) = 2 + \frac{8}{\pi}$

$> 0$

∴ This turning point gives minimum area ($\simeq 20.16$ cm$^2$)

∴ Need to test the end points

ie $x = 0$ and $x = 6$

When $x = 0$, radius of circle $= \frac{12}{\pi}$

and Area $= \pi\left(\frac{12}{\pi}\right)^2 \doteqdot 45.8$ cm$^2$

When $x = 6$, area of square

$= 36$ cm$^2$

∴ Maximum area occurs when $x = 0$

ie when

$A = 0 + \frac{1}{\pi}(12)^2 = \underline{\underline{\frac{144}{\pi}\text{ cm}^2}}$

c) (2) $\frac{1}{|3-x|} \geq \frac{1}{2}$

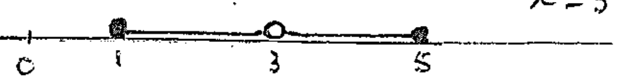
$|3-x| \leq 2$, $3-x \neq 0$

$-2 \leq 3-x \leq 2$, $x \neq 3$

$-5 \leq -x \leq -1$

$5 \geq x \geq 1$

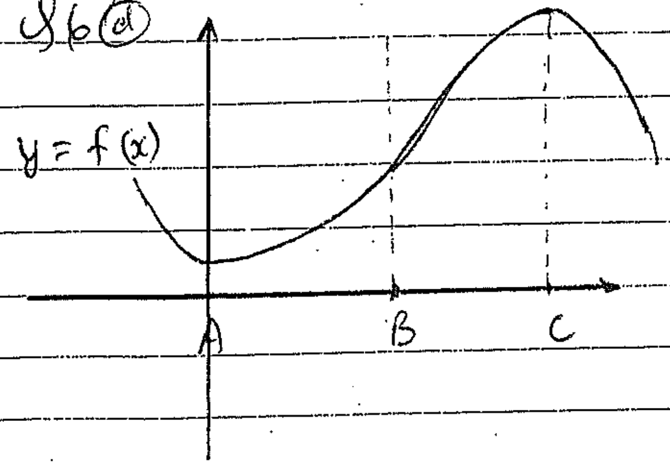
ie $1 \leq x \leq 5$, excluding $x = 3$



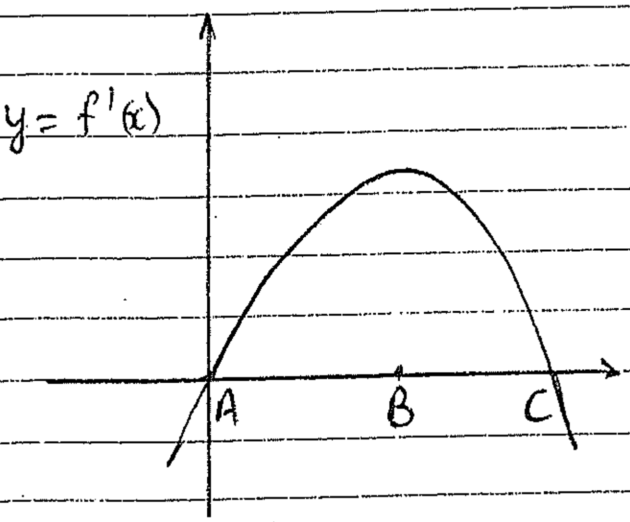
3 ← 2 0 1

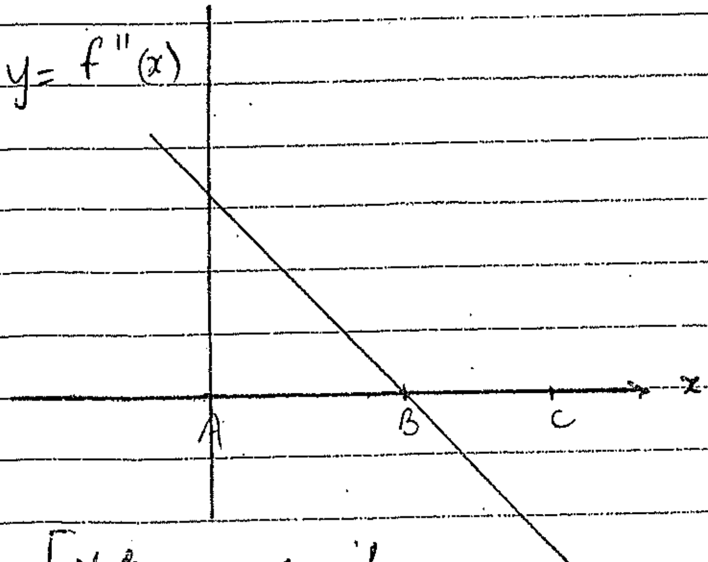
# 3 U Maths Solutions [Page 7]

Q6(d)




Notes Min t point at A
Pt of inflexion at B
Max t. pt at C




Notes: +ve to left of B
0 at B
-ve to right of B

[Not necessarily a straight line]

Question 7. (12)

(3) a) $x^2 + mx + n = 0$

with roots $\alpha, \beta$

i) $\alpha + \beta = -m$

ii) $\alpha\beta = n$

iii) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= \underline{m^2 - 2n}$

iv) $3\alpha^2\beta + 3\alpha\beta^2$

$= 3\alpha\beta(\alpha+\beta)$

$= 3n(-m) = \underline{-3nm}$

b) i) $\int_{-1}^{1} x^2 \cos x \, dx$

(3) $= 2\int_0^1 x^2 \cos x \, dx$ ?

Consider the graph of the function $y = x^2 \cos x$

with key points $(-1, \cos 1)$

When $x = -1$

$y = (-1)^2 \cos(-1)$

$= 1 \cos 1$

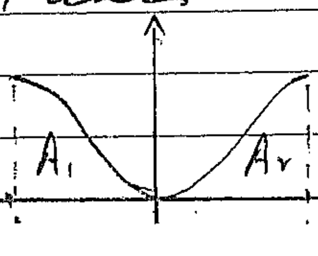
as $\cos(-\theta) = \cos\theta$

When $x = 1$, $y = \cos 1$

When $x = 0$ $y = 0$

$\therefore$ As $f(x) = f(-x)$

the function is even and its graph is symmetrical about the y axis




$\therefore A_1 = A_2$

$\therefore$ This statement is TRUE

ii) $\int_{-3}^{3} (x\cos x + \sin x) = 0$ ?

$f(-3) = -3\cos(-3) + \sin(-3)$

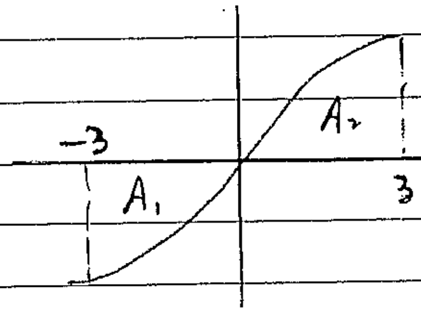
$= -3\cos 3 - \sin 3$

$f(3) = 3\cos 3 + \sin 3$

This function is odd

ie $f(x) = -f(-x)$

so has point symmetry about the origin




So $A_1 = -A_2$

$\therefore A_1 + A_2 = 0$

$\therefore$ This statement is TRUE

[Note: It is sufficient to show that i) graph is even and ii) is odd. The actual shape of each curve is irrelevant]

3 Q7 (c) $\frac{d}{dx}\left[2x\tan^{-1}x - \ln(1+x^2)\right]$

$= \tan^{-1}x \cdot 2 + 2x \cdot \frac{1}{1+x^2} - \frac{1}{1+x^2} \cdot 2x$

$= \underline{2\tan^{-1}x}$

Hence $\int_0^1 \tan^{-1}x \, dx$

$= \left[\frac{1}{2}\left(2x\tan^{-1}x - \ln(1+x^2)\right)\right]_0^1$

$= \frac{1}{2}\left(2\tan^{-1}1 - \ln 2\right) - \frac{1}{2}\left(0 - \ln 1\right)$

$= \tan^{-1}1 - \frac{1}{2}\ln 2$

$= \underline{\frac{\pi}{4} - \frac{\ln 2}{2}}$

3 (d) $x^2 + y^2 - 2x - 14y + 25 = 0$

i) If the line $y = mx$ meets this circle, then

$x^2 + (mx)^2 - 2x - 14 \cdot mx + 25 = 0$

$(1+m^2)x^2 - (2+14m)x + 25 = 0$

If the line is to intersect the circle then this equation will have two real different roots

ie $\Delta > 0$

$b^2 - 4ac > 0$

$\left[-2(1+7m)\right]^2 - 4(1+m^2) \cdot 25 > 0$

$4(1+7m)^2 - 100(1+m^2) > 0$

$\underline{(1+7m)^2 - 25(1+m^2) > 0}$

ii) If $y = mx$ is a tangent to the circle then the above quadratic eqn in $x$ will have equal roots ie $\Delta = 0$

$(1+7m)^2 - 25(1+m^2) = 0$

$1 + 14m + 49m^2 - 25 - 25m^2 = 0$

$24m^2 + 14m - 24 = 0$

$12m^2 + 7m - 12 = 0$

$(4m-3)(3m+4) = 0$

$\underline{m = \frac{3}{4} \text{ or } \frac{-4}{3}}$

NOTE: The following questions were taken from past HSC papers
- All Question 1
- All Question 2
- Question 5 (d)
- Question 7 (d)