Name________________________ Teacher____________________

GOSFORD HIGH SCHOOL

MATHEMATICS

EXTENSION 1

HSC

2016

ASSESSMENT TASK 2

General Instructions

- Reading Time – 5 minutes
- Working Time – 90 minutes
- Write on one side of your paper only.
- Start a new page for each question.
- Write your name on each page you submit.
- Correct setting out ***must*** be shown or full marks may not be awarded.
- Board approved calculators may be used.
- A Reference sheet is provided.

Total Marks – 54

Section 1 (6 marks)
Questions 1- 6

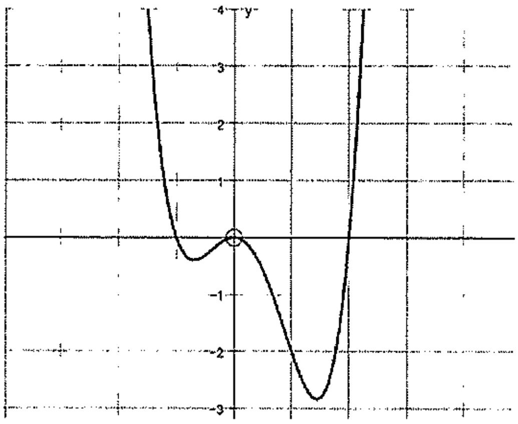
Section 2 (48 marks)
Questions 7 - 9

MULTIPLE CHOICE QUESTIONS TO BE ANSWERED ON SHEET PROVIDED

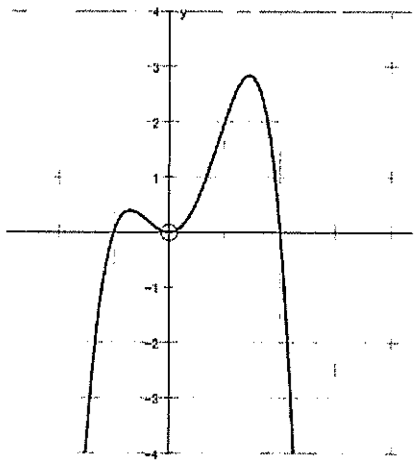
QUESTION 1

Which graph best represents $y = x^4 - x^3 - 2x^2$?

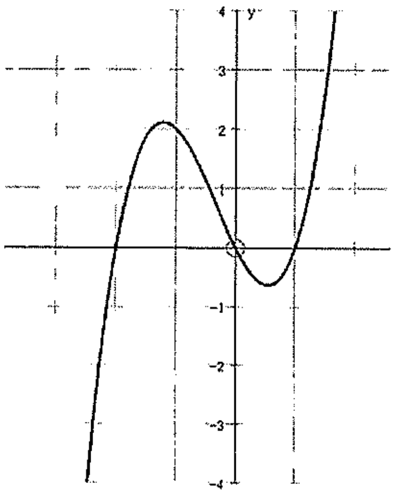
(A)



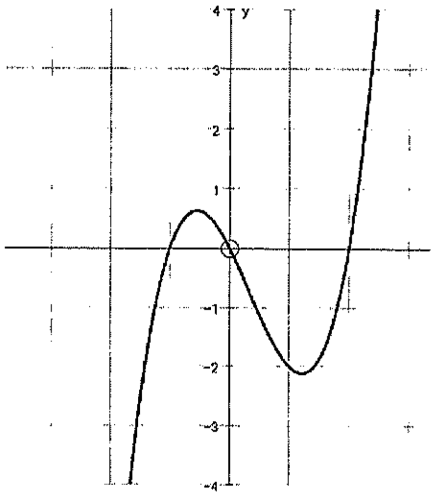
(B)



(C)



(D)



QUESTION 2

A committee of 3 men and 3 women is to be formed from a group of 8 men and 6 women.

In how many ways can this be done?

(A) 48

(B) 1 120

(C) 40 320

(D) 3003

QUESTION 3

Victoria made an error proving that $3^{2n}-1$ is divisible by 8 using mathematical induction (where n is an integer greater than 0).
Part of the proof is shown below.

Step 2: Assume the result true for $n=k$

$3^{2k}-1=8P$ where P is an integer. Line 1

Hence $3^{2k}=8P+1$

To prove the result is true for $n=k+1$

$3^{2(k+1)}-1=8Q$ where Q is an integer. Line 2

$$\begin{aligned}
\text{LHS} &= 3^{2(k+1)}-1 \\
&= 3^{2k}\times 3^2-1 \\
&= (8P+1)\times 3^2-1 && \text{Line 3} \\
&= 72P+1-1 && \text{Line 4} \\
&= 72P \\
&= 8(9P) \\
&= 8Q \\
&= \text{RHS}
\end{aligned}$$

In which line did Victoria make an error?

(A) Line 1 (B) Line 2

(C) Line 3 (D) Line 4

QUESTION 4

What is the indefinite integral for $\int\left(\sin^2 x+x^2\right)dx$?

(A) $x-\frac{1}{2}\sin 2x+\frac{x^3}{3}+c$

(B) $\frac{1}{2}x-\frac{1}{4}\sin 2x+\frac{x^3}{3}+c$

(C) $x-\frac{1}{2}\sin 2x+2x+c$

(D) $\frac{1}{2}x-\frac{1}{4}\sin 2x+2x+c$

QUESTION 5

Using the substitution $u = 1 - x^3$, evaluate $\int_0^1 x^2\sqrt{1-x^3}\,dx$.

(A) $-\frac{1}{9}$ (B) $\frac{1}{9}$

(C) $\frac{2}{9}$ (D) $\frac{1}{3}$

QUESTION 6

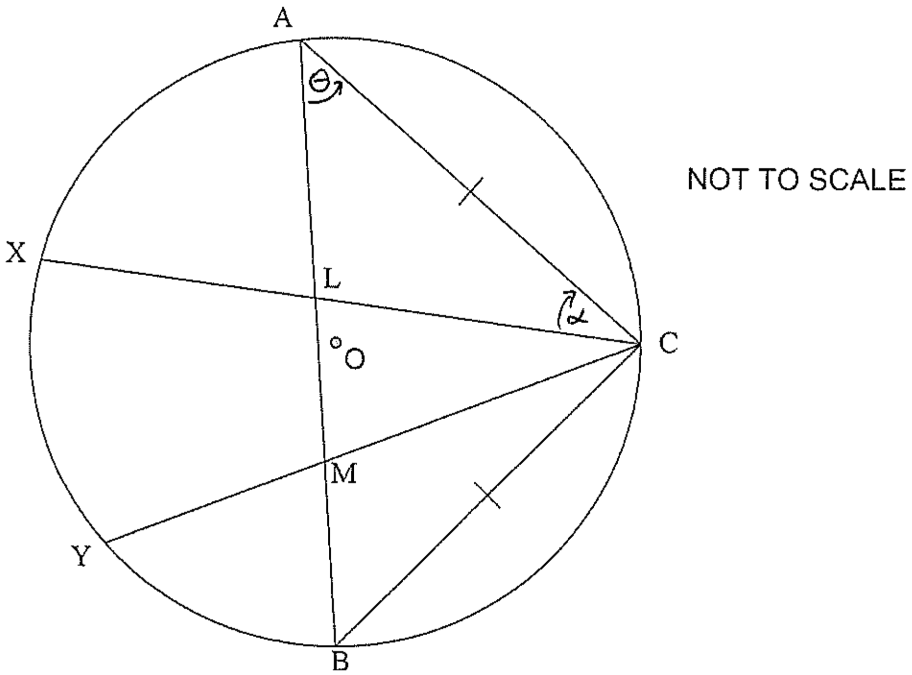
One approximate solution of the equation $f(x) = 4x^3 - 15x^2 + 22x - 12$ is x = 1.3,
What is another approximation to the solution using one of application of Newton's method?

(A) $x = 1.2884$
(B) $x = 1.2885$
(C) $x = 1.2886$
(D) $x = 1.2887$

QUESTION 7 – (17 MARKS) START A NEW PAGE **MARKS**

a) Find the obtuse angle between the lines $2x - 3y + 4 = 0$ and $y = 3x + 7$. (To the nearest minute) **2**

b) In the diagram A, B and C are 3 points on the circle.
CX and CY are chords cutting AB at L and M respectively.
AC=CB, $\angle CAB = \theta$ and $\angle ACX = \alpha$.
Copy the diagram onto your paper.



i) State why $\angle CLB = \theta + \alpha$. **1**

ii) Explain why $\angle AYC = \theta$ and $\angle AYX = \alpha$. **3**

iii) Prove that XYML is a cyclic Quadrilateral. **2**

QUESTION 7 CONTINUED

MARKS

c) The point P divides the interval joining $A(-1,4)$ and $B(2,-2)$ in the ratio k:1.

i) Write down the coordinates of P in terms of k. **1**

ii) Given that P lies on the line $x-2y-1=0$, find k and hence find the coordinates of P. **3**

d) The equation $2x^3 - 6x + 1 = 0$ has roots α, β and γ. Evaluate:

i) $\alpha+\beta+\gamma$ **1**

ii) $\alpha\beta + \alpha\gamma + \beta\gamma$ **1**

iii) $(\alpha + 1)(\beta + 1)(\gamma + 1)$ **2**

e) $\lim\limits_{x\to\infty}\left[\dfrac{x+2}{1-x}\right] = ?$ **1**

QUESTION 8 – (15 MARKS) START A NEW PAGE

MARKS

a) Prove $\dfrac{2\sin^3\theta+2\cos^3\theta}{\sin\theta+\cos\theta} = 2-\sin 2\theta$ **3**

b) Find the exact value of $\sin\dfrac{11\pi}{12}$ **3**

c) By using the 't' method solve $\sqrt{3}\sin x+\cos x=1$ for $0 \le x \le 2\pi$ **3**

d) Evaluate $\int_{3\sqrt{2}}^{6}\sqrt{36-x^2}\,dx$ using the substitution $x = 6\sin u$. **4**

e) Solve $\dfrac{x}{x+2} \ge 3$ **2**

QUESTION 9 – (16 MARKS) **START A NEW PAGE** **MARKS**

a) i) Show that $P(x) = x^3 - 8x^2 + 9x + 18$ is divisible by $x - 3$ and $x + 1$ **2**

ii) Express $P(x)$ in terms of 3 linear factors. **2**

iii) Hence solve $P(x) \geq 0$ **2**

b) i) Show that $P(x) = 3x^4 + 4x^3 - 12x^2 - 1$ has a root between $x = -3$ and $x = -2$. **1**

ii) Use the method of halving the interval twice to show that the root lies between $x = -3$and $x = -2.75$ **2**

c) Consider the polynomial $P(x) = 4x^3 + 2x^2 + 1$
$P(x)$ has a real zero α in the interval $-1 < x < 0$.

i) By sketching the graph of $P(x)$, show that α is the only real zero of $P(x)$. (Hint: Show all critical points) **3**

ii) Use Newton's method with initial value $\alpha = -\frac{1}{4}$ to obtain a second approximation for the root. **2**

iii) Explain from the graph of $P(x)$ why this second approximation is *not* a better approximation to α than $\alpha = -\frac{1}{4}$. **1**

iv) Give *one* value of x that would give a better approximation of the root. **1**

END OF EXAM
☺

Maths Ext1 Task 2 2016

Multiple Choice

1) $y = x^4 - x^3 - 2x^2$

$= x^2(x^2 - x - 2)$

$= x^2(x-2)(x+1)$

double root at $x^2 = 0 \Rightarrow x = 0$

single roots at $x - 2 = 0 \Rightarrow x = 2$

$x + 1 = 0 \Rightarrow x = -1$

$a > 0$ (A)

2) ${}^8C_3 \times {}^6C_3 = 1120$ (B)

3) Line 4 should read

$72P + 9 - 1$ (D)

4) $\int (\sin^2 x + x^2)\,dx$

$\cos 2x = 1 - 2\sin^2 x$

$\sin^2 x = \frac{1}{2} - \frac{1}{2}\cos 2x$

$= \frac{1}{2}\int (1 - \cos 2x)\,dx + \int x^2\,dx$

$= \frac{1}{2}\left[x - \frac{\sin 2x}{2}\right] + \frac{x^3}{3} + c$

$= \frac{x}{2} - \frac{\sin 2x}{4} + \frac{x^3}{3} + c$ (B)

5) $\int_0^1 x^2\sqrt{1-x^3}\,dx$

$u = 1 - x^3$

$\frac{du}{dx} = -3x^2$

$dx = \frac{du}{-3x^2}$

$x^3 = 1 - u$

$x = 1 \quad u = 0$

$x = 0 \quad u = 1$

$\frac{1}{3}\int_0^1 u^{1/2}\,du$

$= \frac{1}{3}\left[\frac{2u^{3/2}}{3}\right]_0^1$

$\frac{1}{9}(2 - 0)$

$= \frac{2}{9}$ (C)

6) $f(x) = 4x^3 - 15x^2 + 22x - 12$

$f'(x) = 12x^2 - 30x + 22$

$f(1.3) \doteqdot 0.038$

$f'(1.3) \doteqdot 3.28$

$x_1 \doteqdot x_0 - \frac{f(x)}{f'(x)}$

$\doteqdot 1.3 - \frac{0.038}{3.28}$

$\doteqdot 1.2884$ (A)

7a) $2x-3y+4=0$ $\quad m_1=\frac{2}{3}$

$y=3x+7$ $\quad m_2=3$

$\tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|$ (acute ∠)

$=\left|\frac{\frac{2}{3}-3}{1+\frac{2}{3}.3}\right|$ 1 formula

$=\left|\frac{-7}{9}\right|$

$=\frac{7}{9}$

∴ acute ∠ $=37°52'$

∴ obtuse ∠ $=142°8'$ 1

(nearest minute)

$\boxed{2}$

i) In ΔACL

$\angle LAC=\theta$, $\angle ACL=\alpha$ (given)

$\angle CLB=\angle LAC+\angle ACL$

(exterior ∠ of Δ = sum 2 opposite interior ∠'s)

∴ $\angle CLB=\theta+\alpha$ $\boxed{1}$

) In ΔACB

$\angle CAB=\angle CBA=\theta$

(= ∠'s opposite = sides Δ =)

∴ $\angle CBA=\theta$ 1

+ $\angle CBA=\angle AYC=\theta$

(∠'s at circumference from same arc =) 1

$\angle AYX=\angle ACX=\alpha$ 1

(∠'s at circumf from same arc =)

$\boxed{3}$

iii) for XYML

$\angle XYM=\angle AYX+\angle AYC$

(adjacent ∠'s) 1

∴ $\angle XYM=\theta+\alpha$

+ $\angle CLB=\theta+\alpha$ (from above)

∴ $\angle XYM=\angle CLB$

∴ XYML is cyclic quad

(exterior ∠ cyclic quad = opp interior ∠) 1 $\boxed{2}$

c i) $A(-1,4)$ $\quad B(2,-2)$

$k:1$

$x=\frac{2k-1}{k+1}$

$y=\frac{-2k+4}{k+1}$

∴ $P\left(\frac{2k-1}{k+1},\frac{-2k+4}{k+1}\right)$ $\boxed{1}$

ii) sub P into $x-2y-1=0$

$\frac{2k-1}{k+1}-2\left(\frac{-2k+4}{k+1}\right)-1=0$ 1

$\frac{2k-1+4k-8-k-1}{k+1}=0$

∴ $5k-10=0$

$k=2$ 1

∴ $P(1,0)$ 1 $\boxed{3}$

d) $2x^3 - 6x + 1 = 0$

i) $\Sigma\alpha = 0$ $\boxed{1}$

ii) $\Sigma\alpha\beta = \frac{-6}{2} = -3$ $\boxed{1}$

ii) $(\alpha+1)(\beta+1)(\gamma+1)$

$= (\alpha\beta + \alpha + \beta + 1)(\gamma + 1)$

$= \alpha\beta\gamma + \alpha\beta + \alpha\gamma + \alpha + \beta\gamma + \beta + \gamma + 1$

$= \alpha\beta\gamma + \Sigma\alpha\beta + \Sigma\alpha + 1$ 1

$= -\frac{1}{2} - 3 + 0 + 1$

$= -2\frac{1}{2}$ $\boxed{2}$

) $\lim_{x \to \infty} \left(\frac{x+2}{1-x} \right)$

$= \lim_{x \to \infty} \left(\frac{1 + \frac{2}{x}}{\frac{1}{x} - 1} \right)$

$= \frac{1}{-1}$

$= -1$ (must have work) $\boxed{1}$

Question 8

a) $LHS = \dfrac{2\sin^3\theta + 2\cos^3\theta}{\sin\theta + \cos\theta}$

$= \dfrac{2(\sin\theta + \cos\theta)(\sin^2\theta - \sin\theta\cos\theta + c^2)}{\sin\theta + \cos\theta}$

$= 2(\sin^2\theta + \cos^2\theta - \sin\theta\cos\theta)$ (1)

$= 2(1 - \sin\theta\cos\theta)$ (1)

$= 2 - 2\sin\theta\cos\theta$

$= 2 - \sin 2\theta$ (1)

$= RHS$ $\boxed{3}$

b) $\sin\frac{11\pi}{12} = \sin\left(\frac{\pi}{6} + \frac{3\pi}{4}\right)$ (1)

$= \sin\frac{\pi}{6}\cos\frac{3\pi}{4} + \cos\frac{\pi}{6}\sin\frac{3\pi}{4}$

$= \frac{1}{2}\cdot -\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}}$ (1)

$= \frac{-1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}$

$= \frac{\sqrt{3}-1}{2\sqrt{2}}$

$= \frac{\sqrt{6}-\sqrt{2}}{4}$ (1) $\boxed{3}$

c) $\sqrt{3}\sin x + \cos x = 1$

$\sqrt{3}\left(\frac{2t}{1+t^2}\right) + \left(\frac{1-t^2}{1+t^2}\right) = 1$ (1)

$\frac{2\sqrt{3}t + 1 - t^2}{1+t^2} = 1$

$2\sqrt{3}t + 1 - t^2 = 1 + t^2$

$2t^2 - 2\sqrt{3}t = 0$

$t(2t - 2\sqrt{3}) = 0$

$t = 0$ | $2t = 2\sqrt{3}$

| $t = \sqrt{3}$

$\tan\frac{x}{2} = 0$ | $\tan\frac{x}{2} = \sqrt{3}$ (1)

$\therefore x = 0, 2\pi, \frac{2\pi}{3}$ (1)

test $x = \pi$

$\sqrt{3}\sin\pi + \cos\pi = 0 + (-1) \neq 1$

$\therefore$ not a soln.

d) $\displaystyle\int_{3\sqrt{2}}^{6} \sqrt{36-x^2}\,dx$ $\quad x = 6\sin u$

$\frac{dx}{du} = 6\cos u$

$dx = 6\cos u\,du$

$= \displaystyle\int_{\frac{\pi}{4}}^{\pi/2} (36 - 36\sin^2 u).6\cos u\,du$ $\quad x = 6 \quad u = \frac{\pi}{2}$

(1) $\quad x = 3\sqrt{2},\ u = \frac{\pi}{4}$

$= \displaystyle\int_{\frac{\pi}{4}}^{\pi/2} 6\cos u . 6\cos u\,du$ (1)

$= 36\displaystyle\int_{\pi/4}^{\pi/2} \cos^2 u\,du$

$\cos 2\theta$

$=$ $\quad = 2\cos^2\theta - 1$

$\therefore \cos^2\theta = \frac{1}{2} + \frac{1}{2}\cos 2\theta$

$= 18\displaystyle\int_{\frac{\pi}{4}}^{\pi/2} (1 + \cos 2\theta)\,d\theta$ (1)

$= 18\left[\theta + \frac{\sin 2\theta}{2}\right]_{\pi/4}^{\pi/2}$

$= 18\left(\frac{\pi}{2} + 0 - \frac{\pi}{4} - \frac{1}{2}\right)$

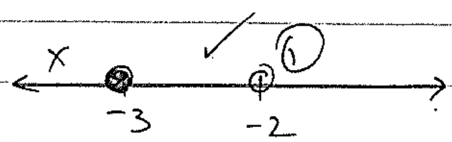
$= 18\left(\frac{\pi}{4} - \frac{1}{2}\right)$ (1) $\boxed{4}$

e) $\frac{x}{x+2} \geq 3$

c.p at

$x + 2 = 0$

$x = -2$



$x = 3x + 6$

$2x = -6$ (1)

$x = -3$

$-3 \leq x < -2$ $\boxed{2}$

Question 9

a) i) $P(x) = x^3 - 8x^2 + 9x + 18$

$P(3) = 3^3 - 8(3^2) + 9(3) + 18$

$= 0$

$\therefore$ by factor theorem $(x-3)$ is factor ①

$P(-1) = (-1)^3 - 8(-1)^2 + 9(-1) + 18$

$= 0$ ①

$\therefore (x+1)$ is factor by factor theorem $\boxed{2}$

ii) check $x = 6$

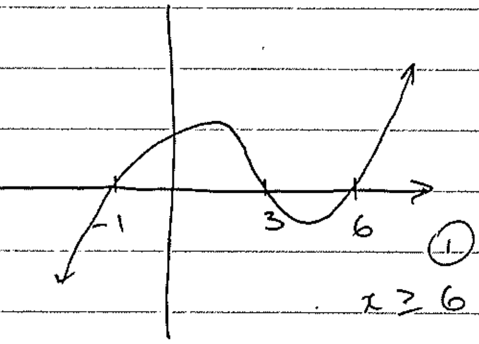
$P(6) = 6^3 - 8 \times 6^2 + 9 \times 6 + 18$

$= 0$ ①

$\therefore (x-6)$ is a factor

$\therefore P(x) = (x-3)(x+1)(x-6)$ ① $\boxed{2}$

iii)



①

$x \geq 6$

$-1 \leq x \leq 3$ $\boxed{2}$

①

b) i) $P(x) = 3x^4 + 4x^3 - 12x^2 - 1$

$P(-3) = 3(-3)^4 + 4(-3)^3 - 12(-3)^2 - 1$

$= 26 > 0$ ①

$P(-2) = 3(-2)^4 + 4(-2)^3 - 12(-2)^2 - 1$

$= -33 < 0$

$\therefore$ root lies between

$x = -3$ & $x = -2$ $\boxed{1}$

ii) $P(-2.5) = 3(-2.5)^4 + 4(-2.5)^3 - 12(-2.5)^2 - 1$

$= -21.3125 < 0$

$\therefore$ root lies between

$x = -2.5$ & $x = -3$

$P(-2.75) = 3(-2.75)^4 + 4(-2.75)^3 - 12(-2.75)^2 - 1$

$= -3.36 < 0$

$\therefore$ root lies between $x = -3$ & $x = -2.75$ $\boxed{2}$

$P(-3) > 0$

c) $P(x) = 4x^3 + 2x^2 + 1$

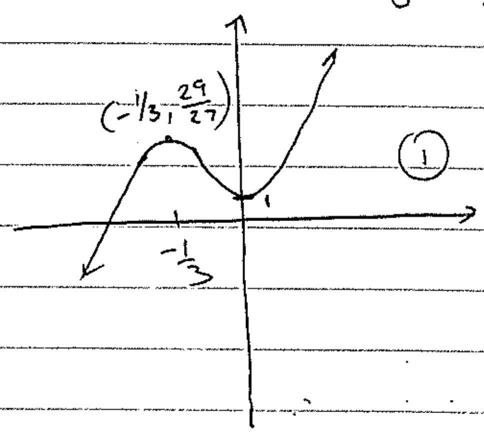
i) $P'(x) = 12x^2 + 4x$

$P''(x) = 24x + 4$

stat pts at $P'(x) = 0$

$4x(3x+1) = 0$

$4x = 0$	$3x + 1 = 0$
$x = 0$	$x = -\frac{1}{3}$ ①
$P'' = 4 > 0$	$P'' = - < 0$ ①
$\therefore$ min turn	$\therefore$ max turn pt
$y = 1$	$y = \frac{29}{27}$



① $\boxed{3}$

i) $P(x) = 4x^3 + 2x + 1$

$P'(x) = 12x^2 + 2$

$P(-\frac{1}{4}) = \frac{17}{16}$ 1

$P'(-\frac{1}{4}) = -\frac{1}{4}$

$\therefore \alpha_1 = \frac{-1}{4} - \frac{\frac{17}{16}}{-\frac{1}{4}}$

$= 4$ 1

$\boxed{2}$

ii) The tangent at $x = -\frac{1}{4}$ has an x-intercept further away from the root 1

$\boxed{1}$

v)

$x = \frac{-2}{3}$

(or any other suitable answer) $\boxed{1}$

in the domain

$-1 < x < -\frac{1}{3}$