NAME:_____________

TEACHER:_____________

HURLSTONE AGRICULTURAL HIGH SCHOOL

2011

Year 12

H S C COURSE

ASSESSMENT TASK 2

HALF YEARLY EXAMINATION

EXTENSION 1

MATHEMATICS

Examiners ~S. Gee, G. Huxley, D. Crancher, B. Morrison, J. Dillon

GENERAL INSTRUCTIONS

- Reading time – 5 minutes.
- Working time – 90 minutes.
- This examination has 5 questions. Attempt all questions.
- Each question is worth 15 marks. Total: 75 marks.
- All necessary working should be shown in each question. Marks may not be awarded for careless or badly arranged work.
- **Start each question in a new answer booklet.** Write your student number on every sheet.
- Board approved calculators and board approved mathematical templates may be used.
- This examination paper must **NOT** be removed from the examination venue.

Question 1 (Commence a new answer booklet) **Marks**

a) Given $x^2 + 4x + 5 \equiv (x+a)^2 + b^2$ find a and b. **(2 mark)**

b) Solve: $\frac{4}{5-x} \geq 1$ **(2 marks)**

c) i) Write, the exact values, for

$\sin 30°, \cos 30°, \sin 45°$ and $\cos 45°$. **(2 marks)**

ii) Hence show $\sin 75° = \frac{\sqrt{2}+\sqrt{6}}{4}$ **(2 marks)**

d) Find the coordinates of the point P which divides the interval internally in the ratio 2:3 where A and B have coordinates $(1, -3)$ and $(6, 7)$ respectively. **(2 marks)**

e) Find, correct to the nearest degree, the size of the acute angle between the line $x+10y-51=0$ and the line $2x+3y-7=0$. **(2 marks)**

f) i) Sketch, on the same diagram (at least $\frac{1}{3}$ of a page) in the domain $0 \leq x \leq 2\pi$, the curves $y = \sin 2x$ and $y = \cos x$. **(1 mark)**

ii) Find, for $0 \leq x \leq 2\pi$, all solutions to the equation $\sin 2x = \cos x$. **(2 marks)**

Question 2 (Commence a new answer booklet) **Marks**

a) If $(x-2)$ is a factor of the polynomial $P(x)=2x^3+x+a$, find the value of a. **(1 mark)**

b) Use the process of polynomial division to find the remainder when $P(x)=x^3-4x$ is divided by $x+3$ **(2 marks)**

c) Consider the polynomial $P(x)=4x^3+2x^2+1$.

You are given the graph of $y=P(x)$ has stationary points at $x=0$ and $x=-\frac{1}{3}$.

i) Show that $P(x)=4x^3+2x^2+1$ has one real root in the interval $-1<x<0$. **(1 mark)**

ii) Let $x=-\frac{1}{4}$ be a first approximation to the root. **(2 marks)**

Apply Newton's Method once to obtain another approximation to the root.

iii) Explain why the application of Newton's Method in part (ii) was NOT effective in improving the approximation to the root. **(2 marks)**

d) If α, β and γ are the roots of the equation $2x^3-4x^2-6x-1=0$ find the value of

i) $\alpha+\beta+\gamma$ **(1 mark)**

ii) $\alpha\beta\gamma$ **(1 mark)**

iii) $\alpha^2+\beta^2+\gamma^2$ **(2 marks)**

e) Find the roots of the equation $x^3-12x^2+12x+80=0$ given that they are three consecutive terms in an arithmetic series. **(3 marks)**

Question 3 (Commence a new answer booklet) **Marks**

a) Find $f(x)$ if $f''(x)=2x-3$, given $f'(-1)=0$ and $f(-1)=12$ **(3 marks)**

b) Find $\frac{d}{dx}\left[\left(3x^2+x\right)^{\frac{3}{2}}\right]$ and hence integrate $(6x+1)\sqrt{3x^2+x}$. **(3 marks)**

c) i) Without the use of calculus sketch the curve

$y=(x+1)(x-1)(x-4)$ indicating the x intercepts. **(1 mark)**

ii) Show that $\int_{-1}^{4}(x+1)(x-1)(x-4)\,dx=\frac{-125}{12}$. **(2 marks)**

iii) Explain why this does not equal the total area of the regions bounded by the curve and the x-axis. **(1 mark)**

d) Peter found the area of a region by evaluating the following definite integral

$$\text{Area} = \int_0^{\frac{\pi}{2}} \sin x\,dx = 1\,\text{unit}^2$$

Use Simpson's rule with three ordinate values to find the approximate area of this region. **(2 mark)**

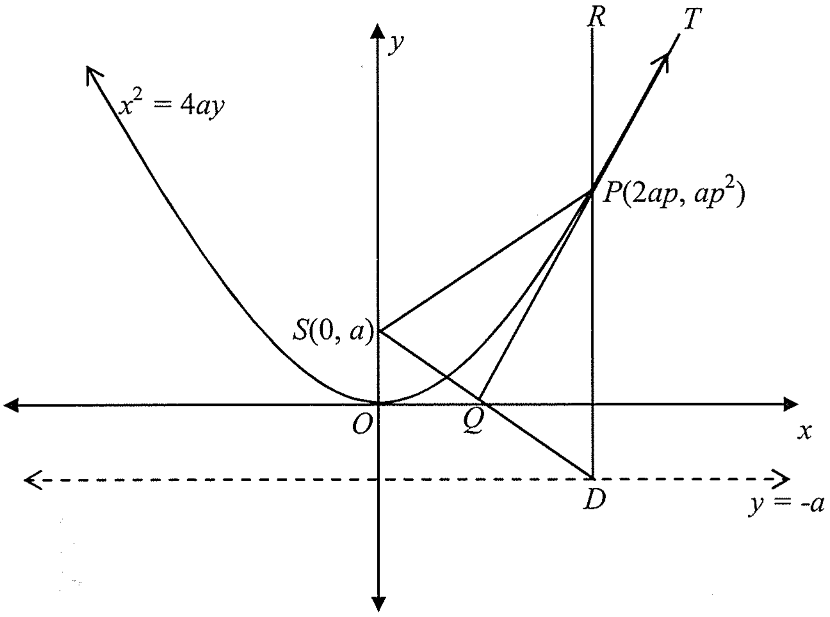
e) A jeweller made a design for a ladies wedding ring by rotating the region **(3 marks)**

bounded by the line $y=1$ and the semi-circle $y=\sqrt{4-x^2}$ about the x-axis.

Find the volume of this wedding ring.

Question 4 (Commence a new answer booklet) **Marks**

a) The diagram shows the parabola $x^2 = 4ay$ with focus $S(0, a)$ and directrix $y = -a$. $P(2ap, ap^2)$ is an arbitrary point on the parabola.
The interval RPD is parallel to the y-axis, meeting the directrix at D.
TPQ is a tangent, meeting SD at Q.



i) Use the locus definition of a parabola to explain why ΔPDS is isosceles. **(2 mark)**

ii) Given that the gradient of the tangent at P is p, prove that $PQ \perp SD$. **(3 marks)**

iii) Hence prove that $SQ = QD$. **(2 marks)**

b) Use calculus to find the gradient of the tangent to the parabola $x^2 = 4ay$ at the point $P(2ap, ap^2)$ and show that the equation of the normal is $x + py = ap^3 + 2ap$ **(3 marks)**

c) Two points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ lie on the parabola $x^2 = 8y$

i) Show that the gradient of the chord PQ is equal to $\frac{p+q}{2}$. **(1 mark)**

ii) Show that the coordinates of M, the midpoint of PQ, are $(2(p+q), p^2 + q^2)$ **(1 mark)**

iii) Let P and Q be variable points on the parabola such that the gradient of $PQ = 2$.
Find and describe the locus of M. **(3 marks)**

Question 5 (Commence a new answer sheet)

Marks

a) Phinh and Nerida plan to each invest a total of \$30 000 over a 20 year period, with their money earning compound interest of 6% per annum using different strategies.

Phinh decided to invest \$10000 at the beginning of the first year. He then added an additional \$1000 at the end of each year, including the first year of the investment. If Phinh's interest rate is 6% per annum compounded annually:

i) Find Phinh's investment at the end of the first year. **(1 mark)**

ii) Show that, at the end of second year, the investment totals $\$10000 \times (1\cdot06)^2 + \$1000 \times (1\cdot06) + \$1000$ **(1 mark)**

iii) Show all working to determine the total of Phinh's account at the end of the 20 years. **(3 marks)**

Nerida decided to invest \$1500 at the beginning of each of the 20 years but her interest rate is 6% per annum compounded six monthly.

iv) Find Nerida's investment at the end of the first year. **(1 mark)**

v) Show that, at the end of the second year , her investment will total $\$1500 \times \left[(1\cdot03)^4 + (1\cdot03)^2\right]$ **(2 marks)**

vi) Determine the total balance of Nerida's account at the end of the 20 years. **(3 marks)**

b) Use the principle of mathematical induction to prove the relationship: **(4 marks)**

$$\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+\ldots\ldots+\frac{1}{n(n+1)}=\frac{n}{n+1}$$

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1} + C, \quad n \neq -1; \; x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x + C, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + C, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax + C, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax + C, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax + C, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax + C, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + C, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a} + C, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right) + C., \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right) + C$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Year 12 Mathematics Extension 1 Half Yearly Examination Task 2 2011

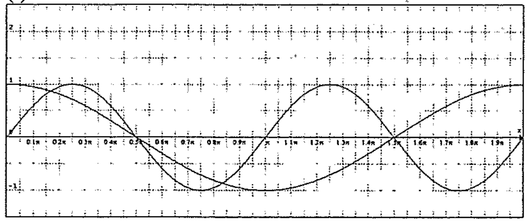
Question No. 1 Solutions and Marking Guidelines

Outcomes Addressed in this Question

P4 chooses and applies appropriate arithmetic, algebraic, graphical, trigonometric and geometric techniques

PE3 **solves problems involving** permutations and combinations, inequalities, polynomials, circle geometry and parametric representations

H5 - Applies appropriate techniques from the study of calculus, geometry, probability, trigonometry and series to solve problems.

Outcome	Solutions	Marking Guidelines
P4	(a) $x^2+4x+5 \equiv (x+a)^2+b^2$ $x^2+4x+5 \equiv (x+2)^2+1^2$ $a=2,\ \ b=\pm 1$	2 marks correct answers 1 mark 1 correct solution
PE4	(b) $\frac{4}{5-x} \geq 1,\ \ x \neq 5$ $\frac{4}{5-x}(5-x)^2 \geq 1(5-x)^2$ $(5-x)^2-4(5-x) \leq 0$ $(5-x)[(5-x)-4] \leq 0$ $(5-x)(1-x) \leq 0$ $x \leq 1,\ x>5$	2 marks correct method leading to correct answer. 1 mark substantial progress towards correct answer.
P4	(c)(i) $\sin 30° = \frac{1}{2}, \cos 30° = \frac{\sqrt{3}}{2},\ \sin 45° = \frac{1}{\sqrt{2}}$ and $\cos 45° = \frac{1}{\sqrt{2}}$	2 marks all exact values correct. 1 mark 2 correct solutions
H5	(ii) $\sin 75° = \sin(45° + 30°)$ $\sin 75° = \sin 45° \cos 30° + \cos 45° \tan 30°$ $\sin 75° = \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2} = \frac{\sqrt{3}+1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$ $\sin 75° = \frac{\sqrt{2}+\sqrt{6}}{4}$	2 marks correct method leading to correct answer. 1 mark substantial progress towards correct answer.
H5	(d) $A(1,-3), B(6,7), k:l = 2:3$ $\left(\frac{kx_2+lx_1}{k+l}, \frac{ky_2+ly_1}{k+l}\right)$ $\left(\frac{2\times 6+3\times 1}{2+3}, \frac{2\times 7+3\times -3}{2+3}\right)$ $(3,1)$	2 marks correct method leading to correct answer. 1 mark substantial progress towards correct answer.
H5	($y = -\frac{1}{10}x + \frac{51}{10}$ $y = -\frac{2}{3}x + \frac{7}{3}$ $\tan \mu = \left\lvert \frac{m_1 - m_2}{1+m_1 m_2} \right\rvert$ $\tan \mu = \left\lvert \frac{-\frac{1}{10} - -\frac{2}{3}}{1+(-\frac{1}{10} \times -\frac{2}{3})} \right\rvert$ $\mu = \tan^{-1}\left(\frac{17}{32}\right)$ $\mu \approx 27°59'$	2 marks correct method leading to correct answer. 1 mark substantial progress towards correct answer.
H5	(f) (i) 	1 mark both graphs correct.
H5	(ii) $\sin 2x = \cos x$ $2\sin x \cos x - \cos x = 0$ $\cos x(2\sin x - 1) = 0$ $\cos x = 0, \sin x = \frac{1}{2}$ $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{2}, \frac{3\pi}{2}$	2 marks correct method leading to correct answer. 1 mark substantial progress towards correct answer. Answer must be in radians

Year 12 **Mathematics Extension 1** **Assessment Task 2 2011**
Question No.2 **Solutions and Marking Guidelines**

Outcome Addressed in this Question

PE3 **solves problems involving** permutations and combinations, inequalities, **polynomials**, circle geometry and parametric representations

Part	Solutions	Marking Guidelines
(a)	$P(2)=0$ $\therefore 2.2^3+2+a=0$ $\therefore a=-18$	**1 mark** – correct answer
(b)	x^2-3x+5 $x+3\overline{)\;x^3-4x}$ x^3+3x^2 $-3x^2-4x$ $-3x^2-9x$ $5x$ $5x+15$ -15	**2 marks** – correct solution **1 mark** – substantial progress towards solution
(c) (i)	$P(-1)=-4+2+1=-1$ $P(0)=1$ Since $P(x)$ is a continuous function, and there is a sign change, there is a root between $x=-1$ and $x=0$.	**1 mark** – for correct explanation
(ii)	$P(x)=4x^3+2x^2+1$ $P'(x)=12x^2+4x$ $x_2=-\frac{1}{4}-\frac{4\left(-\frac{1}{4}\right)^3+2\left(-\frac{1}{4}\right)^2+1}{12\left(-\frac{1}{4}\right)^2+4\left(-\frac{1}{4}\right)}=4$	**2 marks** – correct solution **1 mark** – substantial progress towards solution
(iii)	The initial value is to the right of the turning point meaning the tangent drawn at this point would cut the x – axis on the right hand side of the origin. This means that the new value is not near the root between -1 and 0.	**2 marks** – correct explanation, given in sufficient detail **1 mark** – correct explanation, lacking in detail
(d) (i)	$\alpha+\beta+\gamma=-\frac{-4}{2}=2$	**1 mark** – correct answer
(ii)	$\alpha\beta\gamma=-\frac{-1}{2}=\frac{1}{2}$	**1 mark** – correct answer
(iii)	$\alpha^2+\beta^2+\gamma^2=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\alpha\gamma+\beta\gamma)$ $=2^2-2.\frac{-6}{2}$ $=10$	**2 marks** – correct solution **1 mark** – substantial progress towards solution
(e)	Let the roots be $a-d,\ a,\ a+d$ then $(a-d)+a+(a+d)=3a=12$ $\therefore a=4$ Also $(a-d)\times a\times(a+d)=-80$ $\therefore(4-d)\times 4\times(4+d)=-80$ $\therefore 16-d^2=-20$ $\therefore d^2=36$ $\therefore d=\pm 6$ $\therefore$ Roots are $-2,\ 4,\ 10$.	**3 marks** – correct solution **2 marks** – substantial progress towards solution **1mark** – limited progress towards solution

Year 12 Extension 1 Mathematics Half Yearly Examination 2011

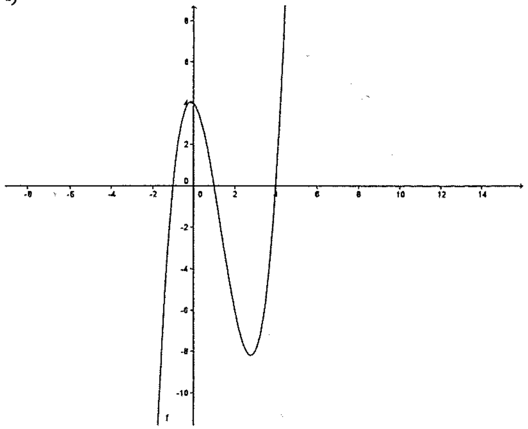
Question No. 3 Solutions and Marking Guidelines

Outcomes Addressed in this Question

H5 - Applies appropriate techniques from the study of calculus, geometry, probability, trigonometry and series to solve problems.

H8- Uses techniques of integration to calculate areas and volumes.

Outcome	Solutions	Marking Guidelines
H5	**3.** **a)** $f''(x)=2x-3$ $f'(x)=x^2-3x+C$ $f(x)=\frac{x^3}{3}-\frac{3x^2}{2}+Cx+K$ now, $f'(-1)=0$ so, $(-1)^2-3(-1)+C=0$ $\therefore C=-4$ and, $f'(x)=x^2-3x-4$ now, $f(-1)=12$ so, $\frac{(-1)^3}{3}-\frac{3(-1)^2}{2}-4(-1)+K=12$ $\therefore K=\frac{59}{6}$ and, $f(x)=\frac{x^3}{3}-\frac{3x^2}{2}-4x+\frac{59}{6}$	3 marks for fully completed correct solution 2 marks for partial correct solution 1 mark for integrating $f''(x)=2x-3$ correctly but forgetting the $+C$
H5	**b)** $\frac{d[(3x^2+x)^{\frac{3}{2}}]}{dx}=\frac{3}{2}(3x^2+x)^{\frac{1}{2}}.(6x+1)$ $=\frac{3}{2}(6x+1)\sqrt{3x^2+x}$ $\int(6x+1)\sqrt{3x^2+x}\ dx=\frac{2}{3}\int\frac{3}{2}(6x+1)\sqrt{3x^2+x}\ dx$ $=(3x^2+x)^{\frac{3}{2}}+C$	3 marks for fully completed correct solution 2 marks for partial correct solution but left of the $+C$ 1 mark for correct differentiation of $(3x^2+x)^{\frac{3}{2}}$

<table>
<tr><td>H5</td><td>i)
 </td><td>1 mark for correct sketch showing the x intercepts</td></tr>
<tr><td>H8</td><td>ii)
$\int_{-1}^{4}(x+1)(x-1)(x-4)\,dx$
$=\int_{-1}^{4}(x^3-4x^2-x+4)\,dx$
$=\left[\frac{x^4}{4}-\frac{4x^3}{3}-\frac{x^2}{2}+4x\right]_{-1}^{4}$
$=\left(\frac{(4)^4}{4}-\frac{4(4)^3}{3}-\frac{(4)^2}{2}+4(4)\right)-\left(\frac{(-1)^4}{4}-\frac{4(-1)^3}{3}-\frac{(-1)^2}{2}+4(-1)\right)$
$=-\frac{40}{3}-\frac{35}{12}$
$=-\frac{125}{25}$</td><td>2 marks for complete correct solution with substitution shown [must show substitution]
1 mark for integrating correctly</td></tr>
<tr><td>H8</td><td>iii)
The definite integral does not equal the total area bounded by the curve and the x-axis since the curve lies below the x-axis between <i>x</i>=1 and <i>x</i>=4.\</td><td>1 mark for correct explanation.</td></tr>
<tr><td>H8</td><td>d)
<table>
<tr><td>x</td><td>0</td><td>$\frac{\pi}{4}$</td><td>$\frac{\pi}{2}$</td></tr>
<tr><td>$f(x)$</td><td>0</td><td>$\frac{1}{\sqrt{2}}$</td><td>1</td></tr>
</table>
$h=\frac{\frac{\pi}{2}-0}{2}=\frac{\pi}{4}$
$A\approx\frac{\frac{\pi}{4}}{3}\left[0+4\left(\frac{1}{\sqrt{2}}\right)+1\right]$
$\approx\frac{\pi}{12}(1+2\sqrt{2})$</td><td>2 marks for complete correct solution
1 mark for partial correct solution</td></tr>
</table>

H8	**e)** Region bounded by $y=\sqrt{4-x^2}$ $\quad y=1$ is rotated about x-axis Find the intersection of the curves $y=\sqrt{4-x^2}$ *and* $y=1$ when $y=1\ \ x=\sqrt{4-1^2}=\pm\sqrt{3}$ Therefore the functions intersect at $(1,\sqrt{3})$ *and* $(1,-\sqrt{3})$ $V=\pi\int_{-\sqrt{3}}^{\sqrt{3}}\left((4-x^2)-(1)\right)dx$ $=\pi\int_{-\sqrt{3}}^{\sqrt{3}}\left(3-x^2\right)dx$ $=\pi\left[3x-\frac{x^3}{3}\right]_{-\sqrt{3}}^{\sqrt{3}}$ $=\pi\left\{\left(3(\sqrt{3})-\frac{(\sqrt{3})^3}{3}\right)-\left(3(-\sqrt{3})-\frac{(-\sqrt{3})^3}{3}\right)\right\}$ $=4\sqrt{3}\pi\ \ units^2$	3 marks for fully completed correct solution 2 marks for partial correct solution leading towards a correct solution but with incorrect intersection values. 1 mark for any correct working that could lead to a possible solution

---

Year 12 Mathematics Extension 1 Task 2 2011 HALF YEARLY EXAM

Question No. 4 Solutions and Marking Guidelines

Outcomes Addressed in this Question

PE4 Uses the parametric representation together with differentiation to identify geometric properties of parabolas.

Outcome	**Solutions**	**Marking Guidelines**
PE4	**(a) (i)** Point P is equidistant from the focus and the parabola. PD is the perpendicular distance to the directrix, so PD = PS. $\therefore \Delta PDS$ is isosceles, because two sidelengths are equal. **(ii)** PQ: $m_1=p$ SD: $m_2=\frac{-a-a}{2ap-0}=\frac{-2a}{2ap}=\frac{-1}{p}$ $m_1\times m_2=p\times\frac{-1}{p}=-1$ Therefore $SD\perp PQ$ **(iii)** The perpendicular from the apex to the base of an isosceles triangle is a bisector. (Property of an isosceles triangle.)	**(a) (i)** **2 marks:** Uses the definition of a parabola <u>and</u> the definition of an isosceles triangle. **1 mark:** Includes one of the above in proof. **(ii)** **3 marks:** Performs substitution for gradient of *SD*; calculates $m_2=\frac{-1}{p}$; and shows the product of the gradients is -1. **2 marks:** 2 of the above included. **1 mark:** 1 of the above. **(iii)** **2 marks:** Reasoning must include that $PQ\perp SD$ and ΔPDS is isosceles. **1 mark:** Includes only one of the above facts.
	(b) $x^2=4ay$ $\therefore y=\frac{x^2}{4a}$ and $y'=\frac{x}{2a}$ When $x=2ap, y'=\frac{2ap}{2a}=p$ Eqn of normal: $y-y_1=m(x-x_1)$ $y-ap^2=\frac{-1}{p}(x-2ap)$ $\rightarrow\quad x+py=ap^3+2ap$	**(b)** **3 marks:** Substitution and evaluation of derivative, and correct substitution into straight line equation. **2 marks:** Substantial progress towards the correct solution. **1 mark:** Performs one of the above steps correctly.
	(c) (i) $m_{PQ}=\frac{aq^2-ap^2}{2aq-2ap}=\frac{a(q-p)(q+p)}{2a(q-p)}$ $=\frac{p+q}{2}$	**(c) (i)** **1 mark:** Correct simplification of fraction.
	(ii) Parabola is $x^2=8y$. Therefore *a=2.* $M=\left(a(p+q),\frac{a}{2}(p^2+q^2)\right)$ $=\left(2(p+q),(p^2+q^2)\right)$ Since *a=2*	**(ii)** **1 mark:** Correct substitution using *a=2.*
	(iii) If $\frac{p+q}{2}=2$ then $p+q=4$ $\therefore M=\left(8,p^2+q^2\right)$ So the locus of *M* is the (vertical) line, *x=8*	**(iii)** **3 marks:** • Coordinates of *M* • Description of straight line • Equation of straight line. **2 marks:** 2 steps correct. **1 mark:** 1 step correct.

Outcomes Addressed in this Question

Year 12 Mathematics Extension 1 Half Yearly Exam 2011

HE2	uses inductive reasoning in the construction of proofs
H5	applies appropriate techniques from the study of calculus, geometry, probability, trigonometry and series to solve problems.

Outcome	Solutions	Marking Guidelines
HE2 H5	(a) Phinh compounding yearly	
	Beginning of year 1 $10 000	**1 mark:**
	End of year 1 $\$10\,000(1.06) + 1\,000$	
	(i) $11 600	**1 mark:**
	End of year 2 $\$10\,000(1.06) + \$1\,000)1.06 + \$1\,000$ $= \$10\,000x1.06^2 + \$1\,000x1.06 + \$\,1\,000$ (ii) $= \$10\,000x1.06^2 + \$1\,000(1.06 + 1)$ End of year: $3[\$10\,000x1.06^2 + \$1\,000x1.06 + \$1\,000]1.06 + \$1\,000$ $= \$10\,000x1.06^3 + \$1\,000x1.06^2 + \$1\,000x1.06 + \$1000$ $= \$10\,000x1.06^3 + \$1\,000(1.06^2 + 1.06 + 1)$ End of year n: $\$10\,000x1.06^n + \$1\,000(1.06^{n-1} + 1.06^{n-2} + \ldots\ldots 1.06 + 1)$ $= \$10\,000x1.06^n + \$1\,000(1 + 1.06 + 1.06^2 + \ldots\ldots + 1.06^{n-1})$ Note that $(1 + 1.06 + 1.06^2 + \ldots\ldots + 1.06^{n-1})$ is a GP with $T_1 = 1$, r=1.06, and n terms	
	End of year n $\$10\,000\times1.06^n + \$1000\frac{(1.06^n - 1)}{(1.06 - 1)}$ $= \$10\,000\times1.06^{20} + \$100\,000\frac{(1.06^{20} - 1)}{6}$ $= \$10\,000\times1.06^{20} + \$100\,000\times\frac{(1.06^{20} - 1)}{6}$ (iii) At the end of year 20 Phinh's investment was $\$10\,000\times1.06^{20} + \$100\,000\frac{(1.06^{20} - 1)}{6}$, $= \$68\,856.95$	**3 marks**: Complete solution **2 marks:** Substantial progress **1 mark:** Some progress
	Nerida compounding half yearly Nerida's initial $1 500 will compound to $1 500x 1.03 at the end of the first 6 months.	
	(i) At the end of year 1 this initial $1 500 will compound to $(\$1\,500x1.03)x1.03$ or $\$1\,500x1.03^2 = \$1591.35$	**1 mark:**
	At the beginning of the second year, Nerida adds another $1 500 and has $\$1\,500x1.03^2 + \$1\,500$. Half way through year 2 this total has compounded to $(\$1\,500x1.03^2 + \$1\,500)x1.03$ This amount continues to compound in the second half of year 2 to $[(\$1\,500x1.03^2 + \$1\,500)x1.03]x1.03$	
	(ii) At the end of year 2 she has $\$1\,500x\,1.03^4 + \$1\,500x1.03^2$ or $\$1\,500[1.03^4 + 1.03^2]$	**2 marks:** Note: reasoning required.

Outcome	Solutions	Marking Guidelines
	At the beginning of year 3 she adds another $1 500. At the end of year 3 the investment compounds to $\$1\,500[1.03^6 + 1.03^4 + 1.03^2]$ At the end of year n the investment becomes $\$1\,500[1.03^{2n} + 1.03^{2(n-1)} + 1.03^{2(n-2)} + \ldots\ldots + 1.03^2]$ Note that $1.03^2 + 1.03^4 + 1.03^6 + \ldots\ldots + 1.03^{2n}$ can be rewritten as $1.03^2(1 + 1.03^2 + \ldots\ldots + 1.03^{2n-2})$ and $(1 + 1.03^2 + \ldots\ldots + 1.03^{2n-2})$ is a GP with $T_1 = 1$, $r = 1.03^2$ and n terms. Hence Nerida's investment after n years is $\$1500x1.03^2\,\frac{(1.03^{2n} - 1)}{(1.03^2 - 1)}$ Nerida's balance at the end of 20 years is $\frac{\$1500 * 1.03^2 \times (1.03^{40} - 1)}{(1.03^2 - 1)}$ or $59 108.27	**3 marks:** **2 marks:** Substantial progress **1 mark:** Some progress
	b) P(n): $\sum_{i=1}^{n}\frac{1}{i(i+1)} = \frac{n}{n+1}$ (i) Test for n = 1. LHS = $\frac{1}{2}$ and the RHS = $\frac{1}{2}$ LHS = RHS so P(n) is true for n =1 (ii) Assume true for n = k i.e. $\frac{1}{1\times2} + \frac{1}{2\times3} + \frac{1}{3\times4} + \ldots\ldots\ldots\ldots + \frac{1}{k(k+1)} = \frac{k}{k+1}$ (iii) Test for n = k + 1 i.e. Is $\frac{1}{1\times2} + \frac{1}{2\times3} + \frac{1}{3\times4} + \ldots\ldots + \frac{1}{k(k+1)} + \frac{1}{(k+1)(k+2)} = \frac{k+1}{k+2}$? LHS = $\frac{1}{1\times2} + \frac{1}{2\times3} + \frac{1}{3\times4} + \ldots\ldots + \frac{1}{k(k+1)} + \frac{1}{(k+1)(k+2)}$ LHS = $\frac{k}{k+1} + \frac{1}{(k+1)(k+2)}$ LHS = $\frac{1}{k+1}(k + \frac{1}{k+2})$	

	LHS = $\frac{1}{(k+1)(k+2)} \times (k^2+2k+1)$ LHS = $\frac{1}{(k+1)(k+2)} \times (k+1)^2$ LHS = $\frac{k+1}{k+2}$ LHS = RHS therefore P(k+1) is true whenever P(k) is true (iv) P(n) is true for n =1, and is true for n = k + 1 whenever it is true for n = k. P(n) is therefore true for $n \geq 1$ Proof by Mathematical Induction	**4 marks:** Complete proof **2 marks:** Substantial attempt.