HURLSTONE AGRICULTURAL HIGH SCHOOL

YEAR 12 2012

HSC COURSE
EXTENSION 1 MATHEMATICS

ASSESSMENT TASK 2
HALF YEARLY EXAMINATION

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General Instructions

- Reading time : 5 minutes
- Working time : 90 minutes
- This exam paper has three questions. Attempt all questions.
  -Question 1 is worth 18 marks - three multiple choice questions worth one mark each and free response questions worth 15 marks.
  - Question 2 and question 3 are worth 17 marks each - two multiple choice questions worth one mark each and free response questions worth 15 marks.
  Total: 52 marks
- **Start a new answer booklet for each question making sure your student number is written at the top of each page.**
- All necessary working should be shown in each question. Marks may not be awarded for careless or badly arranged work.
- Board approved calculators and mathematical templates may be used.
- This examination paper must **not** be removed from the examination room.

Students Name:________________________

Teacher:________________________

QUESTION 1. *Start a new answer booklet.*

For parts a), b) and c) choose the correct answer from A, B, C or D and write your chosen answer in the answer booklet.

a) How many real roots must the following equation have? **1**

$$x^4\left(x^2-4\right)+9\left(x^2-4\right)=0$$

A 1 B 2

C 4 D none

b) If $P(x)=x^3-2x^2+9x-2$, which of the following statements are true? **1**

I. $x-3$ is a factor of $P(x)$

II. $x=3$ is a root of $P(x)=0$

III. $P(3)=34$

IV. $P(-3)=34$

A I only B III only

C I and II only D I and III only

For parts c), d), e), f), and g) show all necessary working out in the answer booklet.

c) A, B and P are the points $(-1,8)$, $(6,-6)$ and $(4,-2)$ respectively.

The point P divides the interval AB internally in thr ratio k:1.

Find the value of k. **2**

d) If a, b and c are the roots of $x^3-3x+2=0$, find $a^2+b^2+c^2$. **2**

e) Find the acute angle between the lines $y=2x-1$ and $3x-2y=5$.
Give your answer in radians correct to two decimal places. **2**

f) The polynomial $Q(x)=x^3+ax+b$ has a factor of $(x+2)$. **2**

When $Q(x)$ is divided by $(x-2)$ the remainder is 12.

Find the values of a and b.

g) Let $f(x)=x^3+5x^2+17x-10$. The equation $f(x)=0$ has only one real root.

i) Show that the root lies between 0 and 2. **3**

ii) Use one application of the 'halving the interval' method to find a smaller interval containing the root. **2**

iii) Which end of the smaller interval found in part ii) is closer to the root? Briefly justify your answer. **2**

QUESTION 2. *Start a new answer booklet.*

For parts a), b) and c) choose the correct answer from A, B, C or D and write your chosen answer in the answer booklet.

a) If $1^2+3^2+5^2+\ldots+(2n-1)^2=\frac{1}{3}n(2n-1)(2n+1)$,
then what is the $(k+1)^{\text{th}}$ term? **1**

A $4k^2$

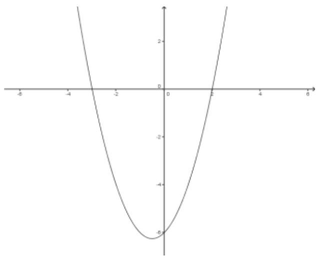
B $(2k+1)^2$

C $\dfrac{(k+1)2k(2k+2)}{3}$

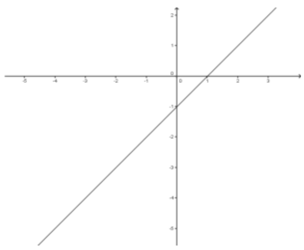
D $\dfrac{(k+1)(2k+1)(2k+3)}{3}$

b) In order to solve the inequality $\dfrac{(x+2)(x-3)}{x+1}<0$, which graph would be the most useful? **1**

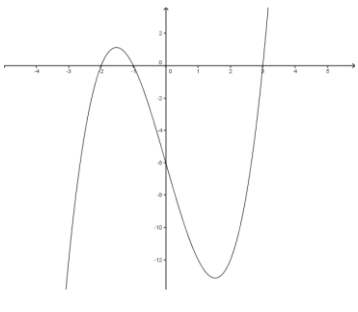
A



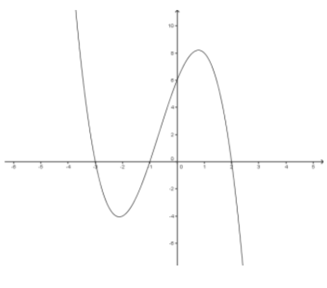
B



C



D



c) Which of the following is NOT equal to $\sin 75^{\text{O}}$? **1**

A $\sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$

B $\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$

C $\frac{\sqrt{3}+1}{\sqrt{2}}$

D $\cos 60^\circ \cos 45^\circ + \sin 60^\circ \sin 45^\circ$

For parts d), e), f), g), and h) show all necessary working out in the answer booklet.

d) Solve the inequality $\frac{x^2}{x-2} \geq -1$ **3**

e) Solve the equation $\sin 2\theta = \cos\theta$ for $0 \leq \theta \leq 2\pi$ **3**

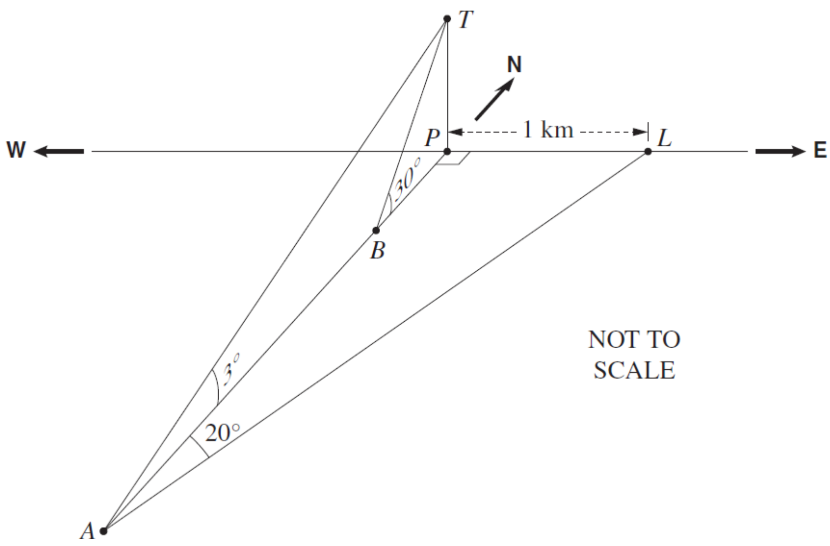
f) Find an expression for $\sin 5x$ in terms of $\sin x$ and $\cos x$ **3**

g) Prove, by mathematical induction, that $9^{n+2} - 4^n$ is divisible by 5 for any positive integer n. **3**

h) A boat is sailing due north from a point A towards a point P on the shore line. The shore line runs from west to east.

In the diagram, T represents a tree on a cliff vertically above P, and L represents a landmark on the shore. The distance PL is 1 km.

From A the point L is on a bearing of 020°, and the angle of elevation to T is 3°. After sailing for some time the boat reaches a point B, from which the angle of elevation to T is 30°.



Show that $BP = \dfrac{\sqrt{3}\tan 3°}{\tan 20°}$. **3**

QUESTION 2. *Start a new answer booklet.*

For parts a), b) and c) choose the correct answer from A, B, C or D and write your chosen answer in the answer booklet.

a) In a game of Poker, 5 cards are dealt to each player. **1**

The deck has 4 suits with 13 cards in each suit.

How many different hands are possible?

A $^{52}C_5$

B $^{13}C_5$

C $^{52}P_5$

D $^{13}P_5$

b) In the same game of Poker, what is the probability of being dealt a "flush" (all five cards from the same suit)? **1**

A $\dfrac{5}{52}$

B $\dfrac{1}{52^5}$

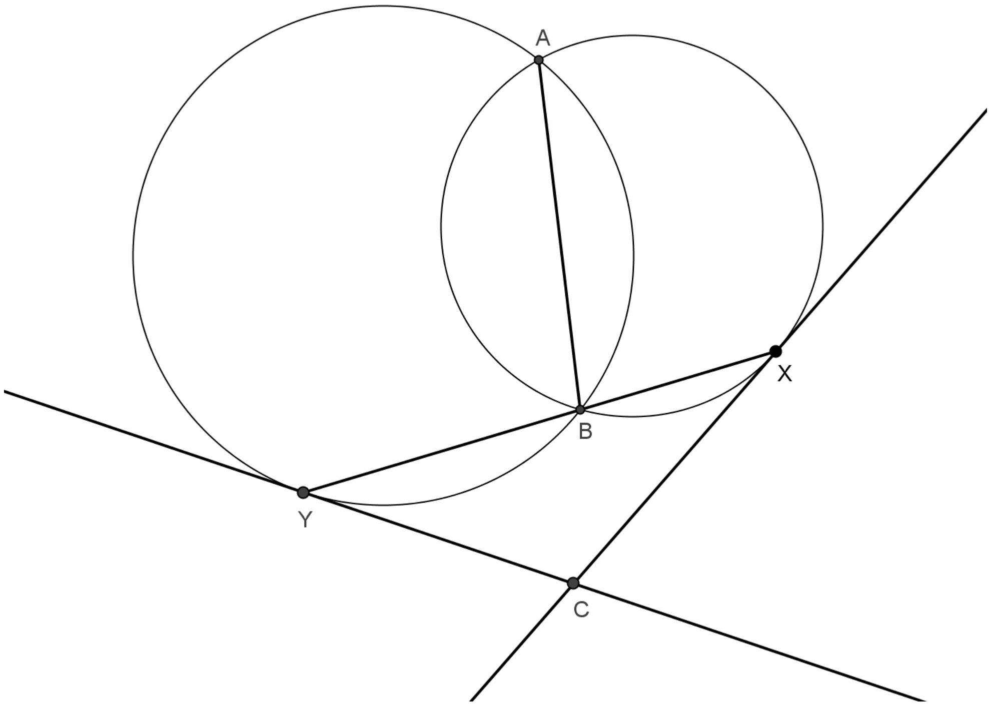
C $\dfrac{33}{66640}$

D $\left(\dfrac{5}{52}\right)^5$

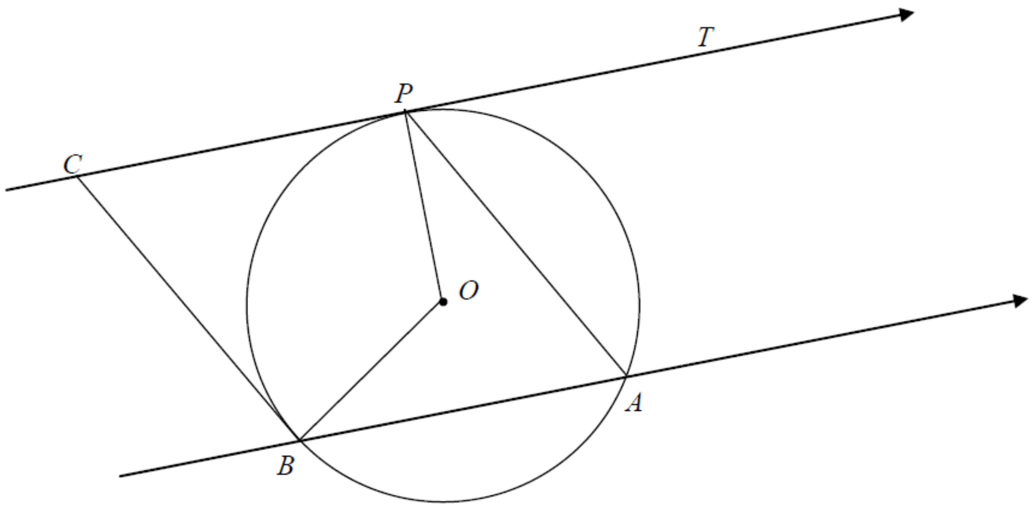
For parts c), d), e), f), g), and h) show all necessary working out in the answer booklet.

c) AB is a common chord of two circles and a straight line through B cuts the circles in X and Y. Tangents to the circles at X and Y meet at C.

Prove that AXCY is a cyclic quadrilateral. **4**



d) *CT* is a tangent to the circle, centre *O*, touching at *P*. *PABC* is a rhombus and *CT* is parallel to *AB*.



(i) Let $\angle TPA = \alpha$ and prove that $\angle POB = 2\alpha$. **2**

(ii) Find the value of α such that POBC is a cyclic quadrilateral, giving reasons. **2**

e) How many nine letter arrangements can be made using the letters of the word *ISOSCELES*? **2**

f) A club has 9 male and 7 female members. How many ways can a team of four be chosen if it is to consist of 2 men and 2 women? **2**

g) In how many ways can 6 boys and 5 girls be arranged in a row if 3 particular girls are kept together? **2**

h) Debby and John and six other people go through a doorway one at a time.

In how many ways can the eight people go through the doorway if John goes through the doorway after Debby with no-one in between? **1**

END OF EXAMINATION

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1} + C, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x + C, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax} + C, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax + C, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax + C, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax + C, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax + C, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + C, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a} + C, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right) + C, \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right) + C$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Year 12	Extension 1 Mathematics	Task 2 HSC 2012
Question No. 1	Solutions and Marking Guidelines	

Outcome Addressed in this Question

PE3-Solves problems involving permutations and combinations, inequalities, polynomials, circle geometry and parametric representations.

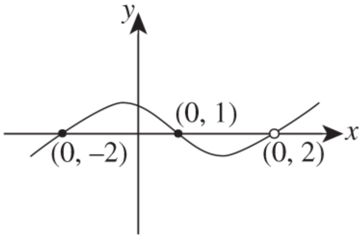
	Solutions	Marking Guidelines
PE3	**Question 1.** a) $(x_1,y_1),(x_2,y_2)$ and (x,y) ratio $m:n$ $(-1,8),\ (6,-6)$ and $(4,-2)$ ratio $k:1$ $x=\dfrac{mx_2+nx_1}{m+n}$, $4=\dfrac{k(6)+1(-1)}{k+1}$, $4k+4=6k-1,$ $10=2k,$ $\therefore k=\dfrac{5}{2}$	**2 marks complete correct solution** **1 marks for partial correct solution**
	b) $a^2+b^2+c^2$ $=(a+b+c)^2-2(ab+bc+ac)$ $=(0)^2-2\left(\dfrac{-3}{1}\right)$ $=6$	**2 marks complete correct solution** **1 marks for partial correct solution**
	c) $\tan\theta=\dfrac{m_1-m_2}{1+m_1m_2}$ $y=2x-1$ has gradient $m_1=2$ $3x-2y=5$ has gradient $m_2=\dfrac{3}{2}$ $\therefore \tan\theta=\dfrac{2-\frac{3}{2}}{1+(2).\left(\frac{3}{2}\right)}$ $=\dfrac{\frac{1}{2}}{4}$ $=\dfrac{1}{8}$ $\therefore \theta=7°8'$ The angle between the two lines is $7°8'$	**2 marks complete correct solution** **1 marks for partial correct solution**

PE3	d) $Q(x)=x^3+ax+b$ $Q(-2)=(-2)^3+a(-2)+b=0$ $\therefore -8-2a+b=0$ $\therefore -2a+b=8.....................................(A)$ $Q(2)=(2)^3+2(2)^2+a(2)+b=12$ $\therefore 8+2a+b=12$ $\therefore 2a+b=4.....................................(B)$ $(A)+(B)$ $2b=12$ $\therefore b=6$ $\therefore a=-1$	**3 marks complete correct solution** **2 marks partial correct solution with only one error.** **1mark for any correct work that could lead to a solution.**
PE3	e) i) $f(0)=(0)^3+5(0)^2+17(0)-10=-10$ $f(2)=(2)^3+5(2)^2+17(2)-10=52$ Since $f(0)$ is negative and $f(2)$ is positive and the polynomial is continuous between $x=0$ and $x=2$, then the root lies between 0 and 2.	**2 marks complete correct solution** **1 mark for partial correct solution**
PE3	ii) $f\left(\frac{0+2}{2}\right)=f(1)=(1)^3+5(1)^2+17(1)-10=13>0$ Hence, the roots lies between 0 and 1.	**2 marks complete correct solution** **1 marks for partial correct solution**
PE3	iii) $f\left(\frac{0+1}{2}\right)=f(0\cdot5)=-0.125$ or $-\frac{1}{8}$ Therefore, the root lies between 0.5 and 1. The root is close to 1 than 0.	**2 marks complete correct solution** **1 marks for partial correct solution**

Year 12	Mathematics Extension 1	H/Y Exam 2012
Question No. 2	Solutions and Marking Guidelines	

Outcomes Addressed in this Question

HE2 - uses inductive reasoning in the construction of proofs
PE3 - solves problems involving permutations and combinations, inequalities, polynomials, circle geometry and parametric representations

Outcome	Solutions	Marking Guidelines
PE3	(a) $\frac{x^2}{x-2} \geq -1$ $x^2(x-2) \geq -(x-2)^2$ $x^2(x-2)+(x-2)^2 \geq 0$ $(x-2)(x^2+x-2) \geq 0$ $(x-2)(x+2)(x-1) \geq 0$  $-2 \leq x \leq 1,\ x > 2$	**<u>3 marks:</u>** correct solution **<u>2 marks:</u>** Substantially correct **<u>1 mark:</u>** Makes some progress Eg indicates $x \neq 2$ OR multiplies both sides by $(x-2)^2$ OR solves separately for $x < 2$ and $x > 2$.
	(b) $\sin 2\theta = \cos\theta$ $\sin 2\theta - \cos\theta = 0$ $2\sin\theta\cos\theta - \cos\theta = 0$ $\cos\theta(2\sin\theta - 1) = 0$ $\cos\theta = 0$ or $\sin\theta = \frac{1}{2}$ $\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$	**<u>3 marks:</u>** correct solution (must be in radians for full marks) **<u>2 marks:</u>** Substantially correct (finds three correct solutions OR makes a minor error) **<u>1 mark:</u>** Makes some progress (uses $\sin 2\theta = 2\sin\theta\cos\theta$)
	(c) $\sin 3x = \sin(2x+x)$ $= \sin 2x\cos x + \cos 2x \sin x$ $= 2\sin x\cos x\cos x + (\cos^2 x - \sin^2 x)\sin x$ $= 3\sin x\cos^2 x - \sin^3 x$ $= 3\sin x(1-\sin^2 x) - \sin^3 x$ $= 3\sin x - 4\sin^3 x$	**<u>3 marks:</u>** for any form that includes only powers of sin*x* **<u>2 marks:</u>** for incomplete expansion **<u>1 mark:</u>** if started using any valid breakup of sin3*x*

HE2	(d) show true for $n = 1$. $9^{1+2} - 4^{1} = 729 - 4$ $= 725$ $= 5(145)$ which is true Assume true for $n = k$ ie $9^{k+2} - 4^{k} = 5M$ where M is an integer so $9^{k+2} = 5M + 4^{k}$ $\quad ...(1)$ prove true for $n = k + 1$ $9^{k+3} - 4^{k+1} = 9.9^{k+2} - 4.4^{k}$ $= 9\left(4^{k} + 5M\right) - 4.4^{k}$ $\quad \left(\text{from } (1)\right)$ $= 5\left(4^{k}\right) + 9(5M)$ $= 5\left(4^{k} + 9M\right)$ $= 5N$ where N is an integer $\therefore$ true for all positive integers n by induction (e) $\dfrac{PL}{PA} = \tan 20°$ $PA = \dfrac{PL}{\tan 20°} = \dfrac{1}{\tan 20°}$ $\dfrac{PT}{PA} = \tan 3°$ $PT = PA.\tan 3° = \dfrac{\tan 3°}{\tan 20°}$ $\dfrac{PT}{BP} = \tan 30° = \dfrac{1}{\sqrt{3}}$ $BP = \sqrt{3}.PT$ $= \dfrac{\sqrt{3}\tan 3°}{\tan 20°}$	**<u>3 marks:</u>** correct solution **<u>2 marks:</u>** substantially correct **<u>mark:</u>** partial progress towards correct solution **<u>3 marks:</u>** correct solution **<u>2 marks:</u>** substantially correct **<u>1 mark:</u>** partial progress towards correct solution

Year 12 Half Yearly 2012 – Extension 1 Mathematics

Question No: 3 **Solutions and Marking Guidelines**

Outcomes Addressed in this Question:
PE2 - uses multi-step deductive reasoning in a variety of contexts
PE3 - solves problems involving permutations and combinations, inequalities, polynomials, circle geometry and parametric representations

Outcome	Sample Solution	Marking Guidelines
PE2	a) $\angle CXY = \angle BAX = \theta$ (alternate segment theorem) $\angle CYX = \angle YAB = \alpha$ (alternate segment theorem) $\alpha + \theta + \angle YCX = 180°$ (angle sum of $\Delta CYZ = 180°$) $\therefore \angle YCX = 180° - (\alpha + \theta)$ $\angle YAX + \angle YCX = \alpha + \theta + 180° - (\alpha + \theta)$ $= 180°$ $\therefore AYCX$ is cyclic (opposite angles are supplementary)	**4 marks** – correct solution with reasons **3 marks** – three of four correct deductions with correct reasons **2 marks** – two of four correct deductions with correct reasons **1 mark** – one of four correct deductions with correct reasons
PE2	b) i) $\angle PAB = \angle TPA = \alpha$ (alternate angles, $PC \parallel AB$) $\angle POB = 2\angle PAB$ (angle at the centre equals twice the angle at the circumference standing on arc PB) $=2\alpha$	**2 marks** –correct answer **1 mark** – substantial progress towards correct answer
PE2	b) ii) $\angle PCB = \angle PAB = \alpha$ (opposite angles of a rhombus are equal) $\angle PCB + \angle POB = 180°$(opposite angles of a cyclic quadrilateral are supplementary) $2\alpha + \alpha = 180°$ $\therefore \alpha = 60°$	**2 marks** –correct answer **1 mark** – substantial progress towards correct answer
PE3	c) $\dfrac{9!}{3!2!} = 30240$	**2 marks** –correct answer **1 mark** – substantial progress towards correct answer
PE3	d) ${}^9C_2 \times {}^7C_2 = 756$	**2 marks** –correct answer **1 mark** – substantial progress towards correct answer
PE3	e) $9! \times 3! = 2177280$	**2 marks** –correct answer **1 mark** – substantial progress towards correct answer
PE3	f) $7! = 5040$	**1 mark** – correct answer