Y12 Mathematics Extension 1 | Term 1 Assessment 2011

Question 1 (9 Marks) **Marks**

(a) Differentiate with respect to x:

(i) $\ln(\cos^2 x)$ **2**

(ii) $x^2 \operatorname{cosec}(x-1)$ **2**

(iii) $e^x \cos(3x)$ **2**

(b) On a cultivated farm plot, there are approximately 12 times as many carrot plants as stinging nettle weeds. However, the plot becomes neglected and the number of weeds increases at 4% per day whilst the number of carrots decreases at 8% per day. Find the number of days, to the nearest day, that must elapse before there are more weeds than carrots. **3**

Question 2 (9 Marks) – START A NEW PAGE **Marks**

(a) Find: $\displaystyle\int \frac{3x}{x-1}\,dx$. **2**

(b) The area bound by $y = \sec x$, $y = 1 - 2x$, $y = 0$ and $x = \frac{\pi}{4}$ is rotated about the x-axis. Find the volume of the solid generated. **3**

(c) An observer at A watches a falcon F flying downward to earth in pursuit of its prey. The falcon is descending at 220km/h and the point B directly beneath the falcon is 1.5km from the observer. Find the rate at which the observer's head is tilting, in radians per minute, when the falcon's altitude is 3.3km. **4**

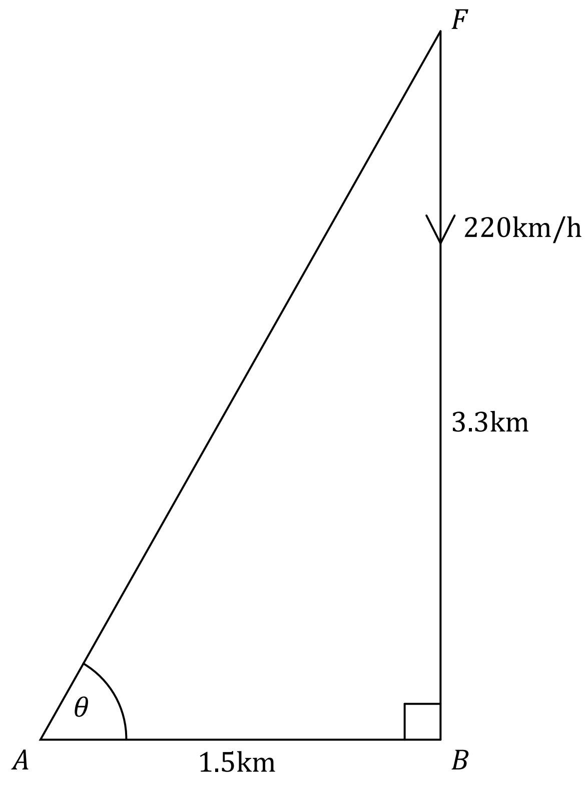

Diagram not to scale

Question 3 (9 Marks) – START A NEW PAGE **Marks**

(a) Express $\log\left(\frac{x^2}{\sqrt[3]{y}}\right)$ in terms of p and q if $p = \log x$ and $q = \log y$. **2**

(b) A geometric series $S(r)$ has a first term of 5, a last term of 20,480 and a common ratio of r.

(i) Find the difference between $S(2)$ and $S(4)$. **2**

(ii) Find the total number of terms in $S(2)$. **1**

(c) Neatly sketch the graph of $y = \sqrt{3}\sin x - \cos x$ for the domain $0 \leq x \leq 2\pi$, clearly showing all intercepts and stationary points. **4**

Question 4 (9 Marks) – START A NEW PAGE **Marks**

(a) Evaluate: $\sum_{k=0}^{4} 3^{2k-3}$ **2**

(b) Evaluate the following integral using the substitution $\theta = \sin^{-1}\left(\frac{x}{2}\right)$: **4**

$$\int_0^{\sqrt{3}} \frac{2x^2}{\sqrt{4-x^2}}\,dx$$

(c) Use the principle of mathematical induction to prove that $5^n + 2(11^n)$ is divisible by 3 for all positive integers n. **3**

Question 5 (9 Marks) – START A NEW PAGE **Marks**

(a) Find: $\int \frac{x-3}{x^2+1}\,dx$. **2**

(b) Find: $\int \sin 2x\,\sqrt{\cos 2x}\,dx$. **2**

(c) The series S_n is given by the following:

$$S_n = 3^{2k} + 3^{2k-3} + 3^{2k-6} + \cdots + 3^{-k}$$

(i) Find the value of n, the number of terms in the series, in terms of k. **1**

(ii) Show that the nth term is given by: **1**

$$T_n = \frac{27 \times 3^{2k}}{3^{3n}}$$

(iii) Calculate the sum of the series. **3**

Question 6 (9 Marks) – START A NEW PAGE **Marks**

(a) Use the principle of mathematical induction to prove that $\frac{d}{dx}x^n = nx^{n-1}$ for all positive integers n. (You may assume the product rule.) **3**

(b) Neatly sketch the graph of $y = \ln(x^2 - 2x + 1)$. **3**

(c) Evaluate $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} (\cot x + \operatorname{cosec}^2 x)dx$. **3**

Question 7 (9 Marks) – START A NEW PAGE **Marks**

(a) A spherical balloon is being inflated at the rate of 85cm^3 per second. Find the rate at which its radius is increasing after one minute (3 decimal places). **3**

(b) A man named Fischer is administered a drug that puts him to sleep. As long as there is 15mg of the drug in his body, he remains asleep. 10% of the drug leaves Fischer's body every 30 minutes. He is given an initial dose of 20mg, and an additional dose of d mg is given at the beginning of each new hour.

(i) Show that immediately after the second additional dose is administered, Fischer has $13.122 + 1.81d$ mg of the drug remaining in his body. **1**

(ii) If R_n is the amount of drug remaining at the end of the nth hour, just before he receives his additional dose, show that R_n is given by: **3**

$$R_n = 20(0.81)^n + d\left[\frac{81}{19} - \frac{(0.81)^n}{0.19}\right]$$

(iii) If Fischer must be kept asleep for at least ten hours, find the minimum hourly dose required (to the nearest 0.01mg). **2**

YR 12 TERM 1 **MATHEMATICS Extension 1 : Question** 1 2011

Suggested Solutions	Marks	Marker's Comments
1 (a)(i) $\frac{d}{dx}\ln(\cos^2 x) = \frac{-2\sin x\cos x}{\cos^2 x}$	1	
$= \frac{-2\sin x}{\cos x}$	$\frac{1}{2}$	
$= -2\tan x$	$\frac{1}{2}$	
1(a)(i) **OR** $\frac{d}{dx}\ln(\cos^2 x) = \frac{d}{dx}(2\ln\cos x)$ for $0 < x < \frac{\pi}{2}$	1	
$= \frac{-2\sin x}{\cos x}$	$\frac{1}{2}$	
$= -2\tan x$	$\frac{1}{2}$	
(ii) $\frac{d}{dx}(x^2\operatorname{cosec}(x-1)) = \frac{d}{dx}\left[\frac{x^2}{\sin(x-1)}\right]$	$\frac{1}{2}$	
$= \frac{\sin(x-1)\times 2x - x^2\times\cos(x-1)}{\sin^2(x-1)}$	1	
$= 2x\operatorname{cosec}(x-1) - x^2\cot(x-1)\operatorname{cosec}(x-1)$ **OR** $= x\operatorname{cosec}(x-1)(2 - x\cot(x-1))$	$\frac{1}{2}$	
(iii) $\frac{d}{dx}\left[e^x\cos(3x)\right] = \cos(3x)e^x + e^x(-3\sin(3x))$ $= e^x\left[\cos(3x) - 3\sin(3x)\right]$	2	1 mark each for the parts of the product rule
(b) Let t be the number of days elapsed $\therefore$ Number of weeds $= n(1.04)^t$ for $n \in R$ Number of carrots $= 12n(0.92)^t$		
Solving $12n(0.92)^t = n(1.04)^t$	1	
$12 = \frac{(1.04)^t}{(0.92)^t}$ $12 = \left(\frac{1.04}{0.92}\right)^t$ $12 = \left(\frac{26}{23}\right)^t$		1 mark lost for $t-1$ as the power
$t = \log_{\left(\frac{26}{23}\right)} 12$ i.e. $\frac{\ln 12}{\ln 26 - \ln 23}$	1	
$t = 20.268022\ldots$	1	
$\therefore$ 21 days must elapse before there are more weeds than carrots.		

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MATHEMATICS Extension 1 : Question 2

Suggested Solutions

(a) $\int \frac{3x}{x-1}\,dx$ or $\int \frac{3x-3+3}{x-1}\,dx$

$= \int\left(3 + \frac{3}{x-1}\right)dx \qquad = \int\left(3 + \frac{3}{x-1}\right)dx$ 1mk

$= 3x + 3\ln(x-1) + C$ 1mk

(b) $V = \pi\int_0^{\pi/4} \sec^2 x\,dx - \frac{1}{3}\pi \times (1)^2 \times \frac{1}{2}$

$= \pi\left[\tan x\right]_0^{\pi/4} - \frac{\pi}{6}$

$= \pi\left(\tan\frac{\pi}{4} - \tan 0\right) - \frac{\pi}{6}$

$= \pi(1-0) - \pi/6$

$= \frac{5\pi}{6}$ units^3

or

$V = \pi\int_0^{\pi/4} \sec^2 x\,dx - \pi\int_0^{1/2}(1-2x)^2\,dx$

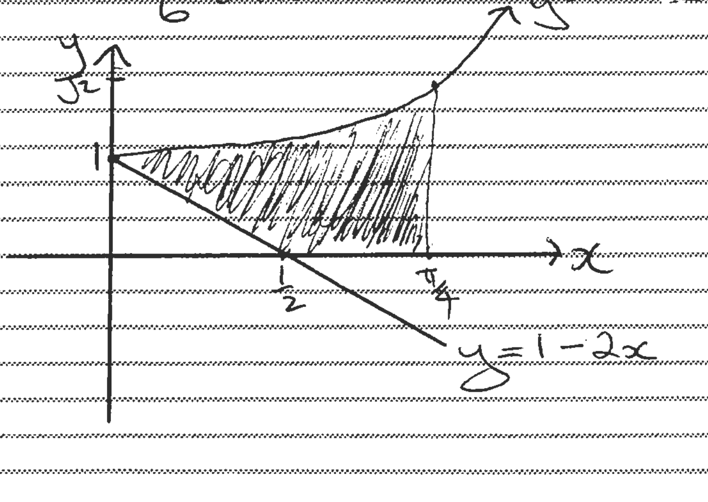
$= \pi\left[\tan x\right]_0^{\pi/4} - \pi\int_0^{1/2}(1+4x^2-4x)\,dx$

$= \pi\left(\tan\frac{\pi}{4} - \tan 0\right) - \pi\left[x - 2x^2 + \frac{4}{3}x^3\right]_0^{1/2}$

$= \pi(1-0) - \pi\left(\frac{1}{2} - \left(2\times\frac{1}{4}\right) + \left(\frac{4}{3}\times\frac{1}{8}\right)\right) - 0$

$= \pi - \pi\left(\frac{1}{2} - \frac{1}{2} + \frac{1}{6}\right)$

$= \frac{5\pi}{6}$ units^3



Marker's Comments

* generally well done. (*Some did it by substitution).

* lost ½ mk if they mixed up the cylinder's height and radius.

* ½ mk off if they couldn't subtract properly.

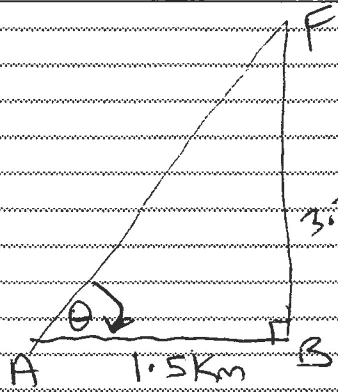
* If they did $\int_0^{\pi/4}(1-2x)^2$ then they lost ½mk.

* no marks for graphing it correctly.

* If they only did $V = \pi\int_0^{\pi/4}\sec^2 x\,dx = \pi$ they got 1mk.

* If they did the area only and made errors they got 0

MATHEMATICS Extension 1 : Question 2 cont.

Suggested Solutions	Marks	Marker's Comments
		
let falcon's altitude $= x$		
$\tan\theta = \frac{x}{1.5}$		
$x = 1.5\tan\theta$		
$\frac{dx}{d\theta} = 1.5\sec^2\theta$		
$= \frac{3}{2\cos^2\theta}$	$\frac{1}{2}$ mk	* If they got $\frac{dx}{dt} = 220$ only, they lost 1 mk; interpretation made question easier.
now $\frac{dx}{dt} = -220$ (falcon is flying down)		
$\frac{d\theta}{dt} = \frac{d\theta}{dx} \times \frac{dx}{dt}$	$\frac{1}{2}$	
$= \frac{2\cos^2\theta}{3} \times -220$	$\frac{1}{2}$	
when $x = 3.3$, $\tan\theta = \frac{3.3}{1.5}$	$\frac{1}{2}$	
$\theta = 1.144168834$	$\frac{1}{2}$	* If they left it in degrees, lost marks.
$\therefore \frac{d\theta}{dt} = \frac{-5500}{219}$	$\frac{1}{2}$	
$\therefore \frac{d\theta}{dt} = \frac{-5500}{219\times 60} = -\frac{275}{657}$	$\frac{1}{2}$	* If they left it in rad/hr, lost $\frac{1}{2}$ mk.
$\therefore$ The observers head is tilting downwards at $\frac{275}{657}$ rad/min.	$\frac{1}{2}$	
OR		
$\tan\theta = \frac{x}{1.5}$		
$\theta = \tan^{-1}\left(\frac{x}{1.5}\right)$	when $x = 3.3$,	
$\frac{d\theta}{dx} = \frac{1.5}{1.5^2 + x^2}$		$\frac{d\theta}{dt} = \frac{-5500}{219}$ rad/hr
$\frac{d\theta}{dt} = \frac{d\theta}{dx} \times \frac{dx}{dt}$		$\frac{d\theta}{dt} = \frac{-275}{657}$ rad/min
$= \frac{1.5}{1.5^2 + x^2} \times -220$		$\therefore$ The observers head ______ etc.
$= \frac{-330}{1.5^2 + x^2}$		

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Y12 MATH EXT ASSESSMENT TASK 2
TERM 1, 2011

MATHEMATICS Extension 1 : Question 3

Suggested Solutions	Marks	Marker's Comments

Q3 (a)

$$\log \frac{x^2}{\sqrt[3]{y}} = \log x^2 - \log \sqrt[3]{y} \quad ; \quad x, y > 0$$

$$= 2\log x - \log y^{\frac{1}{3}} = 2\log x - \frac{1}{3}\log y$$

$$= 2p - \frac{1}{3}q$$

Marker's comments: $\frac{1}{2}$ for $-$; $\frac{1}{2}$ for 2; $\frac{1}{2}$ for $\frac{1}{3}$; $\frac{1}{2}$ for answer. [2]

(b) $T_1 = a = 5$

(i) $S(r) = 5 + 5r + 5r^2 + 5r^3 + \ldots + 5r^{n-1} = \frac{5(r^n - 1)}{r - 1} = \frac{rL - a}{r - 1} = \frac{rL - 5}{r - 1}$ — $\frac{1}{2}$

For $S(2) = 5 + 5.2 + 5.2^2 + 5.2^3 + \ldots + 20480$ ...... 5×2^{12}

$S(4) = 5 + 5.4 + 5.4^2 + 5.4^3 + \ldots + 20480$ ..... 5×4^6

So $S(2) = \frac{5(2^n - 1)}{2 - 1} = 2 \times L - 5 = 2 \times 20480 - 5 = 40\,955$ — $\frac{1}{2}$

$S(4) = \frac{5(4^m - 1)}{4 - 1} = \frac{4 \times L - 5}{3} = \frac{4 \times 20480 - 5}{3} = 27\,305$ — $\frac{1}{2}$

$\therefore S(2) - S(4) = 40955 - 27305 = 13650$ — $\frac{1}{2}$

Marker's comments: Acc $-13\,650$

OR $S(2) - S(4) = 5.2 + 5.2^3 + 5.2^5 + 5.2^7 + 5.2^9 + 5.2^{11}$

$= 5.2[1 + 2^2 + 2^4 + 2^6 + 2^8 + 2^{10}]$

$= 10 \times \frac{[(2^2)^6 - 1]}{2^2 - 1} = 13\,650$

[2]

(ii) As a Geom. series $T_n = ar^{n-1}$

So $20480 = 5 \times 2^{n-1}$

$2^{n-1} = 4096 = 2^{12}$

$\therefore n = 13$

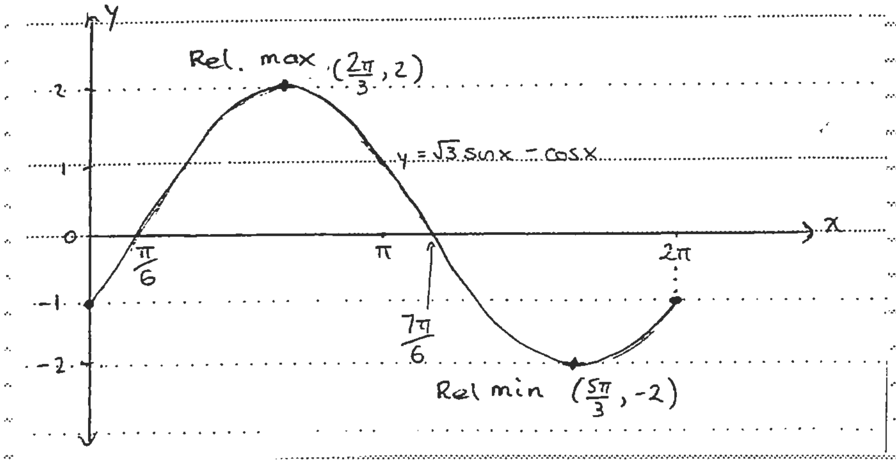
$\therefore$ No. of terms in $S(2)$ is 13

[1]

(c) $y = \sqrt{3}\sin x - \cos x$

$= 2\sin(x - \frac{\pi}{6})$; $2\sin(x + \frac{11\pi}{6})$; $2\cos(x - \frac{2\pi}{3})$; $2\cos(x + \frac{4\pi}{3})$

Marker's comments: $R = 2$ and correct phase $\angle$ (1)



Marker's comments:

$(0, -1)$ ... $(\frac{1}{2})$ $(2\pi, -1)$

x-ints $\frac{\pi}{6}$ and $\frac{7\pi}{6}$ (1)

SPs $\begin{cases} (\frac{2\pi}{3}, 2) & \text{Rel max TP} \\ (\frac{5\pi}{3}, -2) & \text{Rel min TP} \end{cases}$

$\frac{1}{2}$ each

$\frac{1}{2}$ shape

[4]

TERM 1 ASST. 2 2011

MATHEMATICS Extension 1 : Question 4   i of 2

Suggested Solutions	Marks	Marker's Comments
a) $\sum_{k=0}^{4} 3^{2k-3} = 3^{-3}+3^{-1}+3^{1}+3^{3}+3^{5}$	1	
$= \frac{1}{27}+\frac{1}{3}+3+27+243$		
$= \frac{7381}{27} = 273\frac{10}{27}$	1	Either answer got full marks.
b) $\theta = \sin^{-1}\left(\frac{x}{2}\right) \Rightarrow \frac{d\theta}{dx} = \frac{1}{\sqrt{4-x^2}}$		
$\therefore$ " $\frac{dx}{\sqrt{4-x^2}} = d\theta$ "		
Also $x = 2\sin\theta$		
When $x=\sqrt{3}$, $\theta = \sin^{-1}\left(\sqrt{3}/2\right) = \pi/3$	½	(limits)
$x = 0$ $\theta = \sin^{-1}(0) = 0$		
$\therefore$ Integral is $\int_0^{\pi/3} 2.(2\sin\theta)^2\,d\theta$		
$= 8\int_0^{\pi/3} \sin^2\theta\,d\theta$	1	Most people transferred to $x = \sin 2\theta$ before finding $d\theta$
$= \frac{8}{2}\int_0^{\pi/3}(1-\cos 2\theta)\,d\theta$	1	This produced
$= 4\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi/3}$	1	$\sqrt{\cos^2\theta}$ in the denominator.
$= 4\left(\frac{\pi}{3} - \frac{\sqrt{3}/2}{2}\right) - 4(0-0)$		Only a few explained away $\lvert\cos\theta\rvert$ correctly
$= \frac{4\pi}{3} - \sqrt{3}$	½	(or even noticed it!)

MATHEMATICS Extension 1 : Question 4   2 of 2

Suggested Solutions	Marks	Marker's Comments
c) To Prove $5^n + 2(11^n)$ is divisible by 3 for integer $n \geq 1$.		
Step 1 If $n = 1$, $5^n + 2(11^n) = 5 + 22 = 27 = 3 \times 9$	½	Essentially 2 marks for a correctly argued Step 2 plus 1 mark for the structure, including a correct step 1 and step 3.
Thus true for $n = 1$.		
Step 2 Assume result is true for some integer $k \geq 1$.	2	
i.e. $5^k + 2(11^k) = 3A$, some integer A. *		
It is necessary to prove that $5^{k+1} + 2(11^{k+1})$ is divisible by 3.		Step 3 was generally week.
$5^{k+1} + 2(11^{k+1}) = 5 \times 5^k + 2 \times 11 \times 11^k$		
$= 5 \times 5^k + 22 \times 11^k$		
$= 5(5^k + 2(11^k)) + 12 \times 11^k$		Although most people knew the general idea, it was quite poorly presented.
$= 5 \times 3A + 12 \times 11^k$ (By assumption*)		
$= 3(15A + 4(11^k))$		
which is divisible by 3. Thus, if true for k, then also true for k+1.	½	
Step 3 Using steps 1 and 2, we can deduce by P.M.I. that		
$5^n + 2(11^n)$ is divisible by 3 for integer $n \geq 1$.		

MATHEMATICS Extension 1 : Question 5

Suggested Solutions	Marks	Marker's Comments
(a) $\int \frac{x-3}{x^2+1}\,dx = \int \frac{x}{x^2+1}\,dx - 3\int \frac{dx}{x^2+1}$		
$= \frac{1}{2}\ln(x^2+1) - 3\tan^{-1}x + C$	1 each	
(b) $\int \sin 2x\,(\cos 2x)^{\frac{1}{2}}\,dx = -\frac{1}{3}(\cos 2x)^{\frac{3}{2}} + C$	2	1 off each error
(c) (i) $S_n = 3^{2k} + 3^{2k-3} + 3^{2k-6} + \ldots + 3^{-k}$		
$T_n = 3^{2k}\cdot(3^{-3})^{n-1}$		
$= 3^{2k-3n+3}$		
Number terms $2k-3n+3 = -k$	1	
$3n = 3k+3$		
$n = k+1$		
(ii) $T_n = 3^{2k}(3^{-3})^{n-1}$	1	½ off each mistake.
$= 3^{2k}\cdot 3^{-3n}\cdot 3^{3}$		
$= \dfrac{27\cdot 3^{2k}}{3^{3n}}$		
(iii) $S_n = 3^{2k} + 3^{2k-3} + 3^{2k-6} + \ldots + 3^{-k}$		
$3^{-3}S_n = 3^{2k-3} + 3^{2k-6} + \ldots + 3^{-k} + 3^{-k-3}$		
$(1-3^{-3})S_n = 3^{2k} - 3^{-k-3}$	3	1 off each mistake.
$\therefore S_n = \frac{27}{26}\left[3^{2k} - 3^{-k-3}\right]$ OR		/ indice rules.
$= \dfrac{3^{2k+3} - 3^{-k}}{26}$		simplification needed.
OR $S_n = \dfrac{a(1-r^n)}{1-r}$ $\quad n = k+1$		
$= \dfrac{3^{2k}(1-3^{-3(k+1)})}{1-3^{-3}}$		
$= \dfrac{3^{2k+3} - 3^{-3k-3+2k+3}}{27-1}$		

$= \dfrac{3^{2k+3} - 3^{-k}}{26}$

MATHEMATICS Extension : Question 6

Suggested Solutions	Marks	Marker's Comments

Q6(a) Let $P(n)$ be the proposition: $\frac{d}{dx}[x^n] = nx^{n-1}$ for $n = 1, 2, 3, \ldots$

• For $P(1)$:

LHS $= \frac{d}{dx}[x]$; $f(x) = x$ RHS $1.x^{(1-1)}$ $= 1x^0$ $= 1$

$= \lim_{h\to 0} \frac{(x+h) - x}{h}$

$= \lim_{h\to 0} \frac{h}{h}$

$= \lim_{h\to 0} 1$, $h \neq 0$ as $h \to 0$.

$= 1$

$\therefore$ LHS = RHS

$\therefore P(1)$ is true

Marks: 1. Comments: ($\frac{1}{2}$) For $\frac{d}{dx}x = 1$; 1 if explain via First Principles • Geometrical

• Assume $P(n)$ is true up to some integer $k \geq 1$

i.e. $\frac{d}{dx}x^k = kx^{k-1}$ ——— (*)

Marks: $\frac{1}{2}$. Comments: $-\frac{1}{2}$ if state $k \in \mathbb{Z}^+$ "$k \geq 1$"; $P(k)$

RTP $P(n)$ is true for $n = k+1$

i.e. $\frac{d}{dx}x^{k+1} = (k+1)x^k$.

• Proof for $P(k+1)$

Now $\frac{d}{dx}x^{k+1} = \frac{d}{dx}[x^k \times x]$

$= \frac{d}{dx}x^k \times x + x^k \times \frac{dx}{dx}$ — $\frac{1}{2}$ Product Rule

$= kx^{k-1} \times x + x^k \times 1$ — $\frac{1}{2}$ using $P(1)$ and Assumption

$= kx^k + x^k$

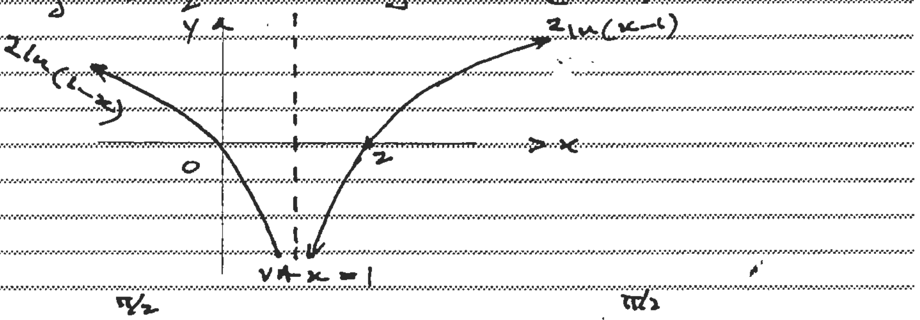
$= (k+1)x^k$

$\therefore P(k+1)$ is True

• Hence by the PMI $P(n)$ is true for $n = 1, 2, 3, \ldots$ — $\frac{1}{2}$

$\boxed{3}$

(b) $y = \ln[x^2 - 2x + 1] = \ln(x-1)^2$



Comments: $2\ln(x-1)$ only if $x > 1$.
1 For VA $x = 1$ with sketch concave downwards
$\frac{1}{2}$ For each intercept
$\frac{1}{2}$ For c.d. shape
Max 2 for $y = 2\ln(x-1)$ sketch.

$\boxed{3}$

(c) $I = \int_{\pi/3}^{\pi/2} (\cot x + \operatorname{cosec}^2 x)\,dx = \int_{\pi/3}^{\pi/2} \left(\frac{\cos x}{\sin x} + \operatorname{cosec}^2 x\right) dx$

$= \left[\ln(\sin x) - \cot x\right]_{\pi/3}^{\pi/2}$ — 1 for each

$= \left[\ln(\sin\frac{\pi}{2}) - \cot\frac{\pi}{2}\right] - \left[\ln(\sin\frac{\pi}{3}) - \cot\frac{\pi}{3}\right]$ — $\frac{1}{2}$ For subst.

$= [\ln 1 - 0] - \left[\ln\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{3}}\right]$

$I = \frac{1}{\sqrt{3}} - \ln\left(\frac{\sqrt{3}}{2}\right)$ — $\frac{1}{2}$ answer (correct)

$\boxed{3}$

T1 2011

MATHEMATICS Extension 1 : Question 7

Suggested Solutions	Marks	Marker's Comments
a) $\frac{dr}{dt}=\frac{dr}{dV}\times\frac{dV}{dt}=\frac{1}{4\pi r^2}\times 85$	1 m	
$t=60$ $\quad V=60\times 85=5100=\frac{4}{3}\pi r^3$		
$r=\left(\frac{3825}{\pi}\right)^{\frac{1}{3}}\doteqdot 10.67809\cdots$	1 m	many get r wrong
$\frac{dr}{dt}=\frac{85}{4\pi(10.67809)^2}=0.05932\cdots$		
radius increasing at 0.059 cm/sec or 3.559 cm/min #	1 m	
bi) After 1st additional dose $=20(0.9)^2+d$		
After 2nd additional dose $=[20(0.9)^2+d]0.9^2+d$	1 m	
$=13.122+1.81d$ #		
ii) Before additional dose		many fudging
$R_1=20(0.81)$		if show consistency
$R_2=20(0.81)^2+d(0.81)$	1 m	in GP get 1 m
$R_3=20(0.81)^3+d(0.81^2+0.81)$		
$\vdots$		
$R_n=20(0.81)^n+d(0.81^{n-1}+0.81^{n-2}+\cdots+0.81)$	1 m	must show the first term & last term, min 3 terms
$=20(0.81)^n+d\frac{0.81}{0.19}(1-0.81^{n-1})$		
$=20(0.81)^n+d\left[\frac{81}{19}-\frac{0.81^n}{0.19}\right]$ #	1 m	
iii) $R_{10}=15$ $\quad 20(0.81)^{10}+d\left(\frac{81}{19}-\frac{0.81^{10}}{0.19}\right)=15$	1 m	Students solve $R_{10}=0$ Get no marks as it makes calculation easier
$d=\frac{15-20(0.81^{10})}{\frac{81}{19}-\frac{0.81^{10}}{0.19}}=3.47$ mg # (nearest 0.01 mg)	1 m	& the answer is -0.67 mg does not make sense