Y12 Mathematics Extension 1 | Term 1 Assessment 2012

Question 1 (9 Marks) **Marks**

(a) Find:

(i) $\int e^{2x-1}dx$ **1**

(ii) $\int_0^1 \sqrt{1-x^2}dx$, where $\theta = \sin^{-1} x$. **3**

(b) Find the exact value of the gradient to the curve $y = \operatorname{cosec} x$ at $x = \frac{5\pi}{6}$ **2**

(c) If $f'(x) = 2\cot x - x$ and $f\left(\frac{\pi}{2}\right) = 0$, find an expression for $f(x)$ **3**

Question 2 (9 Marks) – START A NEW PAGE **Marks**

(a) (i) Find the sum of the series S given: **3**

$$S = 2^n + 2^{n-1} + 2^{n-2} + \ldots + 2^{-2n}$$

(ii) Discuss the existence of a limiting sum as $n \to \infty$ for S, giving reasons. **1**

(b) Write $\sqrt{6}\sin x + \sqrt{2}\cos x$ in the form $R\cos(x+\alpha)$, **3**

where $R > 0$ and $0 < \alpha < 2\pi$

(c) Find k, if $x^{k+3} = e^{7\ln x}$, where $x > 0$ **2**

Question 3 (9 Marks) – START A NEW PAGE **Marks**

(a) A spherical bubble is expanding so that its volume increases at a constant rate of 40mm^3 per second. What is the rate of increase of the surface area when the radius is 10 mm? **3**

(b) Prove by mathematical induction that: **3**

$$1 + 4 + 16 + \ldots + 4^n = \frac{1}{3}(4^{n+1} - 1), \text{ for } n = 0, 1, 2, \ldots$$

(c) Determine all values of x for which **3**

$\log_5(x-2) + \log_5(x-6) = 1$

Question 4 (9 Marks) – START A NEW PAGE **Marks**

(a) (i) Sketch the graph of $y = x\ln x$, showing all important features. **3**

(ii) Explain the nature of the curve as $x \to 0$, giving reasons. **1**

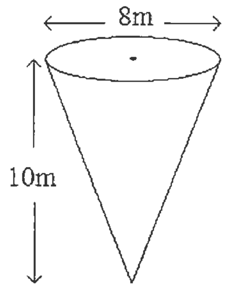
(b) (i) Show that $\frac{d}{dx}\left(\tan^3 x\right) = 3\sec^4 x - 3\sec^2 x$. **2**

(ii) Using (i) or otherwise, evaluate $\int_0^{\frac{\pi}{2}} \sec^4 x\, dx$. **3**

Question 5 (9 Marks) – START A NEW PAGE **Marks**

(a) When the same constant k is added to each of the numbers 60, 100, and 150 respectively, a geometric sequence is obtained. Find the common ratio for the sequence formed. **3**

(b)



A large grain storage container is in the shape of an inverted cone.
The diameter of the top is 8 metres.
The vertical height is 10 metres.

(i) If h (in metres) is the height of the grain at any given time, show that the volume V cubic metres at the time is given by $V = \frac{4}{75}\pi h^3$. **2**

(ii) Grain runs out at the bottom at the rate of 3 cubic metres per second. Find the rate of change of the height of grain in the container at the instant when the height is 5 metres. Give your answer in exact form. **4**

Question 6 (9 Marks) – START A NEW PAGE **Marks**

(a) Use the principle of mathematical induction to prove $3^n > 2n + 4$, for $n = 2, 3, 4, \ldots$. **3**

(b) Andrew takes out a loan for $50,000 to buy a new car. The interest on the loan is 9% per annum, compounded monthly. He agrees to pay the loan by equal monthly instalments over 5 years.

(i) Show that each monthly repayment will be approximately $1 037.92. **3**

(ii) Assuming he repays $1 037.92 per month, how much is still owing after making the 30^{th} repayment? **2**

(iii) After the 30^{th} repayment he makes a lump sum payment of $10,000. If Andrew wishes to still make 30 more repayments, what would be the new value of each repayment? **1**

Question 7 (9 Marks) – START A NEW PAGE **Marks**

(a) A flashlight throws a cone of light with a 60° angle between the outermost rays and the axis of the cone. A man points the light straight at a wall. At what rate is the illuminated area of the wall changing at the moment when the light is 3 metres from the wall and is being brought towards the wall at a rate of $0 \cdot 25$ metre per second? 3

(b) A sector of angle θ in radians is cut from a circular disc of radius 4π cm and used to make the complete curved surface of a right circular cone (with no overlap).

(i) Prove that the volume of this cone is given by: 3

$$V = \frac{8\pi\theta^2}{3}\sqrt{4\pi^2 - \theta^2}$$

(ii) Find the value of θ for which the volume of this cone is a maximum. 3

END OF PAPER

$\frac{1}{2}$

MATHEMATICS Extension 1 : Question 1

Suggested Solutions	Marks	Marker's Comments
(a) (i) $\int e^{2x-1}\,dx = \frac{1}{2}e^{2x} + C$	1	lost $\frac{1}{2}$ a mark if forgot "+C"
(ii) $\int_0^1 \sqrt{1-x^2}\,dx$		
$= \int_0^{\pi/2} \sqrt{1-\sin^2\theta}\,.\cos\theta\,d\theta$	$\frac{1}{2}$ for limits, $\frac{1}{2}$	
$= \int_0^{\pi/2} \cos^2\theta\,d\theta$	$\frac{1}{2}$	
$= \frac{1}{2}\int_0^{\pi/2} (\cos 2\theta + 1)\,d\theta$	$\frac{1}{2}$	
$= \frac{1}{2}\left[\frac{1}{2}\sin 2\theta + \theta\right]_0^{\pi/2}$	$\frac{1}{2}$	
$= \frac{1}{2}\left[0 + \pi/2 - 0\right]$		
$= \pi/4$	$\frac{1}{2}$	
or Area $= \frac{1}{4}$ of unit circle	1	If the radius $=1$ wasn't mentioned, then full marks wasn't awarded.
$= \frac{1}{4}\pi \times 1^2$	1	
$= \pi/4$	1	
(b) $y = \operatorname{cosec} x$		
$\frac{dy}{dx} = \frac{-\cos x}{\sin^2 x}$		
$= -\cot x\,.\operatorname{cosec} x$	1	
when $x = 5\pi/6$		
$m = \frac{-\cos\frac{5\pi}{6}}{\sin^2(\frac{5\pi}{6})}$	$\frac{1}{2}$	
$= \frac{\sqrt{3}/2}{1/4}$		
$= 2\sqrt{3}$		
$\therefore$ gradient is $2\sqrt{3}$	$\frac{1}{2}$	

$2/2$

MATHEMATICS Extension 1 : Question 1

Suggested Solutions	Marks	Marker's Comments
(c) $f'(x) = 2\cot x - x$		
$f(x) = 2\ln(\sin x) - \frac{1}{2}x^2 + C$		$1/2$ off for each error!
when $x = \pi/2$ $f(x) = 0$		
$0 = 2\ln(\sin \pi/2) - \frac{1}{2}(\pi/2)^2 + c$		
$0 = 0 - \frac{\pi^2}{8} + c$	3	
$c = \pi^2/8$		
$\therefore f(x) = 2\ln(\sin x) - \frac{x^2}{2} + \frac{\pi^2}{8}$		

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Yr12 2012 T1 Ext1 **MATHEMATICS: Question** 2

Suggested Solutions	Marks	Marker's Comments
a i) $S = 2^n + 2^{n-1} + 2^{n-2} + \cdots + 2^{-2n}$ (1)		
$\frac{1}{2}S = 2^{n-1} + 2^{n-2} + \cdots + 2^{-2n-1}$ (2)		
$S - \frac{1}{2}S = 2^n - 2^{-2n-1}$		
$\frac{1}{2}S = 2^n - 2^{-2n-1}$		
$S = 2^{n+1} - 2^{-2n}$ #		
or $a = 2^n$ $r = \frac{1}{2}$ # of terms $= 3n+1$		1m for $3n+1$
$S = \dfrac{2^n\left[1 - \left(\frac{1}{2}\right)^{3n+1}\right]}{1 - \frac{1}{2}}$		many students made mistake in # of terms or mixed up the 'n' in formula
$S = 2^{n+1}(1 - 2^{-3n-1})$		
$S = 2^{n+1} - 2^{-2n}$ #		$S_n = \dfrac{a(r^n - 1)}{r-1}$
a ii) as $n \to \infty$ $2^{n+1} \to \infty$, $2^{-2n} \to 0$		Calculation will be made easier
$S \to \infty$ $\therefore$ no limiting sum		max. $1\frac{1}{2}$m
b) $\sqrt{6}\sin x + \sqrt{2}\cos x = R(\cos x \cos\alpha - \sin x \sin\alpha)$		
$R\sin\alpha = -\sqrt{6}$ $R\cos\alpha = \sqrt{2}$ $(R>0)$	1 m	a ii) must give correct reason for no limiting sum.
$R = \sqrt{(\sqrt{6})^2 + (\sqrt{2})^2} = 2\sqrt{2}$	1 m	$\lvert r\rvert < 1$ or $\lvert r\rvert > 1$
$\tan\alpha = -\sqrt{3}$ in 4^{th} Q as $\sin\alpha < 0$, $\cos\alpha > 0$		is irrelevant here
$\alpha = \frac{5\pi}{3}$	1 m	
$2\sqrt{2}\cos\left(x + \frac{5\pi}{3}\right)$ #		forgot $2\sqrt{2}\cos\left(x + \frac{5\pi}{3}\right)$ $-\frac{1}{2}$m
c) $x^{k+3} = e^{\ln x^7} = x^7$	1 m	
$k = 4$	1 m	

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MATHEMATICS Extension 1 : Question 3

Suggested Solutions	Marks	Marker's Comments
a) Given $\frac{dv}{dt} = 40$	3	
$V = \frac{4}{3}\pi r^3 \quad \therefore \frac{dv}{dr} = 4\pi r^2$		
$\therefore \frac{dv}{dr} = \frac{dv}{dt} \times \frac{dr}{dt}$		
$4\pi r^2 = 40 \times \frac{dr}{dt}$		1 $\frac{dr}{dt}$
$\therefore \frac{dr}{dt} = \frac{10}{\pi r^2}$		
$A = 4\pi r^2 \quad \therefore \frac{dA}{dr} = 8\pi r$		
$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$		
$= 8\pi r \times \frac{10}{\pi r^2}$		
$= \frac{80}{r}$		1 $\frac{dA}{dt}$
When $r = 10 \quad \frac{dA}{dr} = 8$		$\frac{1}{2}$ correct answer with units
Surface area is increasing at $8\,mm^2/sec$		$\frac{1}{2}$ answering the question

MATHEMATICS Extension 1 : Question 3

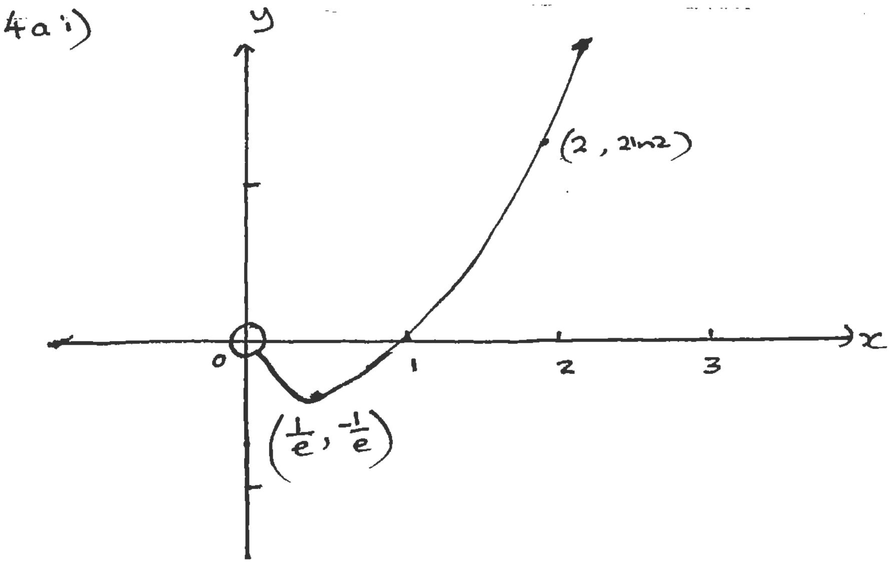
Suggested Solutions	Marks	Marker's Comments
b) Let $P(n)$ be the proposition that: $1+4+16 \cdots 4^n = \frac{1}{3}(4^{n+1}-1)$ for $n=0,1,2\cdots$	3	
Test $P(0)$: LHS $=1$ RHS $= \frac{1}{3}(4^1-1) = \frac{3}{3} = 1$ $\therefore$ LHS $=$ RHS $\therefore P(0)$ is true		1 Must test $n=0$. must show substitution for RHS.
Assume $P(k)$ is true for some $k=0,1,2\cdots$ ie $1+4+16\cdots 4^k = \frac{1}{3}(4^{k+1}-1)$		$\frac{1}{2}$
Required to Prove $P(k+1)$ is true ie $1+4+16 \cdots 4^{k+1} = \frac{1}{3}(4^{k+2}+1)$		
LHS $= 1+4+16 \cdots 4^k + 4^{k+1}$ $= \frac{1}{3}(4^{k+1}-1) + 4^{k+1}$ (by assumption)		$\frac{1}{2}$ "by assumption" $\frac{1}{2}$ substition
$= \frac{4}{3}(4^{k+1}) - \frac{1}{3}$ $= \frac{1}{3}[4 \times 4^{k+1} - 1]$ $= \frac{1}{3}[4^{k+1} - 1]$ $=$ RHS.		1 algebra
$\therefore P(n)$ is true by principle of mathematical induction		no marks for conclusion if incorrect working

MATHEMATICS Extension 1 : Question 3		
Suggested Solutions	**Marks**	**Marker's Comments**
c) $\log_5(x-2) + \log_5(x-6) = 1$	3	
$\log_5[(x-2)(x-6)] = 1$		$\frac{1}{2}$ combine logs
		$\frac{1}{2}$ multiply out
$\therefore x^2 - 8x - 12 = 5$		
$\therefore x^2 - 8x - 7 = 0$		1 Solve quadratic for 2 solutions
$(x-7)(x-1) = 0$		
$\therefore x = 7$ or $x = 1$		
But $x > 6$ $\therefore x \neq 1$		1 reject $x = 1$.
$\therefore x = 7$ (only).		

MATHEMATICS Extension 1 : Question 4

Suggested Solutions	Marks	Marker's Comments

4a i)



Calculus $y = x\ln x$

x int put $y = 0$

$x \ln x = 0$

$x = 1$

stationary points $y' = \ln x + 1$

$\ln x + 1 = 0$

$\ln x = -1.$

$x = e^{-1}$

$\therefore\ y = \frac{1}{e} \times -1 = -\frac{1}{e}$

$\therefore$ SP is $\left(\frac{1}{e}, \frac{-1}{e}\right)$

Test nature $y'' = \frac{1}{x}$

at $x = e^{-1}$ $y'' = \frac{1}{e^{-1}}$

$= e$

> 0 concave up

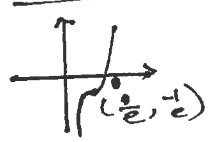
absolute minimum point at $\left(\frac{1}{e}, -\frac{1}{e}\right)$

Marks: 3 marks

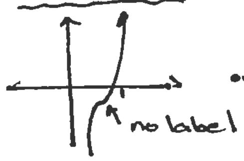
Marker's Comments:

Marks were awarded for:
- x-int
- shape
- open circle at zero
- TP showing $\left(\frac{1}{e}, \frac{-1}{e}\right)$.
- Scale on y-axis.

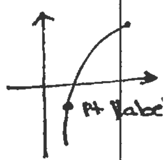
2 marks



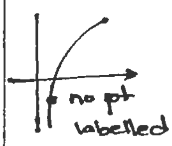
1½ marks

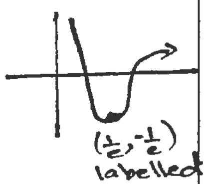


or

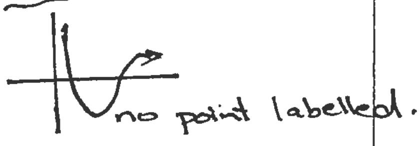


1 mark





½ mark



ii) Question says explain

As $x \to 0$, the gradient becomes negatively very steep (the curve approaches $(0,0)$ with a negative gradient. It approaches a vertical tangent.

Marks: 1 mark

Marker's Comments:

- No marks for statement formulae only
- you had to mention gradient
- No CFE because one had to go back to the original question to look at gradients.

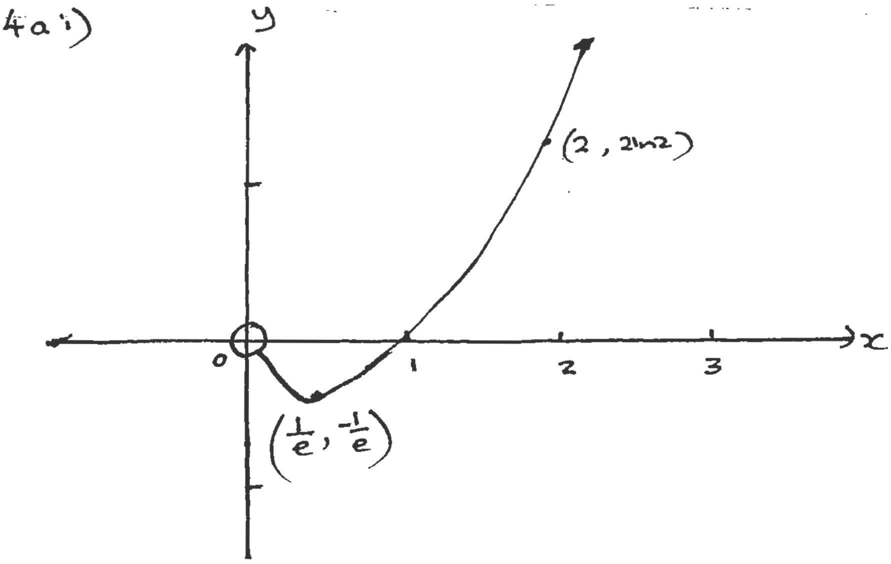
MATHEMATICS Extension 1 : Question 4

Suggested Solutions	Marks	Marker's Comments
4b (i) $\frac{d}{dx}\tan^3 x = 3\tan^2 x \cdot \sec^2 x$ ✓ $= 3(\sec^2 x - 1)\sec^2 x$ ✓ $= 3\sec^4 x - 3\sec^2 x$.	2 marks	Some used Quotient rule $\tan^3 x = \frac{\sin^3 x}{\cos^3 x}$. * Be careful with writing superscripts eg $\tan^3 x$ looked like $\tan 3x$
From (i) $\int_0^{\frac{\pi}{4}} \frac{d}{dx}\left[\tan^3 x\right] = 3\int_0^{\frac{\pi}{4}} \sec^4 x\,dx - 3\int_0^{\frac{\pi}{4}} \sec^2 x.dx$ ✓ $\int_0^{\frac{\pi}{4}} \sec^4 x.dx = \frac{1}{3}\int_0^{\frac{\pi}{4}} \frac{d}{dx}(\tan^3 x) + \int_0^{\frac{\pi}{4}} \sec^2 x.dx$ $= \frac{1}{3}\left[\tan^3 x\right]_0^{\frac{\pi}{4}} + \left[\tan x\right]_0^{\frac{\pi}{4}}$ ✓ $= \frac{1}{3}(1-0) + (1-0)$ $= \frac{4}{3}$ ✓ $\therefore \int_0^{\frac{\pi}{4}} \sec^4 x.dx = \frac{4}{3}$	✓ 3 marks	Some students got $\frac{2}{3}$ because they divided both $\tan^3 x$ and $\tan x$ by 3 ie $\frac{1}{3}\left[\tan^3 x + \tan x\right]_0$

MATHEMATICS Extension 1 : Question 4

Suggested Solutions	Marks	Marker's Comments

4a i)



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$x = e^{-1}$

$\therefore y = \frac{1}{e} \times -1 = \frac{-1}{e}$

$\therefore$ SP is $\left(\frac{1}{e}, \frac{-1}{e}\right)$

Test nature $y'' = \frac{1}{x}$

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$= e$

> 0 concave up

absolute minimum point at $\left(\frac{1}{e}, \frac{-1}{e}\right)$

Marks: 3 marks

Marker's Comments:

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- shape
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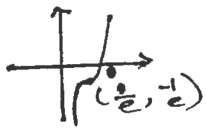
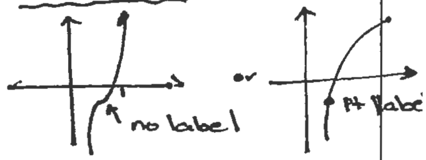
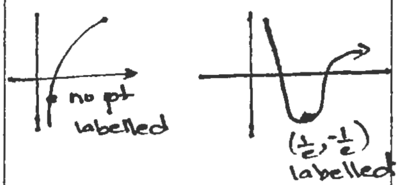
2 marks



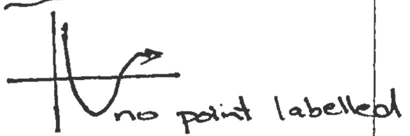
1½ marks



1 mark



½ mark



ii) Question says explain

As $x \to 0$, the gradient becomes negatively very steep (the curve approaches (0,0) with a negative gradient. It approaches a vertical tangent.

Marks: 1 mark

Marker's Comments:

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MATHEMATICS Extension 1 : Question 4

Suggested Solutions	Marks	Marker's Comments
4b(i) $\frac{d}{dx} \tan^3 x = 3\tan^2 x \cdot \sec^2 x$ ✓ $= 3(\sec^2 x - 1)\sec^2 x$ ✓ $= 3\sec^4 x - 3\sec^2 x$.	2 marks	Some used Quotient rule $\tan^3 x = \frac{\sin^3 x}{\cos^2 x}$. * Be careful with writing superscripts eg $\tan^3 x$ looked like $\tan 3x$
From (i) $\int_0^{\pi/4} \frac{d}{dx}[\tan^3 x] = 3\int_0^{\pi/4} \sec^4 x \, dx - 3\int_0^{\pi/4} \sec^2 x \, dx$ ✓ $\int_0^{\pi/4} \sec^4 x \, dx = \frac{1}{3}\int_0^{\pi/4} \frac{d}{dx}(\tan^3 x) + \int_0^{\pi/4} \sec^2 x \, dx$ $= \frac{1}{3}\left[\tan^3 x\right]_0^{\pi/4} + \left[\tan x\right]_0^{\pi/4}$ ✓ $= \frac{1}{3}(1-0) + (1-0)$ $= \frac{4}{3}$ ✓ $\therefore \int_0^{\pi/4} \sec^4 x \, dx = \frac{4}{3}$	✓ 3 marks	Some students got $\frac{2}{3}$ because they divided both $\tan^3 x$ and $\tan x$ by 3 ie $\frac{1}{3}\left[\tan^3 x + \tan x\right]$

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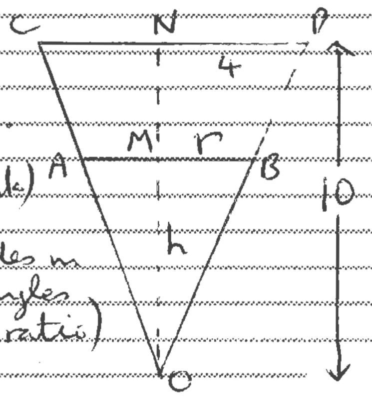
MATHEMATICS Extension 1 : Question 5

Suggested Solutions	Marks	Marker's Comments
a) The amended numbers are in geometric sequence		
$\therefore \dfrac{100+k}{60+k} = \dfrac{150+k}{100+k} = r$	1	
$\therefore (100+k)^2 = (150+k)(60+k)$		
$10000 + 200k + k^2 = 9000 + 210k + k^2$ $\therefore 10k = 1000$ $k = 100$	1	
$\therefore r = \dfrac{100+100}{60+100} = \underline{\underline{5/4}}$	1	
b) i) Let point O, A, B, C, P, M and N be as shown.  $\triangle OMB \parallel\!\!\mid \triangle ONP$ (Equiangular) $\therefore \dfrac{h}{10} = \dfrac{r}{4}$ (Corresponding sides in similar triangles are in the same ratio)		I knocked ½ mark if no mention of similar triangles or equivalent (tan θ etc)
$\therefore \underline{r = \dfrac{2h}{5}}$	1	There was no mark for just knowing the formula for V.
$Vol = V = \dfrac{1}{3}\pi r^2 h = \dfrac{1}{3}\pi\left(\dfrac{2h}{5}\right)^2 h = \underline{\underline{\dfrac{4\pi}{75}h^3}}$	1	
c) $\dfrac{dV}{dt} = -3$ (decreasing)		
$\dfrac{dh}{dt} = \dfrac{dV}{dt} \times \dfrac{dh}{dV}$	1	Chain Rule, explicit or direct.
But $\dfrac{dh}{dV} = 1\Big/\dfrac{dV}{dh} = 1\Big/\dfrac{12h^2\pi}{75} = \dfrac{25}{4h^2\pi}$	1	Calculus
$\therefore \dfrac{dh}{dt} = -3.\dfrac{25}{4h^2\pi} = -\dfrac{3\times 25}{4\pi\times 5^2}$ when $h=5$ $= -\dfrac{3}{4\pi}$	1	Answer
Grain level is decreasing (going down) at $\underline{\underline{\dfrac{3}{4\pi}}}$ m/s	1	Interpretation of − signs. (Lots of ½ docked)

Y12 T1 **MATHEMATICS Extension 1 : Question 6**

Suggested Solutions	Marks	Marker's Comments
a) Step 1 $n=2$ LHS $=3^2 = 9$ RHS $=2\times2+4 = 8$	1	
$\therefore$ LHS > RHS		
$\therefore$ true $n=2$		
Step 2 Assume statement true $n=k$ k +ve integer		
ie $3^k > 2k+4$ or $3^k-2k-4>0$.	$\frac{1}{2}$	
To prove true $n=k+1$		
ie $3^{k+1} > 2(k+1)+4$		
or $3^{k+1} > 2k+6$ or $3^{k+1}-2k-6>0$	$\frac{1}{2}$	
Now $3^{k+1}-2k-6 = 3\cdot3^k - 2k-6$		
$=3\left[3^k-2k-4\right] + 6k+12-2k-6$		
$=3\left[3^k-2k-4\right]+4k+6$	1	
>0 by assumption		
& $4k+6>0$ for $k\geq2$		
$\therefore$ if true $n=k$ it is also true $n=k+1$		
Since true $n=2$ then it is true for $n=1+2=3$		
$n=3+1=4$ and so on for +ve integers ≥2		
b)(i) $A_n=$ amount owing $R=$ repayment $r=0.0075$		To be done 3 times.
Amount owing end of month $=50000\times1.0075-R$ $n=30$	1	
2nd month $=(50000\times1.0075-R)1.0075-R$		
$A_2 = 50000\,1.0075^2-R\left[1+1.0075\right]$		
End 3 months $=\left[50000.1.0075^2-R\left[1+1.0075\right]\right]\times1.0075-R$		
$A_3 = 50000\times1.0075^3-R\left[1+1.0075+1.0075^2\right]$		
$A_{60}=0$		
$\therefore 50000\times1.0075^{60}-R\left[1+1.0075+\ldots1.0075^{59}\right]$		
$50000\times1.0075^{60}=R\left[\frac{1.0075^{60}-1}{1.0075-1}\right]$	1	
$R=\frac{50000\times1.0075^{60}\times0.0075}{1.0075^{60}-1}$	1	Formula for R to be stated or multi decimal value
$=1037.9177$		
$=\$1037.92$		
(ii) $A_{30}=50000\times1.0075^{30}-\left[\frac{1.0075^{30}-1}{0.0075}\right]$	2	
End 30 mths \$27790.26		
(iii) owing \$17790.26.		
$A=\frac{17790.26\times1.0075^{30}\times0.0075}{1.0075^{30}-1}$	1	
$=\$664.43$		

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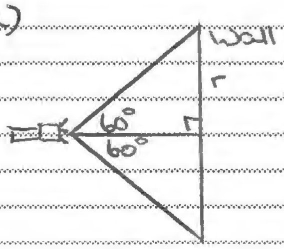
TERM 1 2012 **MATHEMATICS Extension 1 : Question** 7 YEAR 12

Suggested Solutions	Marks	Marker's Comments

(a)



Let the distance from the flashlight to the wall at any instant be x m.

$\therefore$ radius of circle of light $= x \tan 60°$
$= \sqrt{3}x$ m — (1)

The illuminated area (A) is

$A = \pi(\sqrt{3}x)^2$

$A = 3\pi x^2$

Now $\frac{dA}{dt} = \frac{dA}{dx} \times \frac{dx}{dt}$

$= 6\pi x \times -0.25$ given $\frac{dx}{dt} = -0.25$ — (1)

$= 18\pi \times -0.25$ when $x = 3$

$= -\frac{9\pi}{2}$ m²/s

$\therefore$ The illuminated area is decreasing at the rate of $\frac{9\pi}{2}$ m²/s when the light is 3m from the wall. — (1)

Marker's Comments:

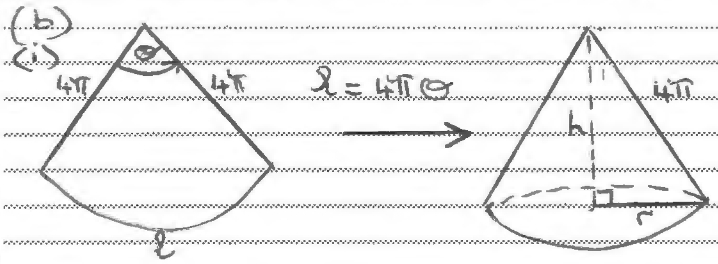
An answer of $\frac{\pi}{2}$ m²/s received max of 2 marks only.

Needed $\frac{dx}{dt} = -0.25$ and the final sentence area is decreasing to gain a full mark.

Wrong units $-\frac{1}{2}$ mark

Having written decreasing at the rate of $-\frac{9\pi}{2}$ m²/s lost $\frac{1}{2}$ mark.

(b)
(i)



$\ell = 4\pi\theta$ and $2\pi r = 4\pi\theta$

$r = 2\theta$ — (1)

Now $V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi \cdot 4\theta^2 \cdot \sqrt{16\pi^2 - R^2}$ — (1)

$V = \frac{4}{3}\pi\theta^2\sqrt{16\pi^2 - 4\theta^2}$

$V = \frac{4}{3}\pi\theta^2 \times 2\sqrt{4\pi^2 - \theta^2}$ — (1)

$\therefore V = \frac{8\pi\theta^2}{3}\sqrt{4\pi^2 - \theta^2}$

Marker's Comments:

Generally well done

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MATHEMATICS Extension 1 : Question 7

Suggested Solutions	Marks	Marker's Comments
(b) (ii) $V = \frac{8\pi\theta^2}{3}\sqrt{4\pi^2-\theta^2}$		
$\frac{dV}{d\theta} = \frac{8\pi\theta^2}{3} \times \frac{1}{2\sqrt{4\pi^2-\theta^2}} \times -2\theta + \sqrt{4\pi^2-\theta^2} \times \frac{16\pi\theta}{3}$	1	1 mark for correct derivative
$\frac{dV}{d\theta} = \frac{8\pi\theta}{3}\left[\frac{8\pi^2-3\theta^2}{\sqrt{4\pi^2-\theta^2}}\right]$		
For stationary pts $\frac{dV}{d\theta}=0$		
$\frac{dV}{d\theta}=0$ when $3\theta^2=8\pi^2$ or $\theta=0$		$\frac{1}{2}$ mark deducted if $\theta=0$ was factored out and not given as a sol^n
$\theta=0$ is rejected since $\theta>0$		
$\therefore \theta = \pm\frac{2\sqrt{2}\pi}{\sqrt{3}} = \pm\frac{2\sqrt{6}\pi}{3}$		
but $0<\theta<2\pi$	1	$\frac{1}{2}$ mark deduction for not considering $\theta=-\frac{2\sqrt{6}\pi}{3}$ and discounting it as $0<\theta<2\pi$
$\therefore \theta = \frac{2\sqrt{6}\pi}{3}$ $\quad (\doteqdot 5.13^c = 294^\circ)$		
Test nature		
Since $V=f(\theta)$ is continuous for all θ in $0<\theta<2\pi$ and $\frac{dV}{d\theta}=0$ has one value in the domain, then there is an absolute maximum value at $\theta=\frac{2\sqrt{6}\pi}{3}$	1	If students used the second derivative test they needed to show the second derivative and the value when $\theta=\frac{2\sqrt{6}\pi}{3}$ is substituted into $\frac{d^2V}{d\theta^2}$

θ	$\frac{\sqrt{6}\pi}{3}$	$\frac{2\sqrt{6}\pi}{3}$	$\sqrt{6}\pi$
$\frac{dV}{d\theta}$	6π >0	0	$-16\pi^4$ <0