Name:	
Class:	

YEAR 12

ASSESSMENT TEST 2
TERM 1, 2013

MATHEMATICS
EXTENSION 1

Time Allowed – 90 Minutes
(Plus 5 minutes Reading Time)

General Instructions:

- *All* questions may be attempted
- *All* questions are of equal value
- Standard Integral Tables will be supplied
- Department of Education approved calculators and templates are permitted
- In every Question, show all relevant mathematical reasoning and/or calculations.
- Marks may not be awarded for careless or badly arranged work
- No grid paper is to be used unless provided with the examination paper

The answers to all questions are to be returned in separate bundles clearly labelled Question 1, Question 2, etc. Each question must show your Candidate Number.

Y12 Mathematics Extension 1 | Term 1 Assessment 2013

Question 1 (9 Marks) **Marks**

(a) Differentiate with respect to x :

(i) $2x^2 log_e x$ **2**

(ii) $e^{2x} sin^2 x$ **2**

(b) Simplify $\dfrac{log_a 4 + log_a 8}{log_a 32}$ **2**

(c) (i) Find the sum of the following series **2**

$$5^{2n} + 5^{2n-2} + \ \dots . + 5^{-2n-2}$$

(ii) Does this series have a limiting sum?

Give reasons for your answer **1**

Question 2 (9 Marks) – START A NEW PAGE **Marks**

(a) The first three terms of a geometric sequence are :

$$4, \frac{8}{3}, \frac{16}{9}$$

Find the exact value of:

(i) Term 10. Give your answer in index notation. **2**

(ii) The sum of the first 6 terms. **2**

(b) Evaluate

(i)

$$\int_1^2 \frac{x^2 + x}{2x^2} \ dx$$ **3**

(ii)

$$\int sin^2 5x \ dx$$ **2**

Question 3 (9 Marks) – START A NEW PAGE **Marks**

(a) (i) Prove that $cos\theta = \frac{1-t^2}{1+t^2}$ and $sin\theta = \frac{2t}{1+t^2}$ where $t = tan\frac{\theta}{2}$ **3**

(ii) Hence, solve $7cos\theta - sin\theta = 5$ for $0° \leq \theta \leq 360°$ **3**

Give your answer correct to 2 significant figures.

(b) Each number in a sequence is obtained by adding the two previous numbers. The 6^{th}, 7^{th} and 8^{th} numbers of the sequence are 29, 47 and 76. Find the third number in the sequence. Show all necessary working to arrive at your answer. **3**

Question 4 (9 Marks) – START A NEW PAGE **Marks**

(a) A cylindrical block of wood is to be turned on a lathe so that its radius will be decreased at the rate of 0.5cm/minute, the height remaining unaltered at 10cm. If the volume of the block is initially $90\pi \text{cm}^3$, find:

(i) The time required for the radius to reduce to 2cm. **2**

(ii) The rate of change of the volume at this time. **2**

(b) (i) Sketch a half page graph of $y = log\,(2x - x^2)$. On your diagram locate the position of any turning points and asymptotes. **4**

(ii) Describe the behaviour of the graph as x approaches zero. **1**

Question 5 (9 Marks) – START A NEW PAGE **Marks**

(a) (i) Write $4cos2\theta + 3sin2\theta$ in the form $Acos(2\theta - \alpha)$ where $A > 0$ and $0° < \alpha < 360°$. Find α to the nearest minute. **2**

(ii) Hence, or otherwise, show that $8cos^2\theta + 6sin\theta cos\theta$ can be expressed in the form $B + Acos(2\theta - \alpha)$. **2**

(iii) Sketch $y = 8cos^2\theta + 6sin\theta cos\theta$ for $0° \leq \theta \leq 360°$ **3**

(iv) Write down the greatest value of $y = 8cos^2\theta + 6sin\theta cos\theta$ and the smallest positive value of θ to the nearest minute for which this can occur. **2**

Question 6 (9 Marks) – START A NEW PAGE **Marks**

(a) (i) Katie borrowed $600,000 in January 2013 to help pay for the purchase of a townhouse. She agreed to pay 6% per annum reducible interest and pay the loan off in 15 years. Calculate the monthly repayments she must pay to the nearest cent. **3**

(ii) After 24 repayments she receives $80,000 from an Uncle's will and pays this to the bank towards the repayment of her loan prior to the 25^{th} repayment. Calculate the amount owing at this time after the lump sum payment has been deducted. **2**

(iv) When will she now pay off the loan and how much will she save by doing this in this manner? **4**

Question 7 (9 Marks) – START A NEW PAGE **Marks**

(a) Using the substitution $\theta = cos^{-1} x$, find the exact value for

$$\int_{\frac{1}{2}}^{1} \frac{\sqrt{1-x^2}}{x^2}\ dx$$ 3

(b) (i) Prove that $\cos[(k-1)\theta] - 2cos\theta cosk\theta = -\cos\,[(k+1)\theta]$ 2

(ii) Use mathematical induction to prove that, if n is a positive integer, then 4

$$1 + cos\theta + cos2\theta + cos3\theta + \ldots + cos(n-1)\,\theta = \frac{1 - cos\theta - cosn\theta + \cos\,(n-1)\theta}{2 - 2cos\theta}$$

END OF PAPER

2013 T1 Ext 1.

Q1 a i) $y = 2x^2 \ln x$

$y' = 4x \ln x + \frac{2x^2}{x}$

$= 4x \ln x + 2x$

$= 2x[2\ln x + 1]$ — 1 + 1

Be careful: A few students factoring wrongly (no halfy mark)

ii) $y = e^{2x} \sin^2 x$

$y' = e^{2x} 2\sin x \cos x + 2\sin^2 x \cdot e^{2x}$

$= 2e^{2x}[\sin x(\cos x + \sin x)]$ — 1 + 1

or $y' = e^{2x}[\sin 2x + 2\sin^2 x]$

b) $\frac{\log_a 4 + \log_a 8}{\log_a 32} = \frac{\log_a 32}{\log_a 32} = 1$ — 1 + 1

or $\frac{2\log_a 2 + 3\log_a 2}{5\log_a 2} = \frac{5\log_a 2}{5\log_a 2} = 1$

Some wrote $\log_a\left(\frac{32}{32}\right) = \log_a 1 = 0$ max 1m

c i) $S = 5^{2n} + 5^{2n-2} + 5^{2n-4} + \dots + 5^{-2n-2}$

$5^2 \cdot S = 5^{2n+2} + 5^{2n} + \dots + 5^{-2n} + 5^{-2n-2}$ — 1

$S[5^2 - 1] = 5^{2n+2} - 5^{-2n-2}$

$S = \frac{5^{2n+2} - 5^{-2n-2}}{24}$ — 1

or use "$S = \frac{a(r^n - 1)}{r-1}$"

$S = \frac{5^{2n}[(5^{-2})^{2n+2} - 1]}{5^{-2} - 1} = \dots$

There are $2n+2$ terms. (1m

$a = 5^{2n}$ $r = 5^{-2}$

many mixed up the n max 1m

some found n negative or fraction max $\frac{1}{2}$m

must simplify $(5^{-2} - 1)$ in denominator

ii) as $n \to \infty$, $5^{2n} \to \infty$, $5^{-2n-2} \to 0$

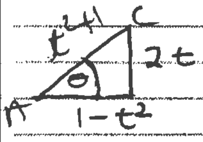
$\therefore$ no limiting sum — 1m

many wrote $|r| < 1 \therefore$ limiting sum exist 0m

must give correct reason for no limiting sum

MATHEMATICS EXTENSION 1: Question 2		
Suggested Solutions	**Marks**	**Marker's Comments**
a) i) $4, \frac{8}{3}, \frac{16}{9} \dots \quad \therefore a = 4, \ r = \frac{2}{3}$.	1	Identifying characteristics of GP
$T_{10} = ar^9$ $= 4 \times \left(\frac{2}{3}\right)^9$ or $\frac{2^{11}}{3^9}$	1	Calculating correct term
ii) $S_6 = \frac{4\left[1 - \left(\frac{2}{3}\right)^6\right]}{1 - \frac{2}{3}}$	1	Sum of a GP
$= 12\left(1 - \frac{64}{729}\right)$		
$= \frac{2660}{243}$ or $10\,\frac{230}{243}$	1	Simplify (exact)
b) i) $\int_1^2 \frac{x^2 + x}{2x^2}\,dx = \frac{1}{2}\int_1^2 \left(1 + \frac{1}{x}\right) dx$	1	Divide through
$= \frac{1}{2}\left[x + \ln x\right]_1^2$	1	Integrate
$= \frac{1}{2}\left[(2 + \ln 2) - (1 + \ln 1)\right]$		
$= \frac{1}{2}(1 + \ln 2)$	1	Evaluate
ii) Identity: $\sin^2 \alpha = \frac{1}{2}(1 - \cos 2\alpha)$		
$\therefore \sin^2 5x = \frac{1}{2}(1 - \cos 10x)$		
So $\int \sin^2 5x\,dx = \frac{1}{2}\int (1 - \cos 10x)\,dx$	1	Substitution
$= \frac{1}{2}\left(x - \frac{1}{10}\sin 10x\right) + c$	1	Integrate
$= \frac{x}{2} - \frac{1}{20}\sin 10x + c$		

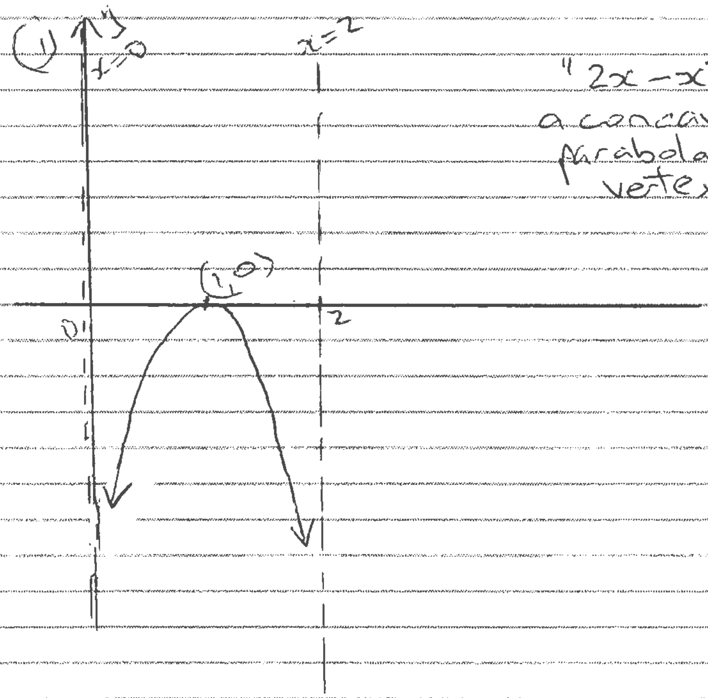
MATHEMATICS Extension 1 : Question 5

Suggested Solutions	Marks	Marker's Comments
(a) This is best solution		
$\cos\theta = \dfrac{\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}}{\cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2}}$		Using a triangle assumes $0° \le \theta \le 90°$ so not valid proof for all θ.
divide by $\cos^2\theta/2$		
$\cos\theta = \dfrac{1 - \tan^2\theta/2}{1+\tan^2\theta/2}$		
$= \dfrac{1-t^2}{1+t^2}$		
$\sin\theta = \dfrac{2\sin\theta/2\cos\theta/2}{\cos^2\theta/2 + \sin^2\theta/2}$		
divide by $\cos^2\theta/2$		
$\sin\theta = \dfrac{2\tan\theta/2}{1+\tan^2\theta/2}$		
$= \dfrac{2t}{1+t^2}$		
Alternative method		
$\tan\theta = \dfrac{2\tan\theta/2}{1-\tan^2\frac{\theta}{2}}$	1 mark	
 by pythagoras		students failed to prove $AC = t^2+1$ just quoted it
$AC^2 = 4t^2 + (1-t^2)^2$		a third approach using triangles was also given full marks.
$= 4t^2 + 1 - 2t^2 + t^4$	1 mark	
$= t^4 + 2t^2 + 1$		
$AC = t^2+1$ $\quad AC > 0$		
from triangle		
$\cos\theta = \dfrac{1-t^2}{1+t^2}$	1 mark	
$\sin t = \dfrac{2t}{1+t^2}$		

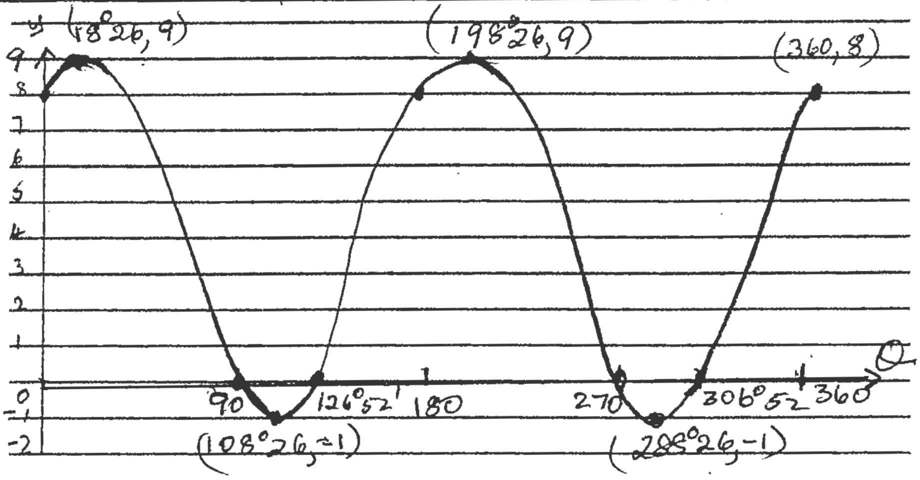
MATHEMATICS Extension 1 : Question……..

Suggested Solutions	Marks	Marker's Comments
(ii) $7\left(\frac{1-t^2}{1+t^2}\right) - \frac{2t}{1+t^2} = 5$		
$7(1-t^2) - 2t = 5+5t^2$		
$12t^2 + 2t - 2 = 0$		
$6t^2 + t - 1 = 0$		
$(3t-1)(2t+1) = 0$	$1\frac{1}{2}$	
$\therefore \tan\frac{\theta}{2} = \frac{1}{3}$ or $-\frac{1}{2}$		• poorly done given in radians or too many answers given
$\frac{\theta}{2} = 18^\circ 26', 153^\circ 26'$		
$0 \le \frac{\theta}{2} \le 180^\circ$.		
$\theta = 37^\circ, 310^\circ$ to 2SF.	1	• the negative called problems.
test $\theta = 180^\circ$		
$7\cos 180 - \sin 180 = 7$ $\therefore$ not a solution	$\frac{1}{2}$	• Many students failed to test for $\theta = 180^\circ$
(iii) Told $T_6 = 29$		
$T_7 = 47$		
$T_8 = 76$		Needed to indicate how the terms related
$T_6 + T_7 = T_8$	1	
$\therefore T_7 = T_5 + T_6$		
$\therefore T_5 = T_7 - T_6$		Fairly well done some students went a long way to gain answer.
$= 47 - 29$	1	
$= 18$		
$T_4 = T_6 - T_5$		
$= 29 - 18$		
$= 11$		
$T_3 = T_5 - T_4$		
$= 18 - 11$	1	
$= 7$ $\therefore$ 7 third number		

MATHEMATICS Extension 1 : Question 4

Suggested Solutions	Marks	Marker's Comments
(a)(i) $\frac{dr}{dt} = -\frac{1}{2}$ cm/min		
$V = \pi r^2 h$	$\frac{1}{2}$	
$t = 0$, $\pi r^2 \times 10 = 90\pi$	$\frac{1}{2}$	
$r = \pm 3$	$\frac{1}{2}$	
$r = 3$ (as $r > 0$)		
∴ It will take 2 minutes	$\frac{1}{2}$	
(ii) $V = 10\pi r^2$		
$\frac{dV}{dr} = 20\pi r$		
$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$	$\frac{1}{2}$	
$= 20\pi r \times -\frac{1}{2}$	$\frac{1}{2}$	
when $r = 2$, $= 40\pi \times -\frac{1}{2}$		
$= -20\pi$	$\frac{1}{2}$	
∴ decreasing at 20π cm³/min	$\frac{1}{2}$	
(b)(i)  "$2x - x^2$" is a concave down parabola with vertex at (1,1)		1 for working $\frac{1}{2}$ for each asymptote $\frac{1}{2}$ for (1,0) 1 for neatness and shape 1 for concave down parabola * If using calculus do it first.
(ii) As $x \to 0$, $y \to -\infty$	1	

MATHEMATICS Extension 1 : Question 5

Suggested Solutions	Marks	Marker's Comments
(a)(i) $4\cos 2\theta + 3\sin 2\theta = A\cos(2\theta - \alpha)$ $A>0$ $0^\circ < \alpha < 360^\circ$ $= A\cos 2\theta\cos\alpha + A\sin 2\theta\sin\alpha$. $A\cos\alpha = 4$, $A\sin\alpha = 3$ $\}$ $A^2(\cos^2\alpha + \sin^2\alpha) = 4^2 + 3^2$ $A^2 = 25$ $A = 5$ $A>0$. $\cos\alpha = 4/5 > 0$ $\sin\alpha = 3/5 > 0$ $\therefore 0 < \alpha < 90^\circ$ $\therefore \alpha = 36^\circ 52'$ nearest minute $A\cos(2\theta - \alpha) = 5\cos(2\theta - 36^\circ 52)$	2	$\frac{1}{2}$ expansion $\frac{1}{2}$ $A = 5$ with explanation $\frac{1}{2}$ α is in First Quadrant $\frac{1}{2}$ α value.
(ii) $8\cos^2\theta + 6\sin\theta\cos\theta$ $= 8\left[\frac{1}{2}(\cos 2\theta + 1) + 3\sin 2\theta\right]$ $= 4\cos 2\theta + 4 + 3\sin 2\theta$ (iii) $= 4 + 5\cos(2\theta - 36^\circ 52)$	2	$\frac{1}{2}$ $\cos 2\theta$ $\}$ expansions $\frac{1}{2}$ $\sin 2\theta$ $\frac{1}{2}$ working $\frac{1}{2}$ "+4"
 $y = 4 + 5\cos(2\theta - 36^\circ 52)$ $= 8\cos^2\theta + 6\sin\theta\cos\theta$	3	$\frac{1}{2}$ start + finish at $y = 8$ 1 θ values of Turning Points 1 θ values of intercepts. $\frac{1}{2}$ max/min values, shape, scale + axes
(iv) Greatest value is 9 which occurs when $\theta = 18^\circ 26'$	2	1 max value $= 9$ 1 $\theta = 18^\circ 26$
Working for Graph. θ intercepts $8\cos^2\theta + 6\sin\theta\cos\theta = 0$ $\cos\theta(8\cos\theta + 6\sin\theta) = 0$ $\cos\theta = 0$ $\therefore \theta = 90^\circ$ or 270° $\tan\theta = -8/6$ $\therefore \theta = 126^\circ 52$ or $306^\circ 52$ Smallest max when $2\theta - 36^\circ 52' = 0$ $\therefore \theta = 18^\circ 26$ $\therefore$ max at $18^\circ 26 + 180 = 198^\circ 26'$ $\therefore$ mins at $18^\circ 26 + 90^\circ = 108^\circ 26'$ $18^\circ 26 + 270^\circ = 288^\circ 56'$ y intercept $x = 0$ $y = 8$ max value $4 + 5 = 9$ min value $4 - 5 = -1$		

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MATHEMATICS Extension 1 : Question 6

Suggested Solutions	Marks	Marker's Comments
i) If principal is P, rate r, repayment.		
After 1 month owes $P(1+r)-R$		
After 2 months owes $(P(1+r)-M)(1+r)-R$		
$=P(1+r)^2-R(1+(1+r))$		
After 3 months owes $(P(1+r)^2-R(1+(1+r)))(1+r)-R$		
$=P(1+r)^3-R(1+(1+r)+(1+r)^2)$		
After n months owes $P(1+r)^n-R(1+(1+r)+\dots+(1+r)^{n-1})$		
$=P(1+r)^n-R\left(\dfrac{(1+r)^n-1}{r}\right)$	1	Derivation of formula
If $P=600000$, $n=180$ and $r=0.005$ and the amount owing is zero, hence		
$R=\dfrac{600000(1.005)^{180}\ 0.005}{(1.005)^{180}-1}$	1	Use of correct numbers in formula
$=5063.140968\ldots$		
∴ Monthly repayment is $5063.14 to nearest cent.	1	Final answer (Some rounded up with explanation which was allowed)
ii) After 24 months owes		
$600000(1.005)^{24}-5063.14\left(\dfrac{(1.005)^{24}-1}{0.005}\right)$		
$=547530.32$	1	
Reduced by 80 000 to $467530.32	1	
iii) Let k be the number of extra months required to pay off.		
$\therefore 467530.32(1.005)^k=5063.14\left(\dfrac{(1.005)^k-1}{0.005}\right)$	1	
$(1.005)^k\,2725.49=5063.14$		
$k\log 1.005=\log 1.857698$		
$k=124.177\ldots$	1	

MATHEMATICS Extension 1 : Question...6...

Suggested Solutions	Marks	Marker's Comments
She must make 148 (124+24) full payments plus one small payment, which would normally be made at end of 149th month.	½	
After 148th payment she owes \$894.82 During months this compounds to $(1.005) \times \$894.82 = \underline{\$899.30}$		
∴ Total payments $148 \times 5063.14 = 749344.72$ $1 \times 899.30 = 899.30$ $1 \times 80000 = 80000.00$ 830244.02		
Would have paid $180 \times 5063.14 = 911365.20$	½	
∴ Gain $911365.20 - 830244.02$		
$= \underline{\$81121.18}$	1	Many people assumed either 148 or 149 <u>full</u> payments. One falls short (7½ marks max) the other is over (8 marks max)
The following gives an alternative (<u>but poorer</u>) answer that assumes that last payment made 0.177 way through the month. Pays off 148.177×5063.14 $= 750240.90$ 80000.00 830240.90		
∴ Gain $911365.20 - 830240.90$	½	
$= \underline{\$81124.30}$	1	

MATHEMATICS Extension 1 : Question 7(a) PAGE 1

Suggested Solutions	Marks	Marker's Comments
QUESTION 7		
a) Let $I = \int_{\frac{1}{2}}^{1} \frac{\sqrt{1-x^2}}{x^2}\,dx$		
$\theta = \cos^{-1}x$		
$\therefore x = \cos\theta$		
$dx = -\sin\theta\, d\theta$	$\frac{1}{2}$	for finding dx
OR $d\theta = \frac{-1}{\sqrt{1-x^2}}\,dx$ $\therefore dx = -\sqrt{1-x^2}\,d\theta$		
when $x = \frac{1}{2}$, $\theta = \frac{\pi}{3}$; when $x = 1$, $\theta = 0$	$\frac{1}{2}$	for changing limits
$\therefore I = \int_{\frac{\pi}{3}}^{0} \frac{\sqrt{1-\cos^2\theta}}{\cos^2\theta}(-\sin\theta\, d\theta)$	$\frac{1}{2}$	for substitution
$= \int_0^{\frac{\pi}{3}} \frac{\sin^2\theta}{\cos^2\theta}\,d\theta$		
$= \int_0^{\frac{\pi}{3}} \tan^2\theta\, d\theta$	$\frac{1}{2}$	For reaching $\tan^2\theta$ or $\sec^2\theta - 1$
$= \int_0^{\frac{\pi}{3}} (\sec^2\theta - 1)\,d\theta$		
$= \left[\tan\theta - \theta\right]_0^{\frac{\pi}{3}}$	$\frac{1}{2}$	integration of $\sec^2\theta - 1$
$= \sqrt{3} - \frac{\pi}{3}$	$\frac{1}{2}$	final correct answer

MATHEMATICS Extension 1 : Question 7(b)(i)

Suggested Solutions	Marks	Marker's Comments
b) (i)		
R.T.P. : $\cos(k-1)\theta - 2\cos\theta\cos k\theta = -\cos(k+1)\theta$		
Proof : $LHS = \cos(k-1)\theta - 2\cos\theta\cos k\theta$	$\frac{1}{2}$	for opening brackets
$= \cos k\theta . \cos\theta + \sin k\theta \sin\theta - 2\cos\theta\cos k\theta$		
$= -\cos\theta\cos k\theta + \sin k\theta\sin\theta$	$\frac{1}{2}$	expansion - double angle formula
$= -(\cos\theta\cos k\theta - \sin k\theta\sin\theta)$		
$= -\cos(k\theta+\theta)$	$\frac{1}{2}$	collecting like terms
$= -\cos(k+1)\theta$	$\frac{1}{2}$	for applying reverse of double angle formula.
$= RHS$		
Alternate:		
$\cos(A+B) = \cos A\cos B - \sin A\sin B$		
$\cos(A-B) = \cos A\cos B + \sin A\sin B$		
$\therefore \cos(A+B) + \cos(A-B) = 2\cos A\cos B$		
$\Rightarrow \cos(k\theta+\theta) + \cos(k\theta-\theta) = 2\cos k\theta\cos\theta$		
i.e $\cos(k-1)\theta - 2\cos\theta\cos k\theta = -\cos(k+1)\theta$		

MATHEMATICS Extension 1 : Question 7 b (ii) PAGE 3

Suggested Solutions	Marks	Marker's Comments
(ii) Let $P(n)$ be the proposition i.e. $P(n) = 1+\cos\theta+\cos2\theta+\cdots+\cos(n-1)\theta$ $= \dfrac{1-\cos\theta-\cos n\theta+\cos(n-1)\theta}{2-2\cos\theta}$		
Step 1: Prove true for $n=1$ $LHS = \cos(1-1)\theta$ $= \cos 0$ $= 1$	$\frac{1}{2}$	for LHS when $n=1$
$RHS = \dfrac{1-\cos\theta-\cos\theta+\cos 0}{2-2\cos\theta}$ $= \dfrac{2-2\cos\theta}{2-2\cos\theta}$ $= 1 = LHS$ $\therefore$ True for $n=1$	$\frac{1}{2}$	for RHS when $n=1$
Step 2: Assume true for $n=k\in Z^+$ i.e. $1+\cos\theta+\cdots+\cos(k-1)\theta = \dfrac{1-\cos\theta-\cos k\theta+\cos(k-1)\theta}{2-2\cos\theta}$	$\frac{1}{2}$	for assumption statement ($-\frac{1}{2}$ if $n=k$ only or $n=k\in Z$ only)
Step 3: Prove true for $n=k+1$ i.e. To prove $1+\cos\theta+\cdots+\cos(k-1)\theta+\cos k\theta$ $= \dfrac{1-\cos\theta+\cos k\theta-\cos(k+1)\theta}{2-2\cos\theta}$		

MATHEMATICS Extension 1: Question 7

Suggested Solutions	Marks	Marker's Comments
$LHS = \underbrace{1+\cos\theta+\cos 2\theta+\cdots+\cos(k-1)\theta}_{\text{from assumption}}+\cos k\theta$		
$= \dfrac{1-\cos\theta-\cos k\theta+\cos(k-1)\theta}{2-2\cos\theta}+\cos k\theta$ (by assumption)	$\frac{1}{2}$	for use of assumption statement
$= \dfrac{1-\cos\theta-\cos k\theta+\cos(k-1)\theta+2\cos k\theta-2\cos\theta\cos k\theta}{2-2\cos\theta}$		$\frac{1}{2}$ for adding the two terms
$= \dfrac{1-\cos\theta+\cos k\theta+[\cos(k-1)\theta-2\cos\theta\cos k\theta]}{2-2\cos\theta}$		
But $[\cos(k-1)\theta-2\cos\theta\cos k\theta]=-\cos(k+1)\theta$ ...		from part (i)
	$\frac{1}{2}$	for use of part (i)
Hence $LHS = \dfrac{1-\cos\theta+\cos k\theta-\cos(k+1)\theta}{2-2\cos\theta}$	$\frac{1}{2}$	for final form of LHS which equate to RHS.
$= RHS$		
Hence true for $n=k+1$		
Since it P(n) is true for $n=1$, and true for $n=k+1$, when $n=k$, the statement must be true for all $n\in Z^+$, by the principle of mathematical induction	$\frac{1}{2}$	for complete conclusion