Student Number: ______________________

KM AM

KAMBALA

Mathematics Extension 1

HSC Assessment Task 2

Half-Yearly Examination

March 2008

General Instructions

- Reading time – 5 minutes.
- Working time – 2 hours.
- Answer all questions in the writing booklets provided. **Start each question in a new booklet.**
- Board-approved calculators may be used.
- A table of standard integrals is provided.
- All necessary working should be shown in every question.
- More marks will be awarded to questions involving higher order thinking.

Total marks – 84

- Attempt Questions 1-7.
- All questions are of equal value.

Question 1 12 marks (Begin a new booklet) **Marks**

(a) Factorise $64a^3+\frac{1}{27}b^3$ **1**

(b) Solve for x: $3^{2x+1}-28(3^x)+9=0$ **3**

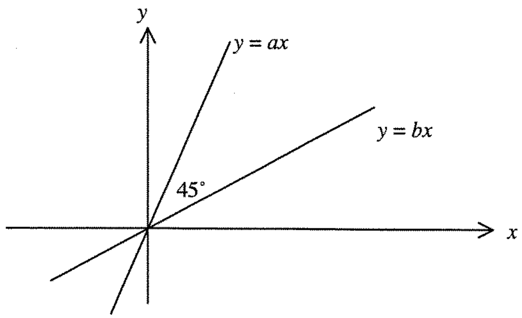
(c) Evaluate $\int_1^4 x\sqrt{5-x}\,dx$ using the substitution $u=5-x$. **3**

(d) Solve $\frac{4}{5-x}\geq 1$ **3**

(e) If $P(x)=x^3-3kx+3$ is divisible by $(x-3)$, find the value of k. **2**

Question 2 12 marks (Begin a new booklet) **Marks**

(a) In the diagram below, the angle between the lines $y = ax$ and $y = bx$ is 45°. **2**



Show that $b = \dfrac{a-1}{a+1}$.

(b) Find $\displaystyle\sum_{a=1}^{n} \frac{n-a}{n}$ **3**

(c) The points A(1, -3), B(10, 9) and P(4, 1) are collinear. In what ratio does the point P divide the interval joining A and B? **3**

(d) Consider the circle $x^2 + y^2 - 2x - 14y + 25 = 0$.

(i) Show that if the line $y = mx$ intersects the circle in two distinct points, then **2**

$$(1+7m)^2 - 25(1+m^2) > 0$$

(ii) For what value(s) of m is the line $y = mx$ a tangent to the curve? **2**

Question 3 12 marks (Begin a new booklet) **Marks**

(a) Find $\displaystyle\int_{-1}^{2} |1-2x|\, dx$ **2**

(b) Given that a root of $x + \ln x = 2$ lies close to $x = 1.5$, use Newton's Method to find an approximation for the root. Answer to two decimal places. **3**

(c) (i) Factorise $3x^3 + 3x^2 - x - 1$. **2**

(ii) Hence or otherwise, solve the equation $3\tan^3\theta + 3\tan^2\theta - \tan\theta - 1 = 0$ for $0^\circ \le \theta \le 180^\circ$. **2**

(d) If $x^4 + 4y^4 = (x^2 + 2y^2 + axy)(x^2 + 2y^2 - axy)$ for all values of x and y, find the value of a. **3**

Question 4 12 marks (Begin a new booklet) Marks

(a) Find the equation of the tangent to the curve $y = 2^x$ at the point $(1, 2)$. **2**

(b) For $x > 0$, the area bounded by the curve $y = \frac{1}{x}$, the x-axis and the lines $x = \mathrm{e}$ and $x = k$ is 1 square unit. **2**

Find the value of k if $k < \mathrm{e}$.

(c) Two geometric series are given by:

(A) $1+\left(\sqrt{3}-1\right)+\left(\sqrt{3}-1\right)^2+\ldots$

(B) $1+\left(\sqrt{3}+1\right)+\left(\sqrt{3}+1\right)^2+\ldots$

A new series is formed by taking the product of corresponding terms:

ie $1\times 1+\left(\sqrt{3}-1\right)\left(\sqrt{3}+1\right)+\left(\sqrt{3}-1\right)^2\left(\sqrt{3}+1\right)^2+\ldots$

(i) Explain whether this new series is arithmetic, geometric or neither. **2**

(ii) Find the n^{th} term of this series. **2**

(d) By Mathematical Induction, prove that for every positive integer n, $13\times 6^n + 2$ is divisible by 5. **4**

Question 5 12 marks (Begin a new booklet) Marks

(a) Given α, β and γ are the roots of the equation $2x^3-5x^2+7x-1=0$, find the values of:

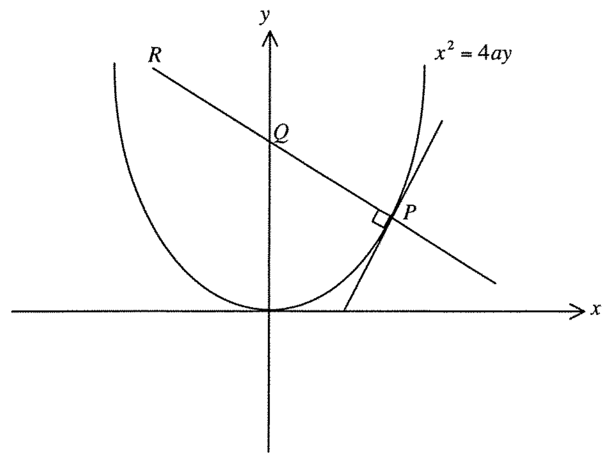
(i) $\alpha+\beta+\gamma$ **1**

(ii) $\alpha\beta+\beta\gamma+\gamma\alpha$ **1**

(iii) $(\alpha-1)(\beta-1)(\gamma-1)$ **2**

(b) If $\cos\alpha = \frac{3}{4}$, $0 < \alpha < 90^\circ$ and $\cos\beta = \frac{2}{3}$, $270^\circ < \beta < 360^\circ$, write down the exact value of $\sin\beta$ and hence find the exact value of $\sin(\alpha-\beta)$. **3**

(c)



The normal at $P\left(2ap, ap^2\right)$ on the parabola $x^2 = 4ay$ cuts the y-axis at Q and is produced to a point R such that $PQ = QR$.

(i) Given that the equation of the normal at P is $x + py = 2ap + ap^3$, find the co-ordinates of Q. **1**

(ii) Show that R has co-ordinates $\left(-2ap, ap^2+4a\right)$. **2**

(iii) Show that the locus of R is a parabola and state its vertex. **2**

Question 6 **12 marks** **(Begin a new booklet)** **Marks**

(a) Differentiate $\dfrac{4x+1}{2x-3}$. Hence evaluate $\displaystyle\int_0^1 \frac{dx}{(2x-3)^2}$. **3**

(b) Consider the function $f(x)=\dfrac{x}{x+2}$.

(i) Show that $f'(x)>0$ for all x in the domain. **2**

(ii) State the equation of the horizontal asymptote of $y=f(x)$. **1**

(iii) Without using any further calculus, sketch the graph of $y=f(x)$. **2**

(iv) Explain why $y=f(x)$ has an inverse function $f^{-1}(x)$. **1**

(v) Find an expression for $f^{-1}(x)$. **2**

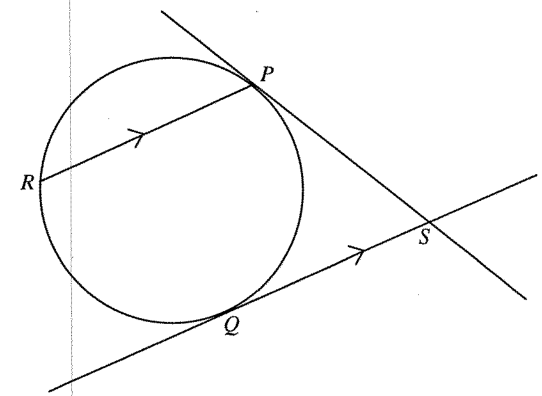
(vi) Write down the domain of $f^{-1}(x)$. **1**

Question 7 **12 marks** **(Begin a new booklet)** **Marks**

(a) (i) Express $\sin A$ and $\cos A$ in terms of t, where $t=\tan\dfrac{A}{2}$. **1**

(ii) Hence, prove that $\dfrac{\sin 2A}{1+\cos 2A}=\tan A$. **3**

(b)



P and Q are points on a circle and the tangents to the circle at P and Q meet at S. R is a point on the circle so that the chord PR is parallel to QS.

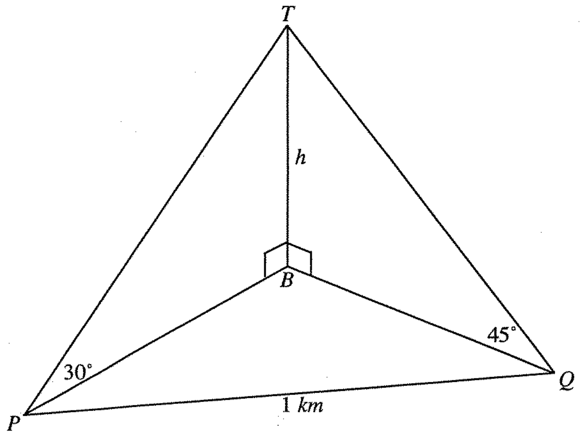
Copy the diagram into your answer booklet.

Prove $QP = QR$. **3**

Question 7 continues next page

Question 7 *continued* **Marks**

(c)



The angle of elevation from a boat at P to a point T at the top of a vertical cliff is 30°. The boat sails 1 km to a second point Q, from which the angle of elevation to T is 45°. Let B be the point at the base of the cliff directly below T and let $h = BT$ be the height of the cliff in metres.

The bearings of B from P and Q are 050°T and 310°T respectively.

(i) Show that $\angle PBQ = 100^\circ$. **1**

(ii) Find expressions for PB and QB in terms of h. **1**

(iii) Hence show that: **2**

$$h^2 = \frac{1000^2}{\cot^2 30 + \cot^2 45 - 2\cot 30 \cot 45 \cos 100}$$

(iv) Calculate the height of the cliff, correct to the nearest metre. **1**

END OF EXAMINATION

Spare Questions

(b) Factorise $250 - 2x^3$ **2**

(c) The line $y = \frac{4\sqrt{3}}{3}$ meets the curve $y = \frac{1}{\sqrt{x}} + \sqrt{x}$ at A and B.

(i) Show that the x co-ordinates of A and B are $\frac{1}{3}$ and 3 respectively. **2**

(ii) Show that the area of the region bounded by the line and the curve is $\frac{8\sqrt{3}}{27}\ u^2$. **2**

(g) Consider the curve $y = \ln x(x+2)$.

(i) State its domain. **2**

(ii) Find its x-intercepts. **2**

(iii) Hence sketch the curve. **1**

Consider the graph of $y = \frac{x^2}{1-x^2}$.

a) Write down the vertical asymptotes of the function.
b) Find any turning points of the function and determine their nature.
c) Show that the function is even.
d) Describe how the function behaves for large x.
e) Sketch the graph

(d) Find $\int \frac{e^x}{1+e^x}\,dx$ **1**

Question	Solutions	Marks	Marking Criteria
1a)	$64a^3 + \frac{1}{27}b^3$		
	$= (4a)^3 + (\frac{1}{3}b)^3$		
	$= (4a + \frac{b}{3})[(4a)^2 - (4a)(\frac{b}{3}) + (\frac{b}{3})^2]$	1	
	$= (4a + \frac{b}{3})(16a^2 - \frac{4ab}{3} + \frac{b^2}{9})$		
1b)	$3^{2x+1} - 28(3^x) + 9 = 0$		
	$(3^x)^2 .3 - 28(3^x) + 9 = 0$	1	
	let $u = 3^x$		
	$\therefore 3u^2 - 28u + 9 = 0$		
	$\therefore (3u-1)(u-9) = 0$		
	$\therefore u = \frac{1}{3}, 9$	1	
	$\therefore 3^x = \frac{1}{3}$ or $3^x = 9$		
	$\therefore 3^x = 3^{-1}$ $\quad 3^x = 3^2$		
	$\therefore x = -1$ $\quad \therefore x = 2$	1	
	$\therefore x = -1, 2$		
1c)	$\int_1^4 x\sqrt{5-x}\,dx$		
	let $u = 5-x$ $\quad \therefore x = 5-u$		
	$\therefore \frac{du}{dx} = -1 \Rightarrow dx = \frac{du}{-1} = -du$		
	$\int_1^4 x\sqrt{5-x}\,dx$ $\quad$ When $x=1, u=4$ When $x=4, u=1$		
	$= \int_4^1 (5-u)u^{\frac{1}{2}} . -du$	1	
	$= \int_1^4 (5-u)u^{\frac{1}{2}}\,du$		
	$= \int_1^4 (5u^{\frac{1}{2}} - u^{\frac{3}{2}})\,du$		

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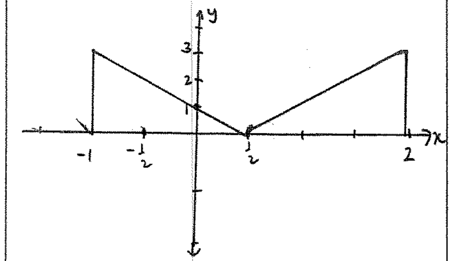
Question	Solutions	Marks	Marking Criteria
1c) ctd	$= \left[\frac{2.5u^{\frac{3}{2}}}{3} - \frac{2u^{\frac{5}{2}}}{5}\right]_1^4$	1	
	$= \left[\frac{10\sqrt{u^3}}{3} - \frac{2\sqrt{u^5}}{5}\right]_1^4$		
	$= \left\{\frac{10\sqrt{64}}{3} - \frac{2\sqrt{1024}}{5}\right\} - \left\{\frac{10}{3} - \frac{2}{5}\right\}$	1	
	$= \left\{\frac{80}{3} - \frac{64}{5}\right\} - \left\{\frac{10}{3} - \frac{2}{5}\right\}$		
	$= \frac{70}{3} - \frac{62}{5}$		
	$= 23\frac{1}{3} - 12\frac{2}{5}$		
	$= 10\frac{14}{15}$		
1d)	$\frac{4}{5-x} \geqslant 1$		
	$4(5-x) \geqslant (5-x)^2, \quad x \neq 5$	1	
	$20 - 4x \geqslant 25 - 10x + x^2$		
	$0 \geqslant x^2 - 6x + 5$		
	$x^2 - 6x + 5 \leqslant 0$		
	$(x-1)(x-5) \leqslant 0$	1	
	 $\therefore$ by inspection, $1 \leqslant x \leqslant 5$ but $x \neq 5$		
	$\therefore 1 \leqslant x < 5$	1	

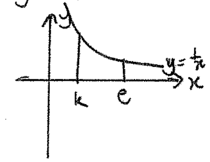
Question	Solutions	Marks	Marking Criteria
1e)	$P(x) = x^3 - 3kx + 3$		
	divisible by $(x-3)$		
	$\therefore P(3) = 0$	1	
	$\therefore (3)^3 - 3k(3) + 3 = 0$		
	$27 - 9k + 3 = 0$		
	$30 - 9k = 0$		
	$k = \frac{30}{9}$		
	$k = \frac{10}{3}$	1	
2a)	$y = ax$ has $m_1 = a$		
	$y = bx$ has $m_2 = b$	1	
	$\tan 45° = 1$ RTP: $b = \frac{a-1}{a+1}$		
	$\tan\theta = \left\lvert \frac{m_1 - m_2}{1 + m_1 m_2} \right\rvert$		
	$1 = \left\lvert \frac{a-b}{1+ab} \right\rvert$		
	$a - b = 1 + ab$		
	$-b - ab = 1 - a$		
	$-b(1+a) = 1 - a$		
	$b(1+a) = a - 1$		
	$b = \frac{a-1}{a+1}$ as required	1	

Question	Solutions	Marks	Marking Criteria
2b)	$\sum_{a=1}^{n} \frac{n-a}{n}$		
	$= \frac{n-1}{n} + \frac{n-2}{n} + \dots \frac{n-(n-1)}{n} + \frac{n-n}{n}$	1	
	$= 1 - \frac{1}{n} + 1 - \frac{2}{n} + \dots + 1 - \frac{n}{n}$		
	$= \underbrace{1+1+1+\dots}_{n \text{ times}} - \frac{1}{n}\underbrace{(1+2+3+\dots+n)}_{AP \text{ with } a=1,\ d=1}$		
	$= n + \left(-\frac{1}{n}\right)\left\{\frac{n}{2}(2a + (n-1)d\right\}$	1	
	$= n + \frac{-1}{n}\left\{\frac{n}{2}(2 + (n-1)1\right\}$		
	$= n + \frac{-1}{n}\left\{\frac{n}{2}(n+1)\right\}$		
	$= n - \frac{1}{n}\left\{\frac{n^2}{2} + \frac{n}{2}\right\}$		
	$= n - \frac{n}{2} - \frac{1}{2}$		
	$= \frac{n}{2} - \frac{1}{2}$	1	
2c)	$A(1,-3)$ $B(10,9)$ $P(4,1)$		
	$\therefore x_1 = 1$ $x_2 = 10$		
	$y_1 = -3$ $y_2 = 9$		
	$P(4,1) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$	1	
	$\therefore 4 = \frac{m(10) + n(1)}{m+n}$; $1 = \frac{m(9) + n(-3)}{m+n}$		
	$4m + 4n = 10m + n$ $\therefore m + n = 9m - 3n$		
	$3n = 6m$ $\therefore 4n = 8m$		
	$\therefore \boxed{m = \frac{1}{2}n}$ $\therefore \boxed{m = \frac{1}{2}n}$	1	
	$\therefore \frac{m}{n} = \frac{1}{2}$		
	$\therefore m : n = \frac{1}{2}n : n = 1 : 2$	1	

Question	Solutions	Marks	Marking Criteria
2d) i)	$x^2+y^2-2x-14y+25=0$		
	$y=mx$		
	intersect when		
	$x^2+(mx)^2-2x-14(mx)+25=0$		
	$x^2+m^2x^2-2x-14mx+25=0$		
	$(m^2+1)x^2-(2+14m)x+25=0$	1	
	$(m^2+1)x^2-2(1+7m)x+25=0$		
	$\therefore \Delta=[-2(1+7m)]^2-4(m^2+1)25$		
	$=4(1+7m)^2-100(m^2+1)$		
	$=4[(1+7m)^2-25(m^2+1)]$		
	for 2 real roots, $\Delta>0$		
	$\therefore 4[(1+7m)^2-25(m^2+1)]>0$		
	$\therefore (1+7m)^2-25(m^2+1)>0$	1	
	as required		
ii)	for the line to be a tangent to the curve, $\Delta=0$		
	$\therefore (1+7m)^2-25(m^2+1)=0$	1	
	$\therefore 49m^2+14m+1-25m^2-25=0$		
	$24m^2+14m-24=0$		
	$\therefore 12m^2+7m-12=0$		
	$(3m+4)(4m-3)=0$		
	$\therefore m=-\frac{4}{3},\ \frac{3}{4}$	1	

Question	Solutions	Marks	Marking Criteria
3a)		1	
	when $x=-1$, $y=3$		
	when $x=2$, $y=3$		
	Area $=\frac{1}{2}(1\frac{1}{2})(3)+\frac{1}{2}(1\frac{1}{2})(3)$		
	$=\frac{1}{2}.\frac{9}{2}+\frac{1}{2}.\frac{9}{2}$		
	$=2.\frac{9}{4}$		
	$=4\frac{1}{2}$	1	
3b)	$x+\ln x<2$		
	$\therefore x+\ln x-2=0$	1	
	$x_0=1.5$		
	$f(x)=x+\ln x-2$		
	$f'(x)=1+\frac{1}{x}$		
	$f'(1.5)=1+\frac{1}{1.5}$		
	$=1+\frac{2}{3}$		
	$=1\frac{2}{3}$	1	
	Newton's method: $x_1=x_0+\frac{f(x_0)}{f'(x_0)}$		
	$\therefore x_1=1.5+\frac{1.5+\ln 1.5-2}{1\frac{2}{3}}$		
	$x_1=1.5+\frac{3}{5}(\ln 1.5-0.5)$		
	$x_1 \doteqdot 1.443279065$		
	$x_1=1.44$ (to 2 dec. pl.)	1	

Question	Solutions	Marks	Marking Criteria
3c) i)	$3x^3+3x^2-x-1$		
	$=3x^2(x+1)-(x+1)$	1	
	$=(3x^2-1)(x+1)$	1	
ii)	$3\tan^3\theta+3\tan^2\theta-\tan\theta-1=0$		
	$0^\circ\le\theta\le180^\circ$		
	$\therefore(3\tan^2\theta-1)(\tan\theta+1)=0$	1	
	$\tan^2\theta=\frac{1}{3}$ or $\tan\theta=-1$		
	$\tan\theta=\pm\frac{1}{\sqrt3}$ $\quad\theta=135^\circ$		
	$\theta=30^\circ,150^\circ$		
	$\therefore\theta=30^\circ,135^\circ,150^\circ$	1	
3d)	$x^4+4y^4=(x^2+2y^2+axy)(x^2+2y^2-axy)$		
	RHS: $(x^2+2y^2+axy)(x^2+2y^2-axy)$		
	$=[(x^2+2y^2)^2-(axy)^2]$	1	
	$=x^4+4x^2y^2+4y^4-a^2x^2y^2$		
	$=x^4+4y^4+4x^2y^2-a^2x^2y^2$		
	$=x^4+4y^4+(4-a^2)x^2y^2$	1	
	$=$ LHS		
	$=x^4+4y^4$		
	$\therefore 4-a^2=0$		
	$\therefore(2-a)(2+a)=0$		
	$\therefore a=\pm2$	1	

Question	Solutions	Marks	Marking Criteria
4a)	$y=2^x$ $\quad P(1,2)$		
	$\frac{dy}{dx}=\ln2.2^x$		
	at $x=1$, $\frac{dy}{dx}=\ln2.2^1=2\ln2$	1	
	eqn of tangent: $y-2=2\ln2(x-1)$		
	$y-2=(2\ln2)x-2\ln2$		
	$y-2=(\ln2^2)x-\ln2^2$		
	$y-2=(\ln4)x-\ln4$		
	$y=x\ln4-\ln4+2$	1	
4b)	$y=\frac{1}{x}$		
			
	$\therefore\int_k^e\frac{1}{x}\,dx=1$		
	$[\ln x]_k^e=1$	1	
	$\ln e-\ln k=1$		
	$1-\ln k=1$		
	$\therefore-\ln k=0$		
	$\therefore\ln k=0$		
	$\therefore k=1$ (since $e^1=0$)	1	

Question	Solutions	Marks	Marking Criteria
4c) i)	$1\times1+(\sqrt{3}-1)(\sqrt{3}+1)+(\sqrt{3}-1)^2(\sqrt{3}+1)^2+\ldots$		
	$=1+2+4+\ldots\ldots$	1	
	$\frac{T_2}{T_1}=\frac{2}{1}$		
	$\frac{T_3}{T_2}=\frac{4}{2}$		
	$\therefore \frac{T_3}{T_2}=\frac{T_2}{T_1}$		
	$\therefore$ the series is geometric with $a=1$, $r=2$	1	
ii)	$T_n=ar^{n-1}$		
	$=(1)(2)^{n-1}$	1	
	$\therefore T_n=2^{n-1}$	1	
4d)	Step 1: test for $n=1$		
	$13\times6^1+2$		
	$=78+2$		
	$=80$, which is divisible by 5		
	$\therefore$ true for $n=1$	1	
	Step 2: assume true for all $n=k$		
	$\therefore 13\times6^k+2=5M$ for some integer M	1	
	Step 3: prove true for $n=k+1$		
	$13\times6^{k+1}+2$		
	$=13\times6^k\cdot6+2$		
	$=6[5m-2]+2$		
	$=30m-12+2$		
	$=30m-10$		

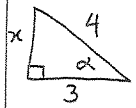
Question	Solutions	Marks	Marking Criteria
	$=5(6m-2)$, which is divisible by 5		
	$\therefore$ true for $n=k+1$	1	
	Step 4: The statement is true for $n=k+1$ whenever it is true for $n=k$		
	But the statement is true for $n=1$		
	$\therefore$ By the Principle of Mathematical Induction the statement is true for all integers $n\geq1$	1	
5a)	$2x^3-5x^2+7x-1=0$		
(i)	$\alpha+\beta+\gamma=\frac{5}{2}$	1	
(ii)	$\alpha\beta+\beta\gamma+\gamma\alpha=\frac{7}{2}$	1	
(iii)	$\alpha\beta\gamma=\frac{1}{2}$		
	$(\alpha-1)(\beta-1)(\gamma-1)$		
	$=(\alpha\beta-\alpha-\beta+1)(\gamma-1)$		
	$=\alpha\beta\gamma-\alpha\beta-\alpha\gamma+\alpha-\beta\gamma+\beta+\gamma-1$	1	
	$=\alpha\beta\gamma-(\alpha\beta+\alpha\gamma+\beta\gamma)+(\alpha+\beta+\gamma)-1$		
	$=\frac{1}{2}-\frac{7}{2}+\frac{5}{2}-1$		
	$=-\frac{3}{2}$	1	

Question	Solutions	Marks	Marking Criteria

5b)

$\cos\alpha = \frac{3}{4}$, $0^\circ < \alpha < 90^\circ$

$\cos\beta = \frac{2}{3}$, $270^\circ < \beta < 360^\circ$

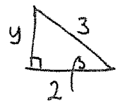


$x^2 + 3^2 = 4^2$ (by Pythagoras' Theorem)

$x^2 + 9 = 16$

$x = \sqrt{7}$

$\therefore \sin\alpha = \frac{\sqrt{7}}{4}$ — 1



$y^2 + 2^2 = 3^2$ (by Pythagoras' Theorem)

$y^2 + 4 = 9$

$y = \sqrt{5}$

$\therefore \sin\beta = \frac{\sqrt{5}}{3}$

but β lies in Quadrant 4

$\therefore \sin\beta = -\frac{\sqrt{5}}{3}$ — 1

$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$= \left(\frac{\sqrt{7}}{4}\right)\left(\frac{2}{3}\right) - \left(\frac{3}{4}\right)\left(\frac{-\sqrt{5}}{3}\right)$ — 1

$= \frac{2\sqrt{7}}{12} + \frac{3\sqrt{5}}{12}$

$= \frac{\sqrt{7}}{6} + \frac{\sqrt{5}}{4}$ (or $\frac{2\sqrt{7} + 3\sqrt{5}}{12}$)

Question	Solutions	Marks	Marking Criteria

5 c)

$P(2ap, ap^2)$

$x^2 = 4ay$

i) normal: $x + py = 2ap + ap^3$

Q lies on y-axis, $\therefore x = 0$

$\therefore 0 + py = 2ap + ap^3$

$py = ap(2 + p^2)$

$\therefore y = a(2+p^2)$, $p \neq 0$

$\therefore Q$ is the point $(0, a(2+p^2))$ — 1

ii) Q is the midpoint of PR

$\therefore (0, a(2+p^2)) = \left(\frac{2ap + x_2}{2}, \frac{ap^2 + y_2}{2}\right)$

$\therefore \frac{2ap + x_2}{2} = 0$

$\therefore 2ap + x_2 = 0$ — 1

$\therefore \boxed{x_2 = -2ap}$

$\frac{ap^2 + y_2}{2} = a(2+p^2)$

$\therefore ap^2 + y_2 = 2a(2+p^2)$

$ap^2 + y_2 = 4a + 2ap^2$

$\boxed{y_2 = 4a + ap^2}$ — 1

$\therefore R$ is the point $(-2ap, ap^2 + 4a)$

iii) $x = -2ap$, $y = ap^2 + 4a$

$\therefore p = \frac{x}{-2a}$ $\qquad \therefore p = \sqrt{\frac{y - 4a}{a}}$

$\therefore \frac{x^2}{4a^2} = \frac{y - 4a}{a}$ — 1

Qn	Solutions	Marks	Comments+Criteria

$$d_{QP}^2 = (2ap-0)^2 + (ap^2 - 2a - ap^2)^2$$

$$= 4a^2p^2 + a^2p^4 - 2a^2p^2 - a^2p^4$$
$$-2a^3p^2 + 4a^2 + 2a^2p^2)$$

$$= 2a^2p^2 + 4a^2$$

Qn	Solutions	Marks	Comments+Criteria

$R(-2ap, ap^2+4a)$

$Q(0, a(2+p^2))$

$P(2ap, ap^2)$

Show $d_{RQ} = d_{PQ}$.

$$d_{RQ}^2 = (-2ap-0)^2 + (ap^2+4a - 2a - ap^2)^2$$

$$= 4a^2p^2 + (ap^2 + 2a - ap^2)^2$$

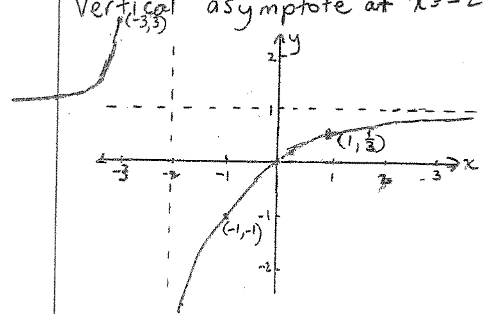
$$= 4a^2p^2 + a^2(p^2 + 2a - ap^2)^2$$

$$= 4a^2p^2 + a^2(p^4 + 2ap^2 - ap^4 + 2ap^2 + 4a^2$$
$$-2a^2p^2 - ap^4 - 2a^2p^2 - a^2p^4)$$

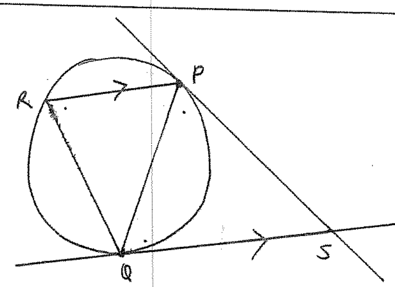
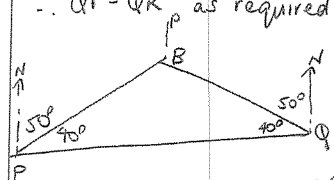
$$= 4a^2p^2 + a^2(p^4 + 4ap^2 - 2ap^4 + 4a^2$$
$$-4a^2p^2 - a^2p^4)$$

$$= 4a^2p^2 + a^2p^4 + 4a^3p^2 - 2a^3p^4 + 4a^4$$
$$-4a^4p^2 - a^4p^4$$

Question	Solutions	Marks	Marking Criteria
5c)iii) ctd)	$x^2 = \frac{4a^2(y-4a)}{a}$ $x^2 = 4a(y-4a)$ ∴ Locus is a parabola with focal length a and vertex $(0, 4a)$	1	
6a)	$\frac{d}{dx}\frac{4x+1}{2x-3}$ $= \frac{(2x-3)(4)-(4x+1)(2)}{(2x-3)^2}$ $= \frac{8x-12-8x-2}{(2x-3)^2}$ $= \frac{-14}{(2x-3)^2}$	1	
	$\therefore \int_0^1 \frac{dx}{(2x-3)^2}$ $= -\frac{1}{14}\int \frac{-14}{(2x-3)^2}\,dx$ $= -\frac{1}{14}\left[\frac{4x+1}{2x-3}\right]_0^1$	1	
	$= -\frac{1}{14}\left\{\left(\frac{4+1}{2-3}\right)-\left(\frac{0+1}{0-3}\right)\right\}$ $= -\frac{1}{14}\left\{\frac{5}{-1}-\frac{1}{-3}\right\}$ $= -\frac{1}{14}\left(-5+\frac{1}{3}\right)$ $= -\frac{1}{14}\left(-4\frac{2}{3}\right)$ $= -\frac{1}{14}\cdot -\frac{14}{3}$ $= \frac{1}{3}$	1	

Question	Solutions	Marks	Marking Criteria
6b)	$f(x) = \frac{x}{x+2}$		
i)	$f'(x) = \frac{(x+2)(1)-x(1)}{(x+2)^2}$	1	
	$= \frac{x+2-x}{(x+2)^2}$ $= \frac{2}{(x+2)^2} > 0$ for all x since $(x+2)^2 > 0$ for all x	1	
ii)	$f(x) = \frac{x}{x+2}$ $= \frac{x+2-2}{x+2}$ $= 1 - \frac{2}{x+2}$ ∴ horizontal asymptote at $y=1$ (since $\frac{2}{x+2} \neq 0$ for all x)	1	
iii)	$f'(x) > 0$ for all x ∴ $f(x)$ is increasing for all x horizontal asymptote at $y=1$ Vertical asymptote at $x=-2$  When $x=0$, $y=0$ When $x=-3$, $y=3$	1	1 for graph 1 for some working

Question	Solutions	Marks	Marking Criteria
6 iv)		1	
v)	$x = \frac{y}{y+2}$ $\therefore xy + 2x = y$ $\therefore xy - y = -2x$ $y(x-1) = -2x$ $y = \frac{-2x}{x-1}$ $\therefore y = \frac{2x}{1-x}$ $\therefore f^{-1}(x) = \frac{2x}{1-x}$	1 1	
vi)	Domain $f^{-1}(x) = \{x : \text{all real } x,\ x \neq 1\}$	1	
7a) i)	$\sin A = \frac{2t}{1+t^2}$, $\cos A = \frac{1-t^2}{1+t^2}$ let $t = \tan A$	1	
ii)	RTP: $\frac{\sin 2A}{1+\cos 2A} = \tan A$ LHS: $\dfrac{\frac{2t}{1+t^2}}{1 + \frac{1-t^2}{1+t^2}}$ $= \dfrac{\frac{2t}{1+t^2}}{\frac{1+t^2+1-t^2}{1+t^2}}$ $= \frac{2t}{2}$, $1+t^2 \neq 0$ $= t = \tan A = \text{RHS}$ $\therefore \frac{\sin 2A}{1+\cos 2A} = \tan A$ as required	1 1 1	

Question	Solutions	Marks	Marking Criteria
7b)	 RTP: $QP = QR$ $PS = QS$ (equal tangents from external point S) $\therefore \angle SPQ = \angle SQP$ (base angles of isosceles $\triangle PQS$ equal) $\angle PQS = \angle QRP$ (angle in alternate segment is equal) $\angle PQS = \angle QPR$ (alternate angles equal; $PR \parallel QS$) $\therefore \angle QRP = \angle QPR$ $\therefore \triangle QPR$ is isosceles (base angles equal) $\therefore QP = QR$ as required	1 1 1	
7c) i)	 $\therefore \angle PBQ + 40° + 40° = 180°$ (angle sum $\triangle PQB$) $\therefore \angle PBQ = 100°$ as required	1	

Question	Solutions	Marks	Marking Criteria
7c) ii)	$\tan 30 = \frac{h}{PB}$ $\tan 45° = \frac{h}{BQ}$ $PB = \frac{h}{\tan 30°}$ $BQ = \frac{h}{\tan 45°}$ $PB = h\cot 30°$ $BQ = h\cot 45°$	1	
iii)	$1000^2 = h^2\cot^2 30° + h^2\cot^2 45° - 2h^2\cot 30°\cot 45°\cos 100°$	1	
	$\therefore h^2 = \frac{1000^2}{\cot^2 30° + \cot^2 45° - 2\cot 30°\cot 45°\cos 100°}$	1	
iv)	$h^2 = \frac{1000^2}{3 + 1 - (2\sqrt{3})(1)(-0.173648177)}$ $h^2 = \frac{1000^2}{3.398465067}$ $h \doteqdot 17.15373098$ $h = 17$ m (to nearest metre)	1	
	$PQ^2 = PB^2 + QB^2 - 2PB.QB \cos\angle PBQ$ $1000^2 = h^2\cot^2 30 + h^2\cot^2 45 - 2h\cot 30 . h\cot 45 \cos 100$		