Name: ____________________________ Student Number: ________________________

2016

HSC ASSESSMENT TASK 2
TERM 1 ASSESSMENT BLOCK

Mathematics Extension 1

Tuesday 22 March (Morning Session)

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black pen
- Board-approved calculators may be used
- A Reference Sheet is provided.
- In Questions 11–16, show relevant mathematical reasoning and/or calculations

Total marks – 70

Section I
10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Section II
60 marks
Attempt Questions 11–14
Allow about 1 hour and 45 minutes for this section

Outcomes to be Assessed:

A student:

H3 manipulates algebraic expressions involving logarithmic and exponential functions.

H5 applies appropriate techniques from the study of calculus, geometry, probability and trigonometry to solve problems.

PE2 uses multi-step deductive reasoning in a variety of contexts

PE3 solves problems involving permutations and combinations, inequalities, polynomials, circle geometry and parametric representations

PE4 uses the parametric representation together with differentiation to identify geometric properties of parabolas

PE5 determines derivatives which require the application of more than one rule of differentiation

HE4 uses the relationship between functions, inverse functions and their derivatives

HE6 determines integrals by reduction to a standard form through a given substitution

HE7 evaluates mathematical solutions to problems and communicates them in an appropriate form

Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the mutiple-choice answer sheet for Questions 1-10.

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1 Which of the following is the correct evaluation of $2\sin\left(\frac{\pi}{7}\right)$ correct to three significant figures?

(A) 0.0157

(B) 0.016

(C) 0.782

(D) 0.868

2 What is the value of $\log_5 200 - 3\log_5 2$?

(A) 1.4

(B) 2.0

(C) 2.5

(D) 3.2

3 Which of the following points divides the line segment from $A(-3, 2)$ to $B(5, -2)$ externally in the ratio $3:1$?

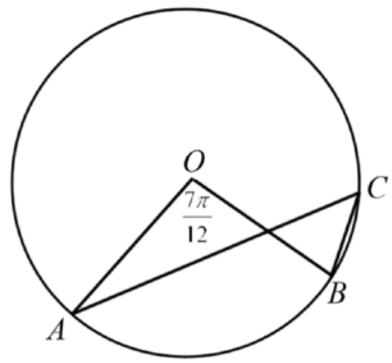
(A) $(-7, 4)$

(B) $(3, -1)$

(C) $(9, -4)$

(D) $(-4, 7)$

4 Points A, B and C lie on a circle centre O. Given that $\angle AOB = \frac{7\pi}{12}$ radians, which of the following is the size of $\angle ACB$?



Not to scale

(A) $\frac{7\pi}{6}$

(B) $\frac{7\pi}{12}$

(C) $\frac{14\pi}{3}$

(D) $\frac{7\pi}{24}$

5 What is the value of the integral $\int e^4 \, dx$?

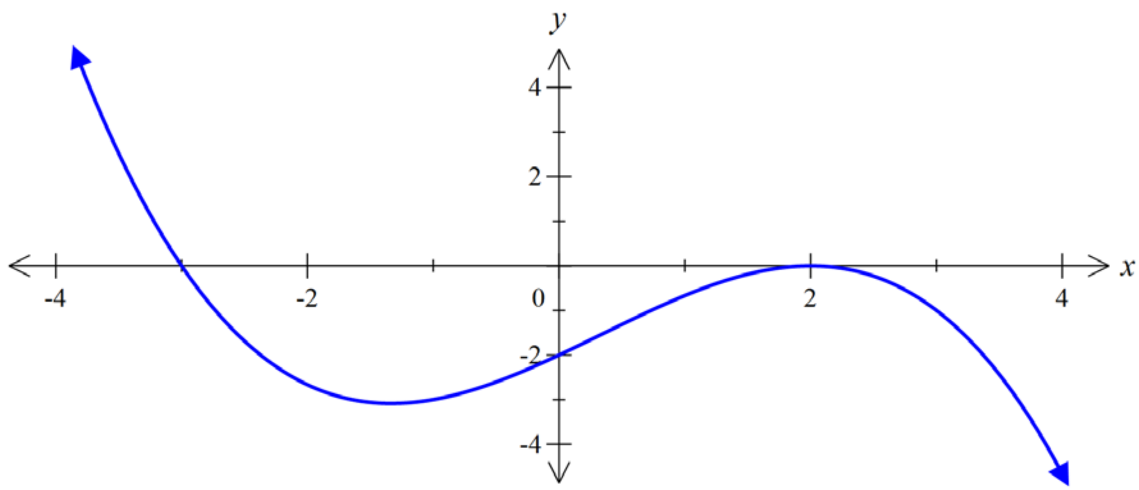
(A) $4e^3 + C$

(B) $\frac{e^4}{4} + C$

(C) 0

(D) $e^4 x + C$

6 The polynomial graph shown below has the equation $y = A(x+B)(x+C)^2$.
What are the values of A, B and C?



(A) $A = \frac{1}{6}, B = -3, C = 2$

(B) $A = -\frac{1}{6}, B = 3, C = -2$

(C) $A = 1, B = 3, C = -2$

(D) $A = -1, B = 3, C = -2$

7 How can the function $f(x) = x - \frac{x^3}{1+x^2}$ be best described?

(A) Even and continuous

(B) Even and discontinuous

(C) Odd and continuous

(D) Odd and discontinuous

8 A curve is defined by the parameters $x = 2\sin\theta$ and $y = \cos 2\theta$. Which of the following represents the curve in Cartesian form?

(A) $x^2 + y^2 = 4$

(B) $y = 1 - \frac{x^2}{2}$

(C) $y = 2x + 1$

(D) $y^2 = \frac{x}{2} - 1$

9 Which of the following equates to $\lim_{(x+y)\to 0}\left\{\frac{\sin x\cos y + \cos x\sin y}{2x + 2y}\right\}$?

(A) $\frac{1}{x+y}$

(B) $\frac{\sin x + \sin y}{2x + 2y}$

(C) 1

(D) $\frac{1}{2}$

10 A piece of string of length 50 cm is cut once. What is the probability that one piece is 30 cm or more longer than the other piece?

(A) $\frac{2}{3}$

(B) $\frac{1}{5}$

(C) $\frac{2}{5}$

(D) $\frac{3}{5}$

Section II

60 marks
Attempt Questions 11-14
Allow about 1 hour and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra paper is available.

In Questions 11-14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a) Consider the functions $y = x^2$ and $y = x^2 - 3x + 2$.

(i) Sketch the two functions on the same axes. **2**

(ii) Hence find the values of x such that $x^2 > (x-1)(x-2)$. **2**

(b) The functions $y = x$ and $y = x^3$ meet at the point (1, 1). What is the acute angle between the tangents to these functions at this point? **3**

(c) Solve $\dfrac{x}{x-5} \geq 2$. **3**

(d) Find $\int x\sqrt{x+1}\,dx$ using the substitution $x = u^2 - 1$. **3**

(e) In a class of 27 students, there are 14 boys and 13 girls. From the class two boys and two girls are chosen to be on the student council.
Find the probability that a particular boy and a particular girl are chosen to be on the council? **2**

Question 12 (15 marks) Use a SEPARATE writing booklet.

(a) (i) Show that $\sin\left(x+\dfrac{\pi}{4}\right)=\dfrac{\sin x+\cos x}{\sqrt{2}}$ **2**

(ii) Hence, or otherwise, solve $\dfrac{\sin x+\cos x}{\sqrt{2}}=\dfrac{\sqrt{2}}{2}$ for $0\le x\le 2\pi$. **3**

(b) Differentiate with respect to x:

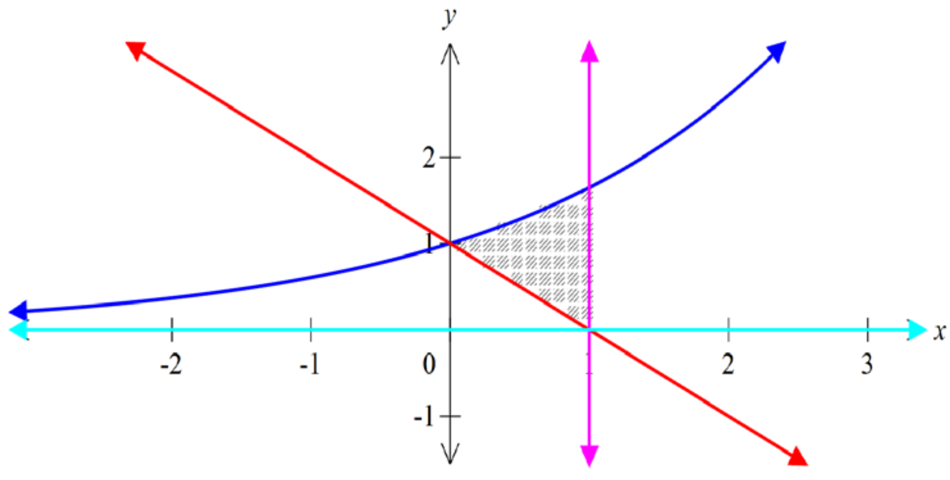
(i) $\tan 5x$ **1**

(ii) $\log_e \sqrt{x}$ **2**

(iii) $\sec x$ **2**

(c) Find $\displaystyle\int_1^3 \frac{x}{2x^2+3}\,dx$ **2**

(d) The graphs of $y=e^{\frac{x}{2}}$, $y=1-x$ and $x=1$ are shown on the diagram. **3**



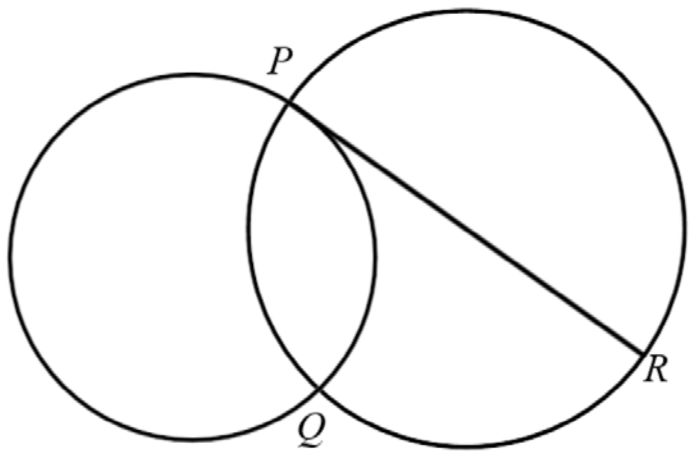
Find the area of the shaded region.

Question 13 (15 marks) Use a SEPARATE writing booklet.

(a) Leon, Anna and five other people go into a cafe one at a time. In how many ways can the seven people go into the shop if Leon goes into the cafe after Anna? **1**

(b) Find the value of $\int_0^{\frac{\pi}{4}} \sin y \cos y \, dy$. **3**

(c) Two circles intersect at P and Q. The diameter of one circle is PR.



Copy this diagram into your writing booklet.

(i) Draw a straight line through P, parallel to QR to meet the other circle at S. Prove that QS is a diameter of the second circle. **2**

(ii) Prove that the circles have equal radii if SQ is parallel to PR. **2**

(d) Find $\int \cos^2 3x \, dx$. **3**

(e) Using the substitution $u = \tan x$ to show that $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sec^2 x}{\tan x} \, dx = \log_e 3$. **4**

Question 14 (15 marks) Use a SEPARATE writing booklet.

(a) Consider the even function $f(x) = \dfrac{1}{1+x^2}$.

(i) Show that the function has a horizontal asymptote. **1**

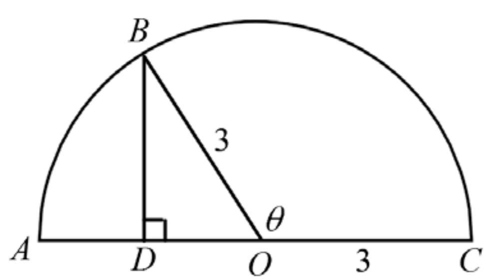
(ii) Show that the $y = f(x)$ has only one stationary point and find its nature. **2**

(iii) Sketch $y = f(x)$ without finding any points of inflexon. **1**

(iv) Explain why $y = f(x)$ for $x \geq 0$ has an inverse function. **1**

(v) Find the inverse function $f^{-1}(x)$. **2**

(b) The point B lies on the circumference of a semicircle of radius 3 units. AC is the diameter of the semicircle and D lies on AC. BD is perpendicular to AC. The arc BC subtends an angle θ at the centre O and the length of arc BC is twice the length of BD.



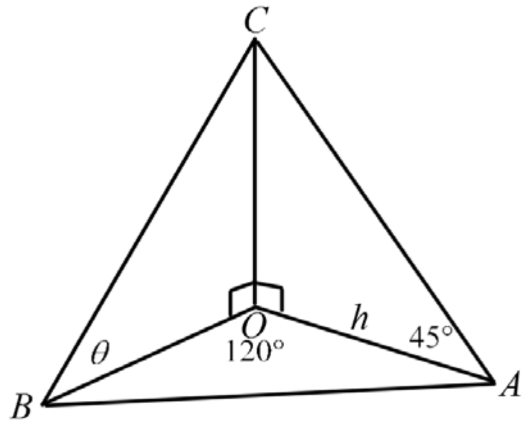
(i) Show that $\theta = 2\sin\theta$. **3**

(ii) Taking $\theta = 1.7$ radians as an approximation for the solution to the equation $\theta = 2\sin\theta$, use one approximation of Newton's method to give a better approximation. Answer correct to two decimal places. **2**

Question 14 continues on the next page

Question 14 (continued)

(c) In the diagram, both $\angle AOC$ and $\angle BOC$ are right angles. $\angle CAO = 45°$, $\angle CBO = \theta$ and $\angle AOB = 120°$. $AO = h$ metres.



Show that $AB^2 = h^2 + \dfrac{h^2}{\tan^2\theta} + \dfrac{h^2}{\tan\theta}$. **3**

End of paper

Student Number

MULTIPLE-CHOICE ANSWER SHEET

Attempt all questions

1	A ⬭	B ⬭	C ⬭	D ⬭
2	A ⬭	B ⬭	C ⬭	D ⬭
3	A ⬭	B ⬭	C ⬭	D ⬭
4	A ⬭	B ⬭	C ⬭	D ⬭
5	A ⬭	B ⬭	C ⬭	D ⬭
6	A ⬭	B ⬭	C ⬭	D ⬭
7	A ⬭	B ⬭	C ⬭	D ⬭
8	A ⬭	B ⬭	C ⬭	D ⬭
9	A ⬭	B ⬭	C ⬭	D ⬭
10	A ⬭	B ⬭	C ⬭	D ⬭

Year 12 Mathematics Extension 1 Solutions to Term 1 Assessment 2016

Section I – Multiple Choice

	A	B	C	D
1	○	○	○	●
2	○	●	○	○
3	○	○	●	○
4	○	○	○	●
5	○	○	○	●
6	○	●	○	○
7	○	○	●	○
8	○	●	○	○
9	○	○	○	●
10	○	○	●	○

Section I – Multiple Choice Worked Solutions

1 $\quad 2\sin\left(\frac{\pi}{7}\right)$ $=0.8677....$ $=0.868\ (3\text{ sf})$ (D) **Note: set calculator to radian mode**	2 $\quad \log_5 200-3\log_5 2$ $=\log_5 200-\log_5 2^3$ $=\log_5\left(\frac{200}{8}\right)$ $=\log_5 25$ $=2$ (B)
3 $\quad A(-3,2) \qquad B(5,-2)$ $3:-1$ $x-\text{coordinate}$ $=\frac{mx_2+nx_1}{m+n}$ $=\frac{-1(-3)+3(5)}{3-1}$ $=\frac{18}{2}$ $=9$ (C) **Note: external division not internal division**	4 $\quad \angle ACB=\frac{7\pi}{24}$ (angle at circumference is half angle at centre standing on same arc) (D)

5 $\int e^4 \, dx$

$= e^4 \, x + C$ (D)

Note: e^4 is a constant not a variable

6 The polynomial has double root at $x = 2$ i.e. factor is $(x-2)^2$

$\therefore C = -2$

The polynomial has single root at $x = -3$ i.e. factor is $(x+3)$

$\therefore B = 3$

The polynomial has y-intercept at -2

Sub. $(0, -2)$ to find A

$-2 = A(3)(-2)^2$

$A = \frac{-2}{12}$

$\therefore A = -\frac{1}{6}$ (B)

7 $f(x) = \frac{x^3}{1+x^2}$

$f(-x) = \frac{(-x)^3}{1+(-x)^2}$

$f(-x) = \frac{-x^3}{1+x^2}$

$f(-x) = -f(x)$

$\therefore f(x)$ is odd

Domain: all real x

$\therefore f(x)$ is continuous (C)

8 $x = 2\sin\theta$

$\therefore \sin\theta = \frac{x}{2}$

$y = \cos 2\theta$

$y = 1 - 2\sin^2\theta$

$y = 1 - 2\left(\frac{x}{2}\right)^2$

$y = 1 - \frac{x^2}{2}$ (B)

9 $\lim\limits_{(x+y)\to 0} \left\{ \frac{\sin x \cos y + \cos x \sin y}{2x + 2y} \right\}$

$= \lim\limits_{(x+y)\to 0} \left\{ \frac{\sin(x+y)}{2(x+y)} \right\}$

$= \frac{1}{2} . \lim\limits_{(x+y)\to 0} \left\{ \frac{\sin(x+y)}{(x+y)} \right\}$

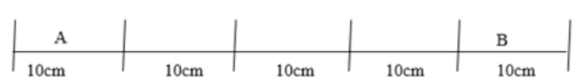
$= \frac{1}{2} .1$

$= \frac{1}{2}$ (D)

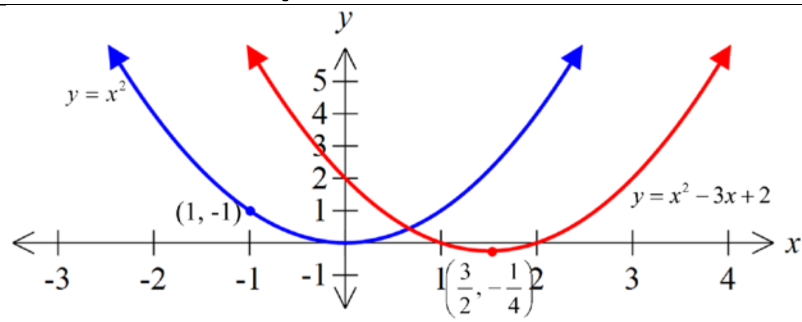
10 To have one piece 30 cm or longer than other piece we would be required to make a cut either in section A or in section B

$P(E) = \frac{20}{50}$

$P(E) = \frac{2}{5}$ (C)



Section II – Working for Questions 11-14

	Question 11	Mks	Marking Criteria
(a)(i)		**2**	Correct graphs showing all correct features
		1	One correct graph showing all features OR both correct graphs without all features shown
(a)(ii)	Solve graphs simultaneously to find pt of intersection $x^2 = x^2 - 3x + 2$ $3x = 2$ $x = \frac{2}{3}$ $x^2 > x^2 - 3x + 2$ for $x > \frac{2}{3}$ $\therefore x^2 > (x-1)(x-2)$ for $x > \frac{2}{3}$	**2**	Correct solution
		1	Correct point of intersection OR Correct inequality from their point of intersection
(b)	$y = x$ $m_1 = 1$ $y = x^3$ $y' = 3x^2$ $x = 1: m_2 = 3$ $\tan\theta = \left\lvert \frac{m_1 - m_2}{1 + m_1 m_2} \right\rvert$ $= \left\lvert \frac{1-3}{1+3} \right\rvert$ $= \frac{1}{2}$ $\theta = \tan^{-1}\left(\frac{1}{2}\right)$ $= 26.565...^\circ$ or $0.4636...$radians	**3**	Correct solution
		2	Correct substitution of correct gradients into correct formula
		1	Correct gradients

<table>
<tr>
<td rowspan="3">(c)</td>
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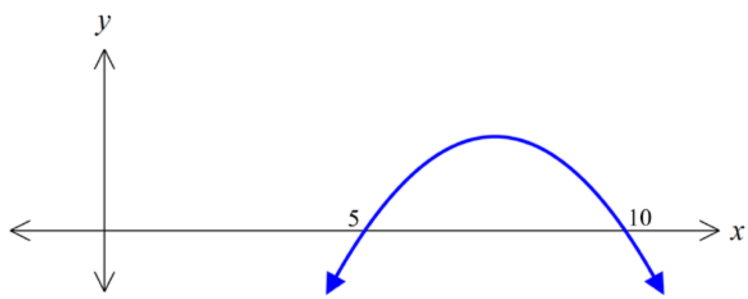
$\dfrac{x}{x-5} \geq 2,\ x \neq 5$

$\dfrac{x-2(x-5)}{x-5} \geq 0$

$\dfrac{10-x}{x-5} \geq 0$

$(10-x)(x-5) \geq 0$ (if $\dfrac{a}{b} \geq 0$ then $ab \geq 0$)

$5 < x \leq 10$



OR

$\dfrac{x}{x-5} \geq 2,\ x \neq 5$

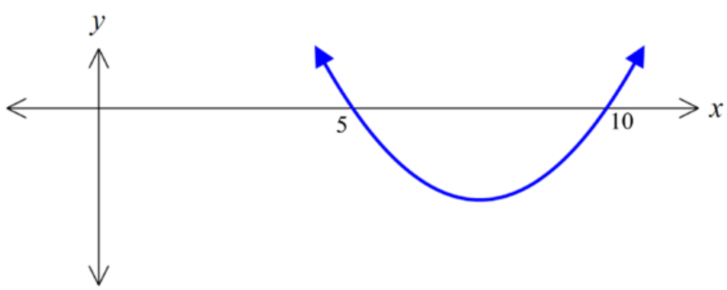
$x(x-5) \geq 2(x-5)^2$

$2(x-5)^2 - x(x-5) \leq 0$

$(x-5)\left[2(x-5)-x\right] \leq 0$

$(x-5)(x-10) \leq 0$

$5 < x \leq 10$



Note: if using critical points method, MUST SHOW SUBSTITUTION into all 3 regions for full marks

</td>
<td>3</td>
<td>Correct solution</td>
</tr>
<tr>
<td>2</td>
<td>Correct domain and correct attempt at correct solution
OR
Substantially correct solution</td>
</tr>
<tr>
<td>1</td>
<td>Correct domain
OR
Correct attempt at correct solution</td>
</tr>
</table>

<table>
<tr><td rowspan="3">(d)</td><td rowspan="3">$\int x\sqrt{x+1}\ dx$
$x = u^2 - 1$
$\frac{dx}{du} = 2u$
$dx = 2u\ dx$
$\int x\sqrt{x+1}\ dx$
$= \int (u^2 - 1)\ .\ u\ .\ 2u\ du$
$= \int (2u^4 - 2u^2)\ du$
$= \frac{2}{5}u^5 - \frac{2}{3}u^3 + C$
$= \frac{2}{5}\sqrt{(x+1)^5} - \frac{2}{3}\sqrt{(x+1)^3} + C$</td><td>3</td><td>Correct solution</td></tr>
<tr><td>2</td><td>Substantially correct solution</td></tr>
<tr><td>1</td><td>Correct change of variable in integral</td></tr>
<tr><td rowspan="2">(e)</td><td rowspan="2">P(particular boy and particular girl are on council)
$= \frac{{}^{13}C_1 \times {}^{12}C_1}{{}^{14}C_2 \times {}^{13}C_2}$
$= \frac{2}{91}$</td><td>2</td><td>Correct solution</td></tr>
<tr><td>1</td><td>Correct total number of ways of choosing two boys and two girls to be on student council</td></tr>
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Question 12		Mks	Marking Criteria
(a)(i)	$\sin(\theta+\phi)=\sin\theta\cos\phi+\cos\theta\sin\phi$ $\sin\left(x+\frac{\pi}{4}\right)=\sin x\cos\frac{\pi}{4}+\cos x\sin\frac{\pi}{4}$ $=\sin x\,.\,\frac{1}{\sqrt{2}}+\cos x\,.\,\frac{1}{\sqrt{2}}$ $=\frac{\sin x+\cos x}{\sqrt{2}}$	**2**	Correct solution
		1	Correct explanation but not justified step by step for "Show that"
(a)(ii)	$\frac{\sin x+\cos x}{\sqrt{2}}=\frac{\sqrt{2}}{2},\quad 0\le x\le 2\pi$ $\sin\left(x+\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}},\quad \frac{\pi}{4}\le x+\frac{\pi}{4}\le 2\pi+\frac{\pi}{4}$ $x+\frac{\pi}{4}=\frac{\pi}{4},\frac{3\pi}{4},2\pi+\frac{\pi}{4}$ $x=0,\frac{\pi}{2},2\pi$	**3**	Correct solution
		2	Substantially correct solution
		1	Correct related acute angle
(b)(i)	$\frac{d}{dx}(\tan 5x)$ $=5\sec^2 5x$	**1**	Correct solution
(b)(ii)	$\frac{d}{dx}\left(\log_e\sqrt{x}\right)$ $=\frac{d}{dx}\left(\frac{1}{2}\log_e x\right)$ $=\frac{1}{2}\cdot\frac{1}{x}$ $=\frac{1}{2x}$ or $\frac{d}{dx}\left(\log_e\sqrt{x}\right)=\frac{d}{dx}\left(\log_e x^{\frac{1}{2}}\right)$ $=\frac{\frac{1}{2}x^{-\frac{1}{2}}}{x^{\frac{1}{2}}}$ $=\frac{\frac{1}{2\sqrt{x}}}{\sqrt{x}}\times\frac{2\sqrt{x}}{2\sqrt{x}}$ $=\frac{1}{2x}$	**2**	Correct solution
		1	Correct use of log law OR Correct attempt at finding derivative

(b)(iii)	$\frac{d}{dx}(\sec x)$ $=\frac{d}{dx}(\cos x)^{-1}$ $=-(\cos x)^{-2}(-\sin x)$ $=\frac{1}{\cos x}\cdot\frac{\sin x}{\cos x}$ $=\sec x\tan x$	**2**	Correct solution, $\sin x\sec^2 x, \frac{\sin x}{\cos^2 x}$ accepted this time
		1	Correct use of chain rule
(c)	$\int_1^3 \frac{x}{2x^2+3}\,dx$ $=\frac{1}{4}\int_1^3 \frac{4x}{2x^2+3}\,dx$ $=\frac{1}{4}\left[\log_e\left\|2x^2+3\right\|\right]_1^3$ $=\frac{1}{4}\left[\log_e 21-\log_e 5\right]$ $=\frac{1}{4}\log_e\left(\frac{21}{5}\right)$	**2**	Correct solution
		1	Primitive of the form $k\log_e\left\|2x^2+3\right\|$

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<td rowspan="3">(d)</td>
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$$A = \int_0^1 \left(e^{\frac{x}{2}} - (1 - x) \right) dx$$

$$= \int_0^1 \left(e^{\frac{x}{2}} - 1 + x) \right) dx$$

$$= \left[2e^{\frac{x}{2}} - x + \frac{x^2}{2} \right]_0^1$$

$$= 2e^{\frac{1}{2}} - 1 + \frac{1}{2} - 2e^0$$

$$= 2e^{\frac{1}{2}} - \frac{5}{2}$$

$$= \frac{1}{2}\left(4e^{\frac{1}{2}} - 5 \right) \text{units}^2$$

OR

$$A = \int_0^1 e^{\frac{x}{2}}\, dx - \frac{1}{2} \times 1 \times 1$$

$$= \left[2e^{\frac{x}{2}} \right]_0^1 - \frac{1}{2}$$

$$= 2e^{\frac{1}{2}} - 2e^0 - \frac{1}{2}$$

$$= 2e^{\frac{1}{2}} - \frac{5}{2}$$

$$= \frac{1}{2}\left(4e^{\frac{1}{2}} - 5 \right) \text{units}^2$$

</td>
<td>3</td>
<td>Correct solution</td>
</tr>
<tr>
<td>2</td>
<td>Correct primitive</td>
</tr>
<tr>
<td>1</td>
<td>Correct expression for the area, must show understanding of $e^0 = 1$</td>
</tr>
</table>

Question 13			Marks
(a)	Number of ways that Leon goes into café after Anna $=\dfrac{\text{Total number of ways 7 people enter cafe}}{2}$ $=\dfrac{7!}{2}$ $=2520$	**1**	Correct solution
(b)	$\int_0^{\frac{\pi}{4}} \sin y \cos y \, dy$ $=\frac{1}{2}\int_0^{\frac{\pi}{4}} \sin 2y \, dy$ $=\frac{1}{2}\left[-\frac{1}{2}\cos 2y\right]_0^{\frac{\pi}{4}}$ $=-\frac{1}{4}\left[\cos\frac{\pi}{2}-\cos 0\right]$ $=-\frac{1}{4}[0-1]$ $=\frac{1}{4}$ OR $\int_0^{\frac{\pi}{4}} \sin y \cos y \, dy$ $=\left[\frac{\sin^2 y}{2}\right]_0^{\frac{\pi}{4}}$ $=\frac{1}{2}\left[\left(\sin\frac{\pi}{4}\right)^2-(\sin 0)^2\right]$ $=\frac{1}{2}\left[\left(\frac{1}{\sqrt{2}}\right)^2-0\right]$ $=\frac{1}{2}\left(\frac{1}{2}\right)$ $=\frac{1}{4}$	**3**	Correct solution
		2	Correct primitive
		1	Correct substitution of expression involving $\sin 2y$ OR Correct primitive of form $k\sin^2 y$ OR Correct primitive of form $k\cos^2 y$ OR Correct change of variable using $u=\sin y$

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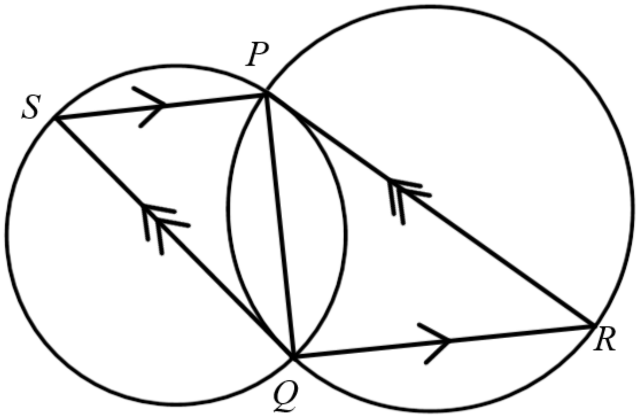
<tr><td rowspan="2">(c)(i)</td><td rowspan="2">

Join PQ
$\angle PQR = 90°$
(angle in a semi-circle is 90°)
$\angle PQR = \angle SPQ = 90°$
(equal alternate angles, $SP \parallel QR$)
$\therefore QS$ is a diameter
($\angle SPQ = 90°$ and angle in a semi-circle is 90°)</td><td>2</td><td>Correct proof</td></tr>
<tr><td>1</td><td>Correctly shows
$\angle PQR = 90°$
OR
Correct proof not fully justified</td></tr>
<tr><td rowspan="2">(c)(ii)</td><td rowspan="2">$SPRQ$ is a parallelogram
(given $SP \parallel QR$ and $SQ \parallel PR$)
$SQ = PR$
(opposite sides in parallelogram $SPRQ$ are equal)
$\therefore$ diameters of both circles, SQ and PR, are equal
$\therefore$ circles have equal radii</td><td>2</td><td>Correct solution</td></tr>
<tr><td>1</td><td>Shows $SQ = PR$ without full justification</td></tr>
<tr><td rowspan="3">(d)</td><td rowspan="3">$\int \cos^2 3x\,dx$
$\cos 6x = 2\cos^2 3x - 1$
$\cos^2 3x = \frac{1}{2}\cos 6x + \frac{1}{2}$
$\int \cos^2 3x\,dx$
$= \int \left(\frac{1}{2}\cos 6x + \frac{1}{2}\right) dx$
$= \frac{1}{2}.\frac{1}{6}\sin 6x + \frac{1}{2}x + C$
$= \frac{1}{12}\sin 6x + \frac{1}{2}x + C$</td><td>3</td><td>Correct solution. ISE.</td></tr>
<tr><td>2</td><td>Correct substitution of expression involving $\cos 6x$
OR
Substantially correct solution</td></tr>
<tr><td>1</td><td>Correct attempt at substitution of expression involving $\cos 6x = 2\cos^2 x - 1$</td></tr>
</table>

<table>
<tr>
<td rowspan="4">(e)</td>
<td rowspan="4">$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sec^2 x}{\tan x}\, dx$
$u = \tan x$
$\frac{du}{dx} = \sec^2 x$
$du = \sec^2 x\, dx$
$x = \frac{\pi}{3} : u = \tan \frac{\pi}{3}$
$u = \sqrt{3}$
$x = \frac{\pi}{6} : u = \tan \frac{\pi}{6}$
$u = \frac{1}{\sqrt{3}}$
$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sec^2 x}{\tan x}\, dx$
$= \int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} \frac{du}{u}$
$= \left[\log_e \lvert u \rvert\right]_{\frac{1}{\sqrt{3}}}^{\sqrt{3}}$
$= \log_e \sqrt{3} - \log_e \frac{1}{\sqrt{3}}$
$= \log_e \left(\sqrt{3} \div \frac{1}{\sqrt{3}}\right)$
$= \log_e \left(\sqrt{3} \times \sqrt{3}\right)$
$= \log_e 3$</td>
<td>4</td>
<td>Correct solution</td>
</tr>
<tr>
<td>3</td>
<td>Correct evaluation of integral but not fully simplified</td>
</tr>
<tr>
<td>2</td>
<td>Correct primitive</td>
</tr>
<tr>
<td>1</td>
<td>Correct change of variable in integral</td>
</tr>
</table>

<table>
<tr><th colspan="2">Question 14</th><th></th><th>Marks</th></tr>
<tr><td>(a)(i)</td><td>

$$\lim_{x\to\infty}\left(\frac{1}{1+x^2}\right)$$

$$=\lim_{x\to\infty}\left(\frac{\frac{1}{x^2}}{\frac{1}{x^2}+1}\right)$$

$$=\frac{0}{0+1}$$

$$=0$$

$\therefore\ f(x)$ has horizontal asymptote at $y=0$

OR

$$y=\frac{1}{1+x^2}$$

$$y(1+x^2)=1$$

$$y\neq 0$$

$\therefore\ f(x)$ has horizontal asymptote at $y=0$

</td><td>1</td><td>Correct solution</td></tr>
<tr><td rowspan="2">(a)(ii)</td><td rowspan="2">

$$f(x)=\left(1+x^2\right)^{-1}$$

$$f'(x)=-\left(1+x^2\right)^{-2}(2x)$$

$$=\frac{-2x}{\left(1+x^2\right)^2}$$

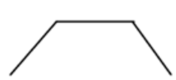
Stationary points $\Rightarrow f'(x)=0$

$$-2x=0$$

$$x=0$$

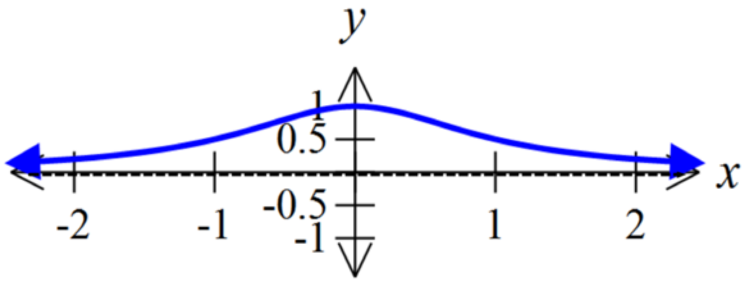
$\therefore y=f(x)$ has only one stationary point $(0,1)$

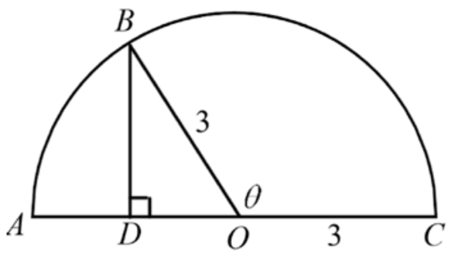
<table>
<tr><td>x</td><td>-1</td><td>0</td><td>1</td></tr>
<tr><td>$f'(x)$</td><td>$\frac{1}{2}$</td><td>0</td><td>$-\frac{1}{2}$</td></tr>
</table>



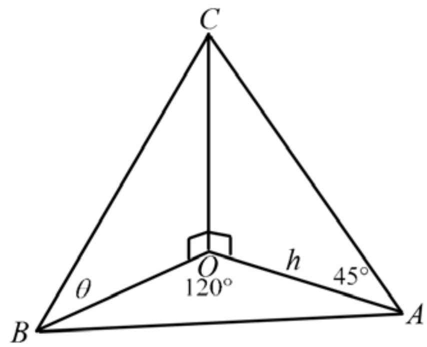
Maximum turning point at (0, 1)

</td><td>2</td><td>Correct solution</td></tr>
<tr><td>1</td><td>Show correctly that there is only one stationary point</td></tr>
</table>

(a)(iii)		**1**	Correct graph
(a)(iv)	As $y = f(x)$ for $x \geq 0$ is a one-to-one mapping, it has an inverse function	**1**	Correct solution
(a)(v)	$y = \dfrac{1}{1+x^2}$ $D: x \geq 0$ and $R: 0 < y \leq 1$ $x = \dfrac{1}{1+y^2}$ $D: 0 < x \leq 1$ and $R: y \geq 0$ $x(1+y^2) = 1$ $x + xy^2 = 1$ $xy^2 = 1 - x$ $y^2 = \dfrac{1-x}{x}$ $y = \pm\sqrt{\dfrac{1-x}{x}}$ As $y \geq 0$, $f^{-1}(x) = \sqrt{\dfrac{1-x}{x}}$	**2**	Correct solution where positive solution must be justified by $y \geq 0$
		1	Substantially correct solution shown by $y^2 = \dfrac{1}{x} - 1$ or $y^2 = \dfrac{1-x}{x}$

<table>
<tr><td rowspan="3">(b)(i)</td><td rowspan="3">

$\angle BOD = 180° - \theta$
$\sin(180° - \theta) = \dfrac{BD}{3}$
$BD = 3\sin(180° - \theta)$
$BD = 3\sin\theta$ as $\sin(180° - \theta) = \sin\theta$
Length of arc $BC = 3\theta$
Length of arc $BC = 2BD$
$3\theta = 2(3\sin\theta)$
$\therefore \theta = 2\sin\theta$</td><td>3</td><td>Correct solution</td></tr>
<tr><td>2</td><td>Substantially correct solution</td></tr>
<tr><td>1</td><td>Correctly finds the length of arc BC
OR
Makes a correct attempt to find the length of BD by showing
$\sin(180° - \theta) = \dfrac{BD}{3}$</td></tr>
<tr><td rowspan="2">(b)(ii)</td><td rowspan="2">Let $f(\theta) = \theta - 2\sin\theta$
$f'(\theta) = 1 - 2\cos\theta$
$x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$
$\theta_1 = 1.7$
$\theta_2 = 1.7 - \dfrac{1.7 - 2\sin 1.7}{1 - 2\cos 1.7}$
$= 1.925...$
$= 1.93$ (2 dp)</td><td>2</td><td>Correct solution</td></tr>
<tr><td>1</td><td>Correct attempt at solution</td></tr>
</table>

(c)	 In ΔOAC, $\tan 45° = \dfrac{OC}{h}$ $OC = h$ as $\tan 45° = 1$ In ΔOBC, $\tan\theta = \dfrac{OC}{OB}$ $OB = \dfrac{h}{\tan\theta}$ as $OC = h$ In ΔABO, $AB^2 = AO^2 + BO^2 - 2(AO)(BO)\cos 120°$ $AB^2 = h^2 + \left(\dfrac{h}{\tan\theta}\right)^2 - 2h\left(\dfrac{h}{\tan\theta}\right)(-\cos 60°)$ $AB^2 = h^2 + \dfrac{h^2}{\tan^2\theta} - \dfrac{2h^2}{\tan\theta}\left(-\dfrac{1}{2}\right)$ $AB^2 = h^2 + \dfrac{h^2}{\tan^2\theta} + \dfrac{h^2}{\tan\theta}$	**3**	Correct solution clearly showing that $\cos 120° = -\dfrac{1}{2}$
		2	Correctly finds both *OC* and *OB* in simplified form and substitutes these values into cosine rule
		1	Explains why $OC = h$ OR Shows $OB = \dfrac{h}{\tan\theta}$ OR Correctly substitutes their *OC* and *OB* into correct cosine rule

Communication:

Question 11

(a)	Showing vertex on graph	1 mark
(c)	Correctly explaining $"\text{If } \frac{a}{b} > 0, \text{ then } ab > 0"$ or showing multiplication of $(x-5)^2$	1 mark

Question 12

(a)(i)	Correct setting out of solution for "show" question	1 mark
(a)(ii)	Correct stating new domain	1 mark
(b)(iii)	Correctly simplifying derivative to $\sec x \tan x$	1 mark
(c)	Showing balance of log by placing $\frac{1}{4}$ of the log	1 mark
(d)	Show set up of area between graphs by subtraction of curves	1 mark

Question 13

(b)	Correct notation used throughout the solution	1 mark
(c)(i)	Correct diagram	1 mark
	Fully justified solution	1 mark
(c)(ii)	Fully justified solution	1 mark
(e)	Correct use of absolute value notation when finding primitive $\ln\|u\|$	1 mark

Question 14

(a)(i)	Correct notation	1 mark
(a)(ii)	Uses first derivative test instead of finding second derivative	1 mark
(a)(iii)	Show asymptote	1 mark
(a)(v)	States clearly both domain for $y = f(x)$ which then is range for $y = f^{-1}(x)$	1 mark
(b)(i)	States that $\sin(180° - \theta) = \sin\theta$ or correctly expands $\sin(180° - \theta)$	1 mark
(b)(ii)	Uses radians not degrees	1 mark