Total marks (75)
Attempt Questions 1 – 6

Answer each question in a SEPARATE writing booklet.

QUESTION 1 (12 Marks)   Use a SEPARATE writing booklet.   **Marks**

(a)   The third term of an arithmetic series is *32* and the sixth term is *17*.   **3**

      (i)   Find the common difference.

      (ii)   Find the sum of the first ten terms.

(b)   The sum of the first *10* terms of the series $(x>0)$ **3**

$\log_2 (\frac{1}{x}) + \log_2 (\frac{1}{x^2}) + \log_2 (\frac{1}{x^3}) + \dots$ is *–440*.

Find the value of x.

(c)   Consider the series $\sin^2 x + \sin^4 x + \sin^6 x + \dots\dots, \quad 0 < x < \frac{\pi}{2}$ **3**

      (i)   Show that a limiting sum exists.

      (ii)   Find the limiting sum expressing the answer in simplest form.

(d)   Use the Principle of Mathematical Induction to show that $9^{n+2} - 4^n$ **3**

is divisible by *5* for all positive integers n.

QUESTION 2 (12 Marks)   Use a SEPARATE writing booklet.   **Marks**

(a)   The polynomial $P(x) = x^3 - 6x^2 + kx + 14$ has a zero at $x = 1$. Determine   **1**
       the value of the constant k, and for this value of k find:

    (i)   the linear factors $P(x)$;   **2**

    (ii)   the roots of the equation $P(x) = 0$;   **1**

    (iii)   the set of values of x for which $P(x) > 0$.   **2**

(b)   The equation $x^3 - 2x^2 + 4x - 5 = 0$ has roots $\alpha, \beta \text{ and } \gamma$.

    (i)   Write down the values of $\alpha\beta + \alpha\gamma + \beta\gamma$ and $\alpha\beta\gamma$.   **2**

    (ii)   Hence find the value of $\alpha^{-1} + \beta^{-1} + \gamma^{-1}$.

                                                                                                                                                                                     1

(c)   Consider the equation $x^3 + 6x^2 - x - 30 = 0$. One of the roots of this   **3**
       equation is equal to the sum of the other two roots. Find the values
       of the three roots.

QUESTION 3   (12 Marks)   Use a SEPARATE writing booklet.   **Marks**

(a)   Differentiate $\log_e\left(\dfrac{x}{x-1}\right)$ with respect to x.   **2**

(b)   Find $\displaystyle\int \dfrac{x+1}{x}\,dx$.   **2**

(c)   Evaluate $\displaystyle\int_0^{\ln 2} \dfrac{e^x}{e^x+1}\,dx$.   **2**

(d)   (i)   Show that $\dfrac{d}{dx}(x\ln x) = 1 + \ln x$.   **1**

       (ii)   Hence, or otherwise, find $\displaystyle\int \ln x\,dx$.   **2**

(e)   Use Simpson's rule with 5 function values to approximate   **3**

$$\int_1^5 x\log_e x\,dx,$$ correct to two decimal places.

QUESTION 4   (12 Marks)   Use a SEPARATE writing booklet.   **Marks**

(a)   Find the exact values of

     (i) $\sin^{-1}\left(\cos\dfrac{\pi}{6}\right)$.   1

     (ii) $\cos\left(2\sin^{-1}\dfrac{3}{7}\right)$.   2

(b)   Consider the function $f(x) = \log_e(2x+1)$.

     (i)   Write down the domain of $f(x)$.   1

     (ii)   Find the inverse function of $f(x)$, and write it in the form   2
           $f^{-1}(x) = \ldots\ldots$

     (iii)   On the same diagrams, draw the graphs of $y = f(x)$ and   2
           $y = f^{-1}(x)$.

(c)   Consider the function $f(x) = 2\sin^{-1}\left(\dfrac{x}{3}\right)$.

     (i)   State the domain and range.   2

     (ii)   Sketch the graph of the function.   2

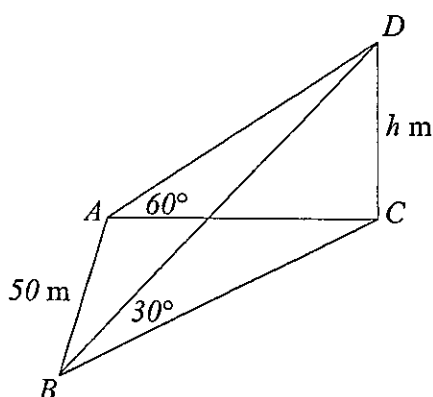**QUESTION 5**   (12 Marks)   Use a SEPARATE writing booklet.   **Marks**

(a)   Find $\dfrac{d}{dx}\left(x\,\cos^{-1}2x\right).$ **2**

(b)   Find $\displaystyle\int \dfrac{1}{\sqrt{4-x^2}}\,dx.$ **1**

(c)   Evaluate $\displaystyle\int_0^{\frac{1}{\sqrt{3}}} \dfrac{1}{1+9x^2}\,dx.$ **3**

(d)   **3**

A vertical flagpole CD of height h metres stands with its base C on horizontal ground. A is a point on the ground due West of C and B is a point on the ground 50 metres due South of A. From A and B the angles of elevation of the top D of the flagpole are 60° and 30° respectively. Find the height of the flagpole correct to the nearest metre.

(e)   Prove $\tan^{-1}\left(\dfrac{2}{3}\right)+\cos^{-1}\left(\dfrac{2}{\sqrt{5}}\right)=\tan^{-1}\left(\dfrac{7}{4}\right).$ **3**

QUESTION 6 (15 Marks)   Use a SEPARATE writing booklet.   **Marks**

(a)   (i)   Express $\sqrt{3}\cos x - \sin x$ in the form $R\cos(x+\alpha)$

where $0 < \alpha < \dfrac{\pi}{2}$ and $R > 0$.   **2**

    (ii)   Hence, solve $\sqrt{3}\cos x - \sin x = 1$ for $0 \le x \le 2\pi$.   **3**

(b)   (i)   Show that $\displaystyle\int_0^{\frac{\pi}{4}} \cos^2 x \, dx = \dfrac{\pi+2}{8}$.   **3**

    (ii)   The region under the curve $y = \cos x + \sec x$, above the x axis   **4**

and between $x = 0$ and $x = \dfrac{\pi}{4}$, makes a revolution about the

x axis.

Show that the volume of the solid traced out is $\dfrac{5\pi(\pi+2)}{8}$ cubic units.

(c)   Prove that $\dfrac{\sin 5\theta}{\sin \theta} - \dfrac{\cos 5\theta}{\cos \theta} = 4\cos 2\theta$.   **3**

End of paper