Total marks – 70

QUESTION 1-10 Objective-response questions. *(10marks)*

Write answers on the Objective response answers booklet

Multiple Choice

1. For what value of k is $x-1$ a factor of $2x^3-3x^2+kx-5$

(A) -10 (B) -4

(C) 6 (D) 10

2. The point P divides the interval from A(–2,2) to B(8,–3) internally in the ratio 3:2 What is the x-coordinate of P?

(A) 4 (B) 2

(C) 0 (D) –1

3. What is the value $\sin 2\theta$, given $\sin\theta = \frac{3}{5}$ and $\sin\theta > 0$?

(A) $\frac{6}{5}$ (B) $\frac{24}{25}$ (C) $\frac{4}{5}$ (D) $\frac{12}{25}$

4. When the polynomial $P(x)$ is divided by x^2-2x-3 the remainder is $2x-5$. What is the remainder when $P(x)$ is divided by $x+1$?

(A) -11 (B) -7 (C) -3 (D) 1

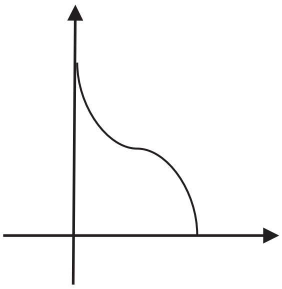
5. $\tan(\alpha-\beta)=$

(A) $\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$ (B) $\dfrac{\tan\alpha\times\tan\beta}{1-\tan\alpha\tan\beta}$

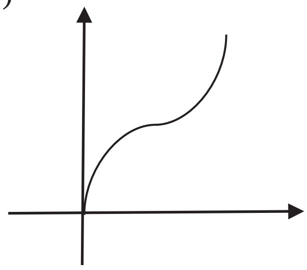
(C) $\dfrac{\tan\beta-\tan\alpha}{1+\tan\beta\tan\alpha}$ (D) $\dfrac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$

6. Which graph best represents $y = \cos^{-1}(1 - x)$

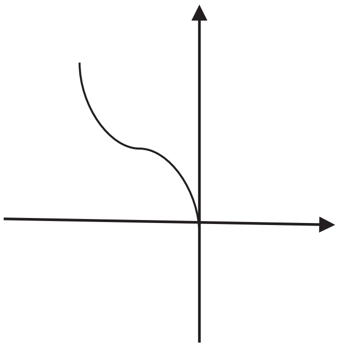
(A)



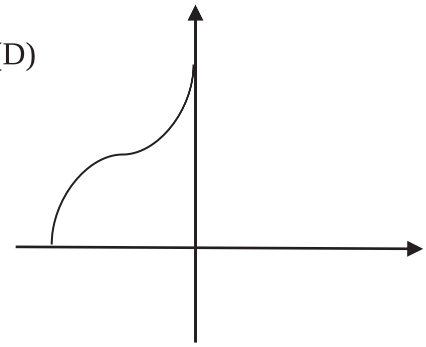
(B)



(C)



(D)



7. $\displaystyle\int_0^{\pi} \frac{\cos x}{\sqrt{1-\sin^2 x}}dx$ is <u>not</u> equal to

(A) $\left[\sin^{-1}(\cos x)\right]_0^{\pi}$ (B) $\displaystyle\int_0^{\pi} 1dx$

(C) $-\pi$ (D) $\left[\ln(\cos x)\right]_0^{\pi}$

8. Which is <u>not</u> a polynomial

(A) $x^3 - 5x + 1$ (B) $x^4 + 3x^3 - \sqrt{3}x^2 + x - 1$

(C) $x^3 + 3x^2 + 2x - 3x^{-1}$ (D) $(3x - 1)^3$

9. The general solution to $\sqrt{3}\sin x - \cos x = 0$ where n is an integer is?

(A) $\frac{\pi}{3} + n\pi$

(B) $\frac{(6n+1)\pi}{6}$

(C) $\frac{\pi}{3} + 2n\pi$ or $\frac{2\pi}{3} + 2n\pi$

(D) $\frac{\pi}{6} + 2n\pi$ or $\frac{2\pi}{6} + 2n\pi$

10. Which expression best represents the primitive function for $y = \cos^2 x$.

(A) $\frac{1}{4}\sin 2x + \frac{x}{2} + c$

(B) $\frac{1}{2}(\sin 2x + x) + c$

(C) $\frac{1}{4}\sin^2 x + c$

(D) $\frac{1}{4}\sin 2x - \frac{x}{2} + c$

End of Question Multiple Choice

QUESTION 11 *(16 marks)* ***START A NEW BOOKLET*** **Mark**

(a) For A(3,–1) and B(a,2), the point (13,4) divides the interval AB externally in the ratio 5:2. Find the value of a. **2**

(b) A curve has parametric equations $x = \frac{t}{3}$, $y = 2t^2$. Write down the Cartesian equation for this curve. **2**

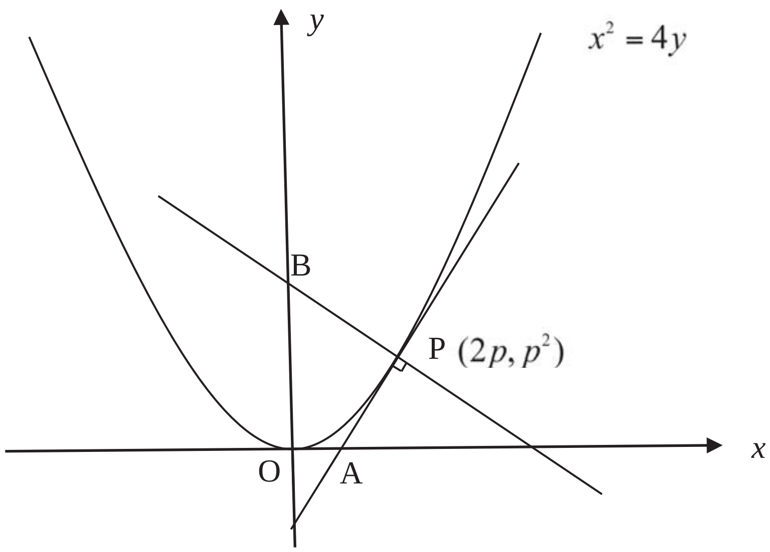
(c) Solve the inequality $\frac{3}{x-2} \leq 1$ **3**

(d) For $n = 1, 2, 3, ...$ let **3**

$$S_n = 1^2 + 2^2 + ... + n^2.$$

Use mathematical induction to prove $S_n = \frac{1}{6}n(n+1)(2n+1)$.

(e) The diagram shows the graph of the parabola $x^2 = 4y$. The tangent to the parabola at $P(2p, p^2)$, $p > 0$, cuts the x axis at A. The normal to the parabola at P cuts the y axis at B



(i) Derive the equation of the tangent AP. **2**

(ii) Show that B has coordinates $(0, p^2 + 2)$. **2**

(iii) Let C be the midpoint of AB. Find the Cartesian equation of the locus of C. **2**

QUESTION 12 *(13 marks)* ***START A NEW BOOKLET*** **Marks**

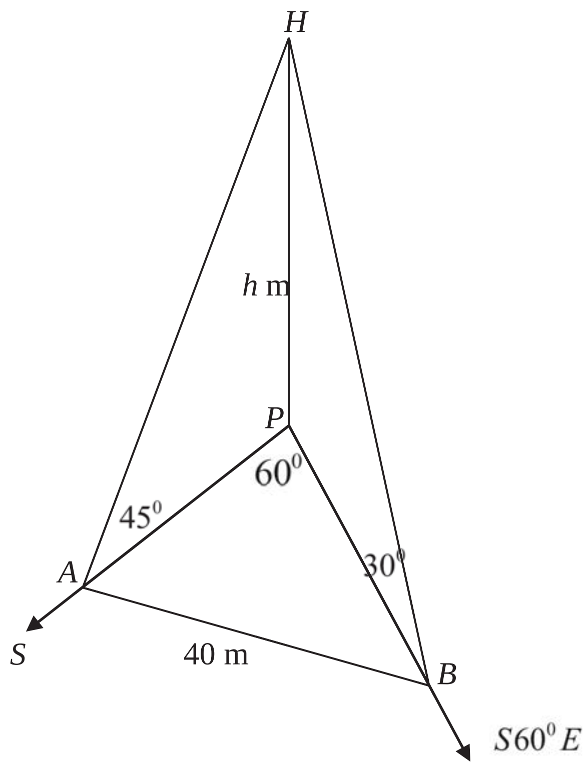
(a) (i) Express $\sin x - \cos x$ in the form $R\sin(x-\alpha)$. **2**

(ii) Hence solve the equation $\sin x - \cos x = \frac{\sqrt{6}}{2}$, $0 \le x \le 2\pi$. **2**

(b) Evaluate $\int_0^{\frac{\pi}{4}} 2\sin^2 x\,dx$. **2**

(c) Using the substitution $t = \tan\frac{\theta}{2}$, show $\frac{\sin\theta}{1-\cos\theta} = \cot\frac{\theta}{2}$. **2**

(d)



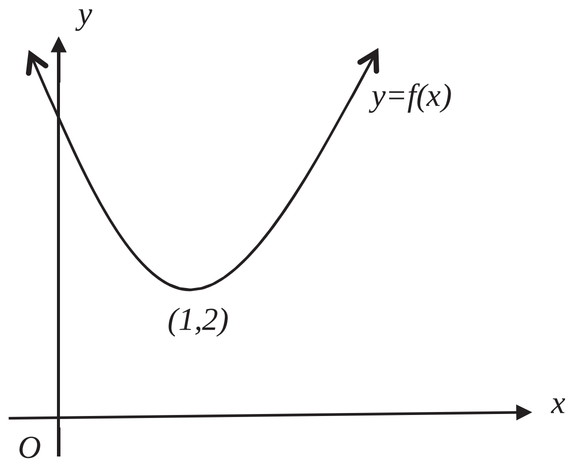
From point *A* the angle of elevation to the top of the tower HP is 45^0. A is due south of *P*. From point *B* the angle of elevation to the top of the tower is 30^0 and *B* is $S60^0E$ of P. AB is a distance of 40 metres.

(i) Show that $BP = \sqrt{3}h$ and $AP = h$, where the tower is *h* metres high. **2**

(ii) Show that $h = \sqrt{\frac{1600}{4-\sqrt{3}}}$ metres. **3**

QUESTION 13 *(15 marks)* ***START A NEW BOOKLET*** **Marks**

(a) The graph of $f(x) = x^2 - 2x + 3$ is shown in the diagram.



(i) Explain why f(x) does not have an inverse function. **1**

(ii) Sketch the graph of the inverse function $g^{-1}(x)$ of $g(x)$, where **1**

$g(x) = x^2 - 2x + 3,\ x \geq 1.$

(b) (i) Find the domain and range for the function $y = 3\sin^{-1}\dfrac{x}{2}$. **2**

(ii) Sketch $y = 3\sin^{-1}\dfrac{x}{2}$. **2**

(c) Find the exact values of each of the following, showing all working.

(i) $\sin^{-1}\left(\sin\dfrac{4\pi}{3}\right)$ **1**

(ii) $\sin\left(2\cos^{-1}\dfrac{1}{3}\right)$. **2**

(d) (i) Differentiate $\sin^{-1}\dfrac{1}{2}x^3$. **2**

(ii) Hence find $\displaystyle\int \frac{x^2}{\sqrt{4 - x^6}}\,dx$ **2**

(e) Evaluate $\displaystyle\int_0^3 \frac{5}{9 + x^2}\,dx$ **2**

QUESTION 14 *(16 marks)* ***START A NEW BOOKLET*** **Marks**

(a) When $P(x) = 3x^3 - x^2 + x + a$ is divided by $(x-1)$ the remainder is 3. Find a. **1**

(b) The polynomial $P(x) = x^3 + bx^2 + cx + d$ has roots -1, 2, 3. Find b, c, and d. **3**

(c) Sketch the following polynomial. Clearly show all intercepts.

$$P(x) = x(x-3)^2(x+2)^3.$$ **3**

(d) The polynomial equation $x^3 + 7x^2 - 2x + 3 = 0$ has 3 roots, α, β, γ

(i) Find $\alpha + \beta + \gamma$ **1**

(ii) Find $\alpha\beta + \beta\gamma + \gamma\alpha$ **1**

(iii) Find $\alpha^2 + \beta^2 + \gamma^2$ **2**

(e) It is known that two of the roots of the equation $2x^3 + x^2 - kx + 6 = 0$ are reciprocals of each other. Find the value of k. **2**

(f) Given that $(x-1)$ and $(x+3)$ are factors of $P(x) = x^4 - 2x^3 - 32x^2 - 30x + 63$, express $P(x)$ in factorised form. **3**

END OF THE PAPER

Y12 Ext 1 Mini Exam Ext1 2013

MC 1/C 2/A 3/B 4/B 5/D 6/B 7/D 8/C 9/B 10/A

Q11

(a) use ratio $5:-2$

$\left(\frac{-2\times 3+5\times a}{5-2},\ \right)=(13,4)$

$\therefore \frac{-6+5a}{3}=13$

$-6+5a=39$

$5a=45$

$a=9$

many did not know formula.

(b) $x=\frac{t}{3}\quad y=2t^2$

$t=3x \quad \therefore y=2(3x)^2$

$y=18x^2$

Done well.

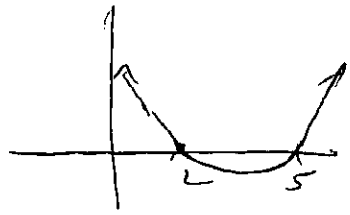
(c) $x(x-2)^2 \quad 3(x-2)\le(x-2)^2$

$x\neq 2 \quad (x-2)^2-3(x-2)\geq 0$

$(x-2)(x-2-3)\geq 0$

$(x-2)(x-5)\geq 0$

$x<2 \quad x\geq 5$



(many forgot $x\neq 2$)

(d) $1^2+2^2+\ldots+n^2=\frac{1}{6}n(n+1)(2n+1)$

Show true for $n=1$

$LHS=1^2$

$RHS=\frac{1}{6}\times 1\times 2\times 3$

$=1$

$LHS=RHS \quad \therefore$ true for $n=1$

Some did not clearly show $LHS=1^2$ and did not get mark.

Let it be true for $n=k$

$\therefore 1^2+2^2+\ldots+k^2=\frac{1}{6}k(k+1)(2k+1) \quad *$

show true for $n=k+1$

$1^2+2^2+\ldots+k^2+(k+1)^2=\frac{1}{6}(k+1)(k+2)(2k+3)$

$LHS=1^2+2^2+\ldots+k^2+(k+1)^2$

$=\frac{1}{6}(k(k+1)(2k+1)+(k+1)^2$

$=\frac{1}{6}(k+1)[k(2k+1)+6(k+1)]$

$=\frac{1}{6}(k+1)(2k^2+k+6k+6)$

$=\frac{1}{6}(k+1)(2k^2+7k+6)$

$=\frac{1}{6}(k+1)(k+2)(2k+3)$

$=RHS$

Many expanded at this point

On the whole poorly done for such an easy induction question.

$\therefore$ By mathematical induction true for all $n\geq 1$

(e) (i) $y = \frac{x^2}{4}$

$\frac{dy}{dx} = \frac{x}{2}$

This whole question was generally done well.

At $x = 2p$

$\frac{dy}{dx} = p$. ✓ – some quoted this result so did not get this mark.

Equation of tangent $y - p^2 = p(x - 2p)$

$y - p^2 = px - 2p^2$

$y = px - p^2$ ✓

(ii) ~~when x =~~ gradient of normal is $-\frac{1}{p}$.

Equation of normal $y - p^2 = -\frac{1}{p}(x - 2p)$ ✓

When $x = 0$ $\quad y - p^2 = -\frac{1}{p}x - 2p$. ✓

$y = p^2 + 2$ $\quad$ (could be shown by using $y = mx + b$).

$\therefore$ B is $(0, p^2+2)$

(iii) A is where $y = 0$ on tangent.

ie $px - p^2 = 0$

$px = p^2$

$x = p$

ie A is $(p, 0)$. ✓

mid point of AB is $\left(\frac{p}{2}, \frac{p^2+2}{2}\right)$

ie $x = \frac{p}{2}$ $\quad y = \frac{p^2+2}{2}$

$p = 2x$ $\quad y = \frac{(2x)^2+2}{2}$

$y = \frac{4x^2+2}{2}$

$y = 2x^2 + 1$. ✓

Q12 (13)

a) i)

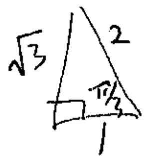
$$R\sin(x-a) = R(\sin x\cos\alpha - \cos x\sin\alpha)$$

$$\sin x - \cos x = \sqrt{2}\left(\sin x \tfrac{1}{\sqrt{2}} - \cos x \tfrac{1}{\sqrt{2}}\right)$$

$$= \sqrt{2}\sin\left(x - \tfrac{\pi}{4}\right) \checkmark$$

$\checkmark$

ii)



$$\sqrt{2}\sin\left(x-\tfrac{\pi}{4}\right) = \frac{\sqrt{6}}{2}$$

$$\sin\left(x-\tfrac{\pi}{4}\right) = \frac{\sqrt{3}}{2}$$

$$\left(x-\tfrac{\pi}{4}\right) = \frac{\pi}{3}, \frac{2\pi}{3} \checkmark$$

$$x = \frac{7\pi}{12}, \frac{11\pi}{12} \checkmark$$

b)

$$\int_0^{\pi/4} 2\left(\tfrac{1}{2}(1-\cos 2x)\right)dx = \int_0^{\pi/4}\left(1 - \cos 2x\right)dx$$

$$= \left[x - \tfrac{1}{2}\sin 2x\right]_0^{\pi/4} \checkmark$$

$$= \frac{\pi}{4} - \frac{1}{2} - (0-0)$$

$$= \frac{\pi}{4} - \frac{1}{2} \checkmark$$

c)

$$\sin\theta = \frac{2t}{1+t^2}, \quad \cos\theta = \frac{1-t^2}{1+t^2}$$

$$\text{LHS} = \frac{\sin\theta}{1-\cos\theta}$$

$$= \frac{\frac{2t}{1+t^2}}{1 - \frac{1-t^2}{1+t^2}} \checkmark$$

$$= \frac{2t}{1+t^2-(1-t^2)} \checkmark$$

$$= \frac{2t}{2t^2}$$

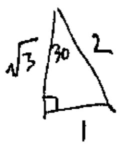
$$= \frac{1}{t}$$

$$= \cot\frac{\theta}{2}$$

$$= \text{RHS}$$

d) i) $\tan 30^\circ = \frac{h}{BP}$ ✓

$BP = \frac{h}{\tan 30}$



$= \sqrt{3}h$

$\tan 45^\circ = \frac{h}{AP}$ ✓

$AP = \frac{h}{\tan 45^\circ}$

$= h$

ii) $AB^2 = AP^2 + BP^2 - 2 \times AP \times BP \cos 60^\circ$ ✓

$40^2 = h^2 + (\sqrt{3}h)^2 - 2 \times h \times \sqrt{3}h \times \frac{1}{2}$ ✓

$1600 = h^2 + 3h^2 - \sqrt{3}h^2$

$1600 = 4h^2 - \sqrt{3}h^2$

$h^2(4 - \sqrt{3}) = 1600$ ✓

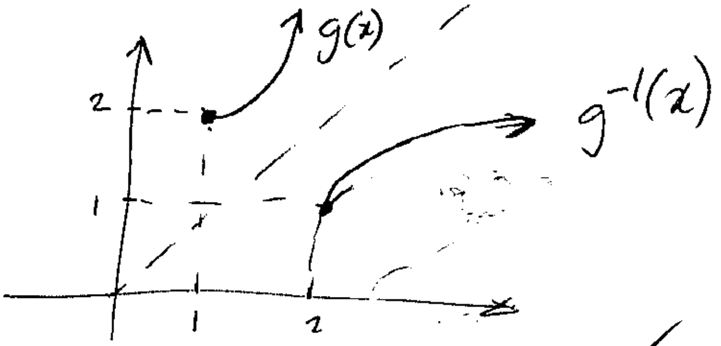
$h^2 = \frac{1600}{4 - \sqrt{3}}$

$h = \sqrt{\frac{1600}{4 - \sqrt{3}}}$

Q13 a) i) – there are y values which have two x values ✓

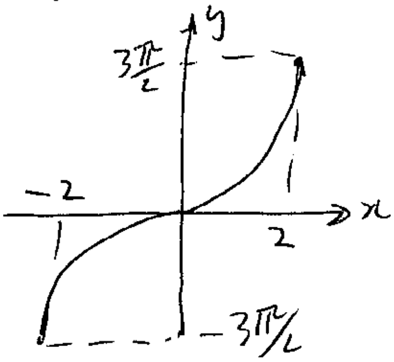
– Horizontal line test

ii)



✓

b) range $-\frac{3\pi}{2} \le y \le \frac{3\pi}{2}$ ✓ domain $-2 \le x \le 2$ ✓



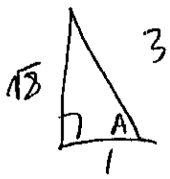
✓ ✓

c) i) $\sin^{-1}\left(\sin\frac{4\pi}{3}\right) = -\frac{\pi}{3}$ ✓

ii) $\sin\left(2\cos^{-1}\frac{1}{3}\right) = 2\sin A\cos A$

$= 2 \times \frac{\sqrt{8}}{3} \times \frac{1}{3}$

$= \frac{4\sqrt{2}}{9}$



d) $\frac{d}{dx}\sin^{-1}\frac{1}{2}x^3 = \frac{1}{\sqrt{1-\left(\frac{x^3}{2}\right)^2}} \times \frac{3x^2}{2}$ ✓

$= \frac{3x^2}{2\sqrt{1-\frac{x^6}{4}}} = \frac{3x^2}{\sqrt{4-x^6}}$ ✓

Q13 d) iv $\displaystyle\int \frac{x^2}{\sqrt{4-x^6}}\,dx = \frac{1}{3}\int \frac{3x^2}{\sqrt{4-x^6}}\,dx$ ✓

$$= \frac{1}{3}\sin^{-1}\frac{1}{2}x^3 + C \checkmark$$

e) $\displaystyle\int_0^3 \frac{5}{9+x^2}\,dx = \frac{5}{3}\left[\tan^{-1}\frac{x}{3}\right]_0^3$ ✓

$$= \tfrac{5}{3}\left(\tan^{-1}1 - \tan^{-1}0\right)$$

$$= \frac{5\pi}{12} \checkmark$$

214 (a) $a = 0$

Ext 1 Mini

(b) $-1+2+3 = -b$

$\therefore b = -4$

$-1.2.3 = -d$

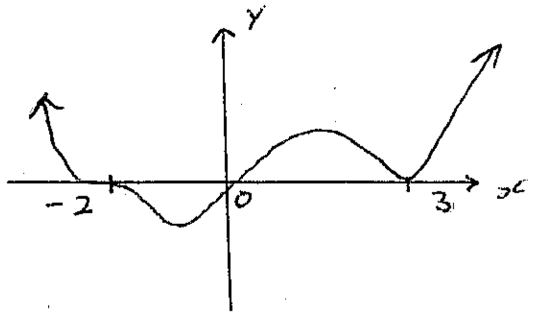
$\therefore d = 6$

$(-1)^3 - 4(-1)^2 + c(-1) + 6 = 0$

$-1 - 4 - c + 6 = 0$

$\therefore c = 1$

(c)



(d) (i) -7

(ii) -2

(iii) $(-7)^2 - 2(-2) = 53$

(e) Let roots be $\alpha, \frac{1}{\alpha}, \beta$

$\alpha \cdot \frac{1}{\alpha} \cdot \beta = -\frac{6}{2}$ ——— ①

$\therefore \beta = -3$

$\alpha + \frac{1}{\alpha} + \beta = -\frac{1}{2}$ ——— ②

$\therefore \alpha + \frac{1}{\alpha} = \frac{5}{2}$

$\alpha \cdot \frac{1}{\alpha} + \alpha \cdot \beta + \frac{1}{\alpha} \cdot \beta = -\frac{k}{2}$

$\therefore k = -2\left(1 + \beta\left(\alpha + \frac{1}{\alpha}\right)\right)$

$= -2\left(1 - 3\left(\frac{5}{2}\right)\right)$

$= 13$

(f) $1; -3, \alpha, \beta = 63 \Longrightarrow \alpha\beta = -21$

$1 - 3 + \alpha + \beta = 2 \Longrightarrow \alpha + \beta = 4$

by inspection, $\alpha = 7, \beta = -3$

$\therefore$

$P(x) = (x-1)(x+3)(x-7)(x+3)$

$= (x-1)(x-7)(x+3)^2$

Marks

(a) 1 mark for answer

(b) 1 mark for each answer.

(c) 1 mark for a "correct" intercept at $x = -2$ or $x = 3$.

2 marks for two "correct" intercepts at $x = 2$ and $x = 3$.

3 marks for correct graph.

(d) (i) 1 mark for answer

(ii) 1 mark for answer

(iii) 1 mark for demonstrating the use of:

$\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \alpha\gamma + \beta\gamma)$

1 mark for "correct" answer.

(e) 1 mark for $\beta = -3$

1 mark for $k = 13$

(f) Most (all?) students used long division.

1 mark for making reasonable progress with long division.

2 marks for correct division of $P(x)$ by $(x-1)(x+3)$

3 marks for correct answer.