Student Number: ....

NEWINGTON COLLEGE

2014
Assessment 2 (HSC mini)

Year 12 Mathematics Extension 1

General Instructions:

- Date of task - Tuesday 8th April (Wk 11A)
- Reading time – 5 mins
- Working time – 120 mins
- Weighting - 30%
- Board-approved calculators may be used.
- A table of standard integrals is provided at the back of the paper.
- Attempt all questions.
- Show all relevant mathematical reasoning and/or calculations.

Total marks – 70

Section I (10 marks)

- Answer questions 1 to 10 on the multiple choice answer sheet provided at the end of this paper.
- Allow about 15 minutes for this section.

Section II (60 marks)

- Answer questions 11 to 14 on the writing paper provided.
- **Start each question on a new page.**
- Each page must show the candidate's computer number.

Outcomes to be assessed:

HE1 Appreciates interrelationships between ideas drawn from different areas of mathematics.

HE4 Uses the relationship between functions, inverse functions and their derivatives.

HE6 Determines integrals by reduction to standard from through a given substitution.

Section I 10 marks

Attempt Questions 1-10

Allow about 15 minutes for this section.

Use the multiple-choice answer sheet (tear off at end of paper).

1 Which of the following is an expression for $\int \cos^2 x \sin x \, dx$?

Use the substitution $u = \cos x$.

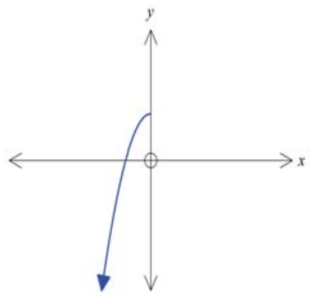
(A) $2\cos x \sin x + c$

(B) $\cos^3 x + c$

(C) $\frac{1}{3}\cos^3 x + c$

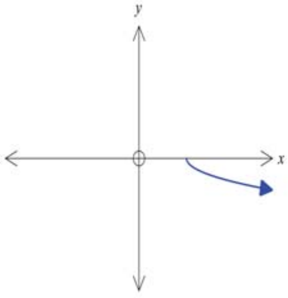
(D) $-\frac{1}{3}\cos^3 x + c$

2 The diagram shows the graph of $y = f(x)$.



Which diagram shows the graph of $y = f^{-1}(x)$?

(A)



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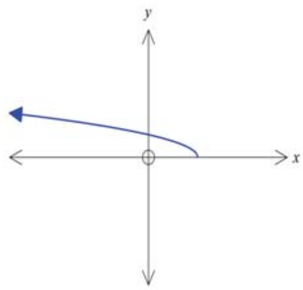

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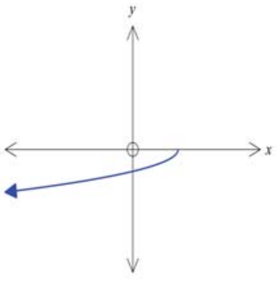

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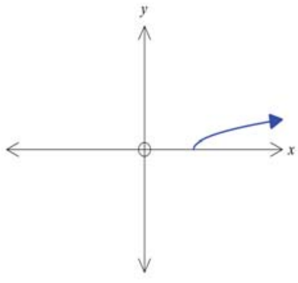


3 What is the solution to the equation $5(\cos(x+\tan^{-1}\frac{4}{3}))=0.5$ for $0\le x\le 2\pi$?

(A) $x=-7.21$ or $x=0.12$

(B) $x=-5.67$ or $x=2.82$

(C) $x=0.54$ or $x=3.89$

(D) $x=1.95$ or $x=4.12$

4 Which of the following is an expression for $\int \frac{e^{2x}}{e^x+1}dx$?

Use the substitution $u=e^x+1$.

(A) $\frac{(e^x+1)^2}{2}-e^x+c$

(B) $\frac{(e^x+1)^2}{2}+e^x+c$

(C) $e^x+1-\log_e(e^x+1)+c$

(D) $e^x+1+\log_e(e^x+1)+c$

5 Which of the following is an expression for $\int \sin^2 6x\,dx$?

(A) $\frac{x}{2}-\frac{1}{24}\sin 6x+c$

(B) $\frac{x}{2}+\frac{1}{24}\sin 6x+c$

(C) $\frac{x}{2}-\frac{1}{24}\sin 12x+c$

(D) $\frac{x}{2}+\frac{1}{24}\sin 12x+c$

6 What is the exact value of the definite integral $\int_0^{\pi}(\cos^2 x+1)dx$?

(A) $\frac{3\pi}{2}$

(B) $\frac{\pi}{2}$

(C) π

(D) $\frac{\pi}{4}$

7 If $f(x)=e^{x+2}$ what is the inverse function $f^{-1}(x)$?

(A) $f^{-1}(x)=e^{y-2}$

(B) $f^{-1}(x)=e^{y+2}$

(C) $f^{-1}(x)=\log_e x-2$

(D) $f^{-1}(x)=\log_e x+2$

8 What is the domain and range of $y=\cos^{-1}(\frac{3x}{2})$?

(A) Domain: $-\frac{2}{3}\le x\le\frac{2}{3}$. Range: $0\le y\le\pi$

(B) Domain: $-1\le x\le 1$. Range: $0\le y\le\pi$

(C) Domain: $-\frac{2}{3}\le x\le\frac{2}{3}$. Range: $-\pi\le y\le\pi$

(D) Domain: $-1\le x\le 1$. Range: $-\pi\le y\le\pi$

9 What is the value of $f'(x)$ if $f(x) = x\tan^{-1} x$?

(A) $\dfrac{1}{1+x^2}$

(B) $\dfrac{x}{1+x^2}$

(C) $\tan^{-1} x + \dfrac{1}{1+x^2}$

(D) $\tan^{-1} x + \dfrac{x}{1+x^2}$

10 Which of the following is the correct expression for $\displaystyle\int \frac{1}{\sqrt{49-x^2}}dx$?

(A) $-\cos^{-1}\dfrac{x}{7} + c$

(B) $-\cos^{-1} 7x + c$

(C) $-\sin^{-1}\dfrac{x}{7} + c$

(D) $-\sin^{-1} 7x + c$

End of Section I

Section II 60 marks

Attempt Questions 11-14

Allow about 1 hour and 45 minutes for this section.

Question 11 (15 marks) – Use a SEPARATE writing booklet.

(a) Find $\lim_{x \to 0} \dfrac{2\sin\frac{x}{2}}{3x}$. 2

(b) Find $\displaystyle\int \frac{1}{\sqrt{36-4x^2}}\,dx$. 2

(c) Use the substitution $u = e^{3x}$ to evaluate $\displaystyle\int_0^{\frac{1}{3}} \frac{2e^{3x}}{e^{6x}+1}\,dx$. 3

(d) Find the acute angle between the lines $2x - 3y + 4 = 0$ and $5x - y - 7 = 0$. 3

(e) Consider the function $f(x) = \tan^{-1}\dfrac{x}{3}$.

(i) Evaluate $f(\sqrt{3})$. 1

(ii) Sketch the function $y = f(x)$. 2

(iii) Find the gradient of the curve at the point where it intersects the x axis. 2

End of Question 11

Question 12 (15 marks) – Use a SEPARATE writing booklet.

(a) (i) Write $\sqrt{3}\cos x - \sin x$ in the form $2\cos(x+\alpha)$, where $0 < \alpha < \frac{\pi}{2}$. **2**

(ii) Hence, or otherwise, give the general solution for the equation **3**

$$\sqrt{3}\cos x - \sin x = 1 .$$

(b) (i) Factorise $a^3 + b^3$. **1**

(ii) Hence, or otherwise, show that **3**

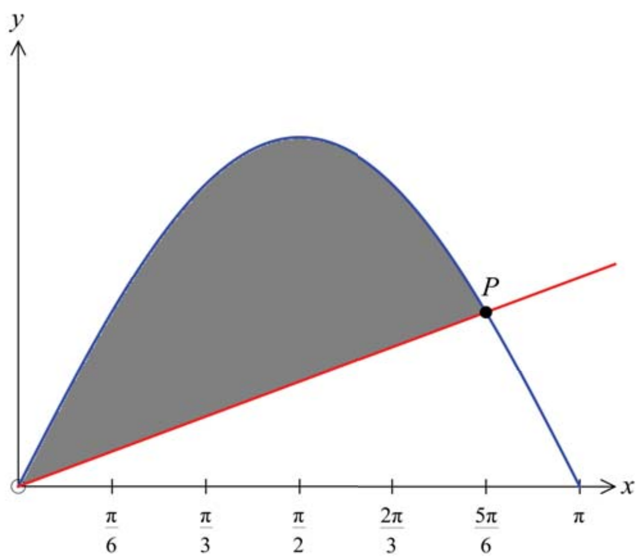
$$\frac{\sin^3\alpha + \cos^3\alpha}{\sin\alpha + \cos\alpha} = \frac{2 - \sin 2\alpha}{2}$$

if $\sin\alpha + \cos\alpha \neq 0$.

(c) Use the table of standard integrals to find the exact value of **2**

$$\int_0^{\frac{\pi}{12}} \sec 3x \tan 3x \, dx .$$

(d)



The curve $y = \sin x$ and the line $y = \frac{3}{5\pi}x$ intersect at the origin and at P for the domain $0 \leq x \leq \pi$.

(i) Show that $x = \frac{5\pi}{6}$ is a solution of the equation $\sin x = \frac{3}{5\pi}x$. **1**

(ii) Find the shaded region shown above in exact form. **3**

End of Question 12

Question 13 (15 marks) – Use a SEPARATE writing booklet.

(a) (i) Prove that $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ for $-1 \le x \le 1$. 2

(ii) Find the volume of the solid formed when the function $y = \sin^{-1} x + \cos^{-1} x$ is rotated about the x-axis between $x = 0$ and $x = 1$. 2

(b) Let $f(x) = \sin^{-1}(x+5)$.

(i) State the domain and range of the function $f(x)$. 2

(ii) Find the gradient of the graph of $y = f(x)$ at the point where $x = -5$. 2

(iii) Sketch the graph of $y = f(x)$. 3

(c) Use the fact that that $\tan\left(\alpha - \beta\right) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$ to show that 4

$$1 + \tan n\theta \tan(n+1)\theta = \cot\theta\left(\tan(n+1)\theta - \tan n\theta\right).$$

End of Question 13

Question 14 (15 marks) – Use a SEPARATE writing booklet.

(a) At any point of on a curve $y = f(x)$ the gradient function is given by

$$\frac{dy}{dx} = \frac{x+1}{x+2}.$$

(i) If $y = -1$, when $x = -1$, then find and expression for y **3**

(ii) Find the value of y when $x = 1$. **1**
Give you answer to 3 significant figures.

(b) Evaluate $\int_0^{\pi/6} \sec^2 x \tan^8 x \, dx$ **2**

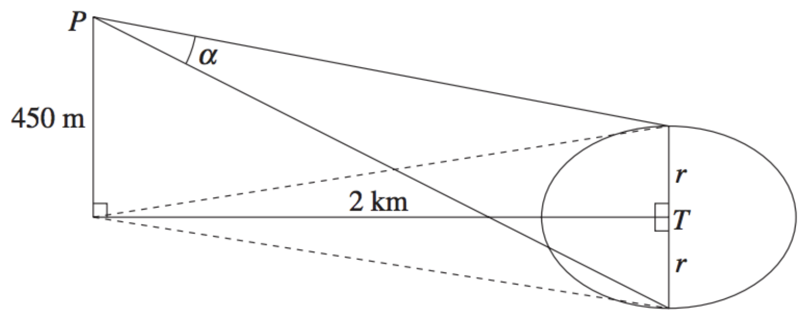
(c) (i) By expanding the left hand side, show that **1**
$\sin(5x+4x) + \sin(5x-4x) = 2\sin 5x \cos 4x$

(ii) Hence find $\int \sin 5x \cos 4x \, dx$ **2**

(d) By making the substitution $t = \tan\frac{\theta}{2}$, or otherwise, show that **2**

$$\operatorname{cosec}\theta + \cot\theta = \cot\frac{\theta}{2}$$

(e) An oil tanker at T is leaking oil which forms a circular oil slick.
An observer is measuring the oil slick from a position P, 450 metres above sea level and 2 kilometres horizontally from the centre of the oil slick.



(i) At a certain time the observer measures the angle, α, subtended by the diameter of the oil slick, to be 0.1 radians. What is the radius, r, at this time? **2**

(ii) At this time, $\frac{d\alpha}{dt} = 0.02$ radians per hour. Find the rate at which the radius of the oil slick is growing. **2**

End of paper

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$