



NORTH SYDNEY BOYS

2013 HSC ASSESSMENT TASK 2

Mathematics Extension 1

General Instructions

- Reading time 5 minutes
- Working time 55 minutes
- Write on the lined paper in the booklet provided
- Write using blue or black pen
- Board approved calculators may be used
- All necessary working should be shown in every question
- Each new question is to be started on a **new page**.
- Attempt all questions

Class Teacher:

(Please tick or highlight)

- O Mr Lucas
- O Mr Berry
- O Mr Lin
- O Mr Fletcher
- O Mr Lam
- O Ms Ziaziaris

Student Name/Number: _____

Question No	1-5	6	7	8	9	10	11	Total	Total %
Mark	5	10	8	6	10	3	7	49	100

(To be used by the exam markers only.)

SECTION A – MULTIPLE CHOICE (5 Marks)

1. If $y = e^{x^2}$ which of the following is an expression for $\frac{d^2y}{dx^2}$

(A) $2xe^{x^2}$ (B) $2e^{x^2}$ (C) $4xe^{x^2}$ (D) $(4x^2 + 2)e^{x^2}$

2. If $e^{x+2} = 4$, the exact value of x is:

(A) 2
(B) e⁴ + 2
(C) log_e2
(D) log_e4 - 2

3. The polynomial $P(x) = ax^3 + bx^2 + cx + d$ has zeros at: -2, -1, 1 and 2. What is the value of b

(A) −2
(B) −1
(C) 0
(D) 2

4. The function $y = x^3 + 2x^2 + 3x + 4$ has a root between -1 and -2. By halving the interval a better approximation can be determined to be between:

(A) -1 and -1.25
(B) -1.25 and -1.5
(C) -1.5 and -1.75
(D)-1.75 and -2

5. The gradient of the normal to the curve $y = log_e x^2$ at x = 2 is:

(A) -1 (B) 0 (C) 1 (D) 4

SECTION B – EXTENDED RESPONSE

Question 6 (10 Marks)

(a) Differentiate

(i)
$$log_{10}x$$
 1
(ii) x^2e^x 2

(b) Find the primitive of:

(i)
$$2e^x + 1$$
 1

$$(ii)\frac{x}{2x^2+1}$$

(iii)
$$\frac{2x^2+1}{x}$$

(c) Evaluate
$$\int_{1}^{2x} dx$$
 correct to two decimal places: 2

Question 7 (8 Marks)

2

(a)	(i) Factorise $x^3 - x^2 - 8x + 12$ completely	3
	(ii) Hence sketch $P(x) = x^3 - x^2 - 8x + 12$ (vou are not required to use calculus)	2

(b) The root of $f(x) = 0.4x - e^{-x^2}$ is near x = 1. Use Newton's Method once to find a 2 decimal place approximation to this root

Question 8 (6 Marks)

- (a) If α , β and γ are the roots of the equation $3x^3 5x^2 + 2x 3 = 0$. Find the values of
 - (i) $\alpha + \beta + \gamma$ 1
 - (ii) $\alpha\beta + \beta\gamma + \alpha\gamma$ 1

(iii)
$$\alpha^2 + \beta^2 + \gamma^2$$
 1

(b) The polynomial $P(x) = x^3 + ax^2 - bx + 6$ has (x-3) as a factor. When P(x) is divided by (x + 1) the remainder is 8. Find the values of *a* and *b*. 3

Question 9 (10 Marks)

(a) Find the volume when $y = \frac{2}{\sqrt{x}}$ is rotated around the <i>x</i> -axis between $x = 3$ and $x = 5$	3
(b) (i) Draw a neat sketch of $y = log_e(x - 1)$ showing all relevant features	2
(ii) Use 2 applications (i.e. five function values) of Simpson's rule to estimate the area enclosed by $y = log_e(x - 1)$, $x = 6$ and the <i>x</i> -axis, in the first quadrant correct to two decimal places.	3
(iii) Hence, or otherwise, approximate the area enclosed by the lines $y = log_e(x - 1)$, $y = log_e 5$ and the coordinate axes.	2

Question 10 (3 Marks)

(a) Prove by mathematical induction that $7^n - 6n - 1$ is divisible by 36 for all positive integers $n \ge 2$.

Question 11 (7 Marks)

(a) Solve for x: $log_2 x + log_2 (x + 7) = 3$ 3

(b) If
$$y = \frac{\log_e x}{x}$$

(i) Show that
$$\frac{dy}{dx} = \frac{1 - \log_e x}{x^2}$$
 1

(ii) Hence or otherwise show that:

$$\int_{e}^{e^{2}} \frac{1 - \log_{e} x}{x \log_{e} x} dx = (\log_{e} 2) - 1$$
3

End of Examination

	MULTIPLE CHOICE	· · · · · · · · · · · · · · · · · · ·
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Question 6

	$(a)(i) \frac{d}{dx} \log_{10} x = \frac{1}{x \log_{e} 10}$	/ correct V answer
Brecharden in die Geschart von die Angeleinen	$(\alpha)(i) y = \infty^2 e^{x}$	a part all a sum a callena annonan na canna a nagardan nagar na callanan na na analasin na callanan sina. Na anno 1, marana anno anno anno anno annonan na annona ca an adarana calla anno anno anno anno anno anno anno
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	y' = u'v + v'u	/ product rule
	$= (2 \operatorname{sc})(e^{\operatorname{sc}}) + (e^{\operatorname{sc}})(x^2)$	Vanswer.
	$= x e^{x} (x + 2)$	المعاد العالم ومعالمة المعاد المع المعاد المعاد
	(b)(i) $\int (2e^{x} + 1) dsc = 2e^{x} + sc + c$	/ correct answer
	$(b)(ii) \int \frac{x}{2r^2 + 1} dx$	
	$=\frac{1}{4} \begin{pmatrix} 4 \\ 2x^2 + 1 \end{pmatrix} dx$	Vany log as the primitive
	$= \frac{1}{4} \log(2x^2 + 1) + C$	V curriect answer
	(b) (iii) $\int \frac{2x^2 + 1}{x} dx$	
	$= \int \left(2x + \frac{1}{x}\right) dx$	/breaking ap fraction
	$= x^2 + \log e x + C$	/ correct a nswer
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 $C \int_{1}^{3} e^{2x} dx$ $= [\frac{1}{2}e^{2x}]_{1}^{3}$ / currect primitive $= \frac{1}{2} (e^{6} - e^{2})$ = 198.02 (2 d.p.) / currect answer • •

Question 7

(a)(i) let $P(x) = x^3 - x^2 - 8x + 12$ $P(2) = 2^3 - 2^2 - 8 \times 2 + 12$ = 0 finding (x-2) is o factor $\frac{x^2 + x - 6}{x^3 - x^2 - 8x + 12}$ Polynumial division $\frac{2x^3-2x^2}{x^2-8x}$ $\frac{x^2 - 2x}{-6x + 12}$ -6x + 12 $P(x) = (x-2)(x^2 + x - 6)$ -/ facturised P(x) = (x-2)(x+3)(x-2)expression $P(x) = (x-2)^2(x+3)$ (a) (i i) l'correct singt d'duble rout 12 V shape -4 2

(b) $f(x) = 0.4 \times -e^{-x^2}$ $f'(x) = 0.4 + 2x e^{-x^2}$ / correct derivative a. = 1 $\frac{f(a_{\upsilon})}{f'(a_{\upsilon})}$ $a_1 = q_0 =$ / subbing qo=1 into formula $\frac{0.4}{0.4} - e^{-1}$ I final answer $\therefore a_1 = 0.97 (2 d. p.)$ ern Ad

Question 8. (a)(i) d + B + Y = -b/qCorrec = 5/3answer $(c)(i)dB+\alpha r+BY = \frac{1}{a}$ V currect = 2/3 answe/ $(a)(iii) d^2 + \beta^2 + \gamma^2 = (d + \beta + \gamma)^2 - 2(d\beta + d\beta + \beta\gamma)$ $= (5/3)^2 - 2(2/3)$ 1 correct answer $= \frac{13}{9}$ (b) P(3) = 0 $3^{3} + a \times 3^{2} - 3b + 6 = 0$ 33 + 99 - 3b = 0/ simultoneus) V equations 9a - 3b = -33, $\widehat{}$ P(-1) = 8 $(-1)^{3} + q(-1)^{2} + b + 6 = 8$ -1 + a + b + 6 = 8a+b=3. b = 3-9 . . . D sub (2) -> ()

 $q_a - 3(3 - a) = -33$ 9q - 9 + 3q = -33 $12\alpha = -24$ / solution to q q = -2sub into Q b = 3 - 93 - (-2) = b = 5 v solution to 6 (-----

Question 9 (a) $V = \pi \int_{a}^{b} y^2 dx$ $= \pi \int_{3}^{5} \left(\frac{2}{3} \right)^2 dx$ / subbing = $\pi \int_{3}^{5} \frac{4}{3} dx$ = $4\pi \int_{3}^{5} /z dz$ / correct integral 4 T [luge X]3 4π [luge5 -luge3] = $4\pi \log(\%)$ / final onswer a=1 (b)(i)/ shape √ asymptok AND 2-interrept) J (b)(ii)yo Tugel - y1 y2 y3 y4 - lug2 lug3 lug4 lug5 / correct for mulg $A = \frac{1}{3} \left[(y_0 + y_n) + 4(y_1 + y_3 ...) + 2(y_2 + y_4 ...) \right]$ = 1/3 [(log1 + log5) + 4(log2 + log4) + 2(log3)] / correct values = 4.04 (2 d. p.)V tinul answer.

(b)(iii) luye5 /area of rectangle 6 V final answer (ony roanding 15 sk) $A \doteq (6 \times \log 5) - 4.04$ = 5.615(3d.p.)

Question 10 1/when n = 2/ proving $7^2 - 6 \times 2 - 1 = 36$ case which is divisible by 36 2./ assume true for n=k where k is an integer $i.e. 7^{R} - 6k - 1 = 36 m$ where mis an integer 3./ for n=k+1 7 k+1 -6 (k+1) -1 = 7.7 k - 6k - 6 - 1 = 7x7 R - 6R - 7 = 7 × 7 × - 42 × - 7 - 6 × + 42 × $= 7(7^{k} - 6k - 1) + 36k$ / substitution from assumption = 7×36m + 36 k, from assumption = 36(7m + R)contration de manacementes 1 currectly which is divisible by 36 since V showing m & k are both integers divisibility 4./ therefore the proposition is true by the principle of mathematical induction

Question (a) $\log_2 x + \log_2(x+7) = 3$ Ro-suraciona / combining into 1 log $\log_2(x^2+7x)=3$ re-urranging to get exponential $\therefore x^2 + 7x = 2^3$ $\alpha^2 + 7\alpha = 8$ $x^2 + 7x - 8 = 0$ (x+8)(x-1)=0X. x = -8 or x = 1/ correct final answar must have only I final solution 1. x=1 as x70 (b)(i) y = <u>logex</u> let a = loyex & v = x $(', u' = /\alpha & v' = 1$ $\frac{dy}{dx} = \frac{y^2 - y^2 y}{y^2}$ $- (1/x)(x) - (1) \log x$ all necessary steps shown 1 - luger as regid

(b)(ii) pe² <u>I-loye</u> X Je xloye X $\int_{-\frac{1-\log x}{2}}^{\frac{1-\log x}{2}}$ / dividing by 22 da $k \left(\frac{x \log e x}{2} \right)$ $\int \frac{e^2}{1-\log e^2}$ と (loge X) = $\left[\log e \left(\frac{\log x}{2} \right) \right]^{e^2}$ (from (i)) / correct integral = loge (luge e²) - loge (luge e) $= \log \left(\frac{2}{e^2}\right) - \log \left(\frac{1}{e}\right)$, (luge2 - luge e² - luge + lugee = loye2 - 2 + 0 + l successfully getting to final answer. = (luge2) -1, as required