



Name:

SCEGGS Darlinghurst

2005
HIGHER SCHOOL CERTIFICATE
Assessment Task
Tuesday, 29 March

Extension 1 Mathematics

Task Weighting: 25%

General Instructions

- Time allowed - 75 minutes
- Answer on the pad paper provided
- Write your name at the top of each page
- Start each question on a new page
- Attempt **all** questions
- Marks may be deducted for careless or badly arranged work
- Mathematical templates and geometrical equipment may be used
- Approved scientific calculators should be used
- A table of standard integrals is provided

	Com	Calc	Reas	Marks
Question 1	/3		/2	/12
Question 2			/2	/10
Question 3	/3		/3	/10
Question 4	/3		/5	/10
TOTAL	/9		/12	/42

Question 1 (12 marks)

Marks

- (a) The polynomial equation

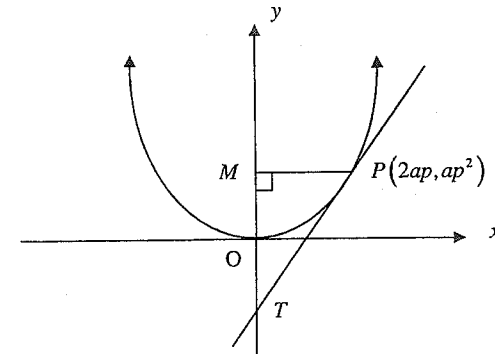
2

$$P(x) = x^3 - 2x^2 - 2x$$

has a solution near $x = 2.7$.

Use this value of x and Newton's Method once to find a more accurate solution, correct to 3 significant figures.

- (b) The tangent to the parabola $4ay = x^2$ at the point $P(2ap, ap^2)$ meets the y axis at T . M is the foot of the perpendicular from P to the y axis.



- (i) Prove that the equation of the tangent is:

2

$$px - y - ap^2 = 0$$

- (ii) Hence prove that the origin O is the midpoint of the interval MT .

2

- (c) (i) How many different arrangements are there for the letters of the word COMMITTEE?

1

- (ii) If one arrangement is chosen at random, find the probability that the Es are together.

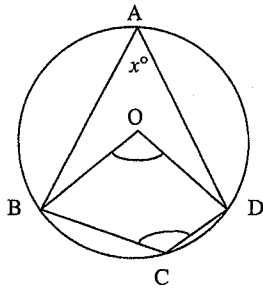
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Real

Question 1 continues on the next page

Question 1 (continued)

(d)



Not to scale

O is the centre of the circle $\angle BOD = \angle BCD$.

If $\angle BAD = x^\circ$, evaluate x° giving clear reasons.

Marks

3 *Comm*

Question 2 (10 marks)

Marks

(a) The polynomial $P(x) = 2x^3 + 4x^2 - 3x - 6$ has roots α , β and γ .
Evaluate:

(i) $\alpha\beta\gamma$

1

(ii) $(\alpha - 1)(\beta - 1)(\gamma - 1)$

3

(b) A committee of 5 is to be formed from a group of 5 boys and 6 girls.
How many committees are possible if:

(i) there are no restrictions?

1

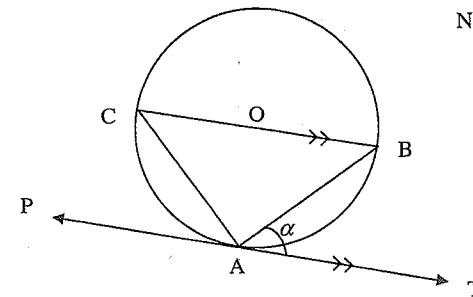
(ii) there must be a majority of girls?

2 *reas*

(c)

Not to scale

3



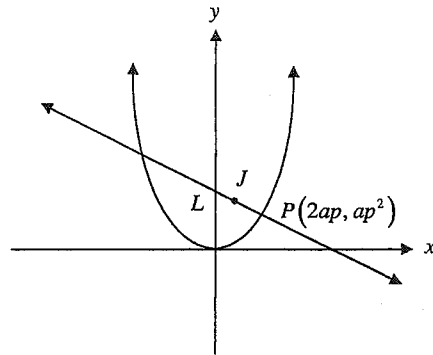
A, B and C are points on the circle centre O.
PT is a tangent to the circle at A.
Diameter BC is parallel to PT.
 $\angle BAT = \alpha$.

Find the value of α giving clear reasons.

Question 3 (10 marks)

Marks

(a)



PL is the normal to the parabola $4ay = x^2$ at the point $P(2ap, ap^2)$.

The equation of PL is $x + py = 2ap + ap^3$.

L is on the y axis.

- (i) Find the co-ordinates of L . 1
- (ii) Find the co-ordinates of J , the midpoint of PL . 1
- (iii) Prove that the locus of J is a parabola and find its vertex. 3 *Reas.*

(b) (i) Prove that the coefficient of x^3 in the expansion of $\left(x + \frac{1}{x}\right)^7$ is 21. 2

(ii) A student was asked to find the coefficient of x^3 in the expansion of $(2x^2 + 3)\left(x + \frac{1}{x}\right)^7$. 3 *Common*

$$(2x^2 + 3)\left(x + \frac{1}{x}\right)^7$$

Her incorrect answer was 63.

Why did she think this was the answer?

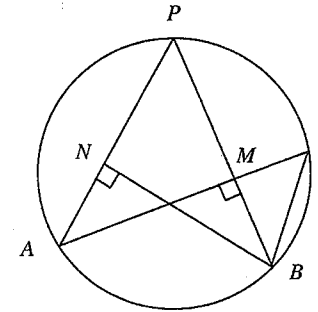
Without further calculation, explain the method she should have used.

Question 4 (10 marks)

Marks

(a) At Virginia's birthday party, her 9 guests are to be seated with her around a circular table. How many arrangements are possible if Virginia must sit next to her special friend Annabel? 2

(b)



Not to scale

Given $BN \perp AP$ and $AF \perp PB$ at point M , prove that:

- (i) $MNAB$ is a cyclic quadrilateral. 1
- (ii) PB bisects $\angle NBF$. 2 *Common*

(c) The polynomial $P(x) = (x-2)(x+3)Q(x) + Ax - 1$ gives a remainder of 8 when divided by $x+3$.

Find the remainder when $P(x)$ is divided by $(x-2)(x+3)$. 2 *Reas*

(d) Use the expansion of $(2+x)^n$ to prove that 3 *Reas*

$$2^n = \binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \dots + \binom{n}{r} + \dots + \binom{n}{n}$$

End of Assessment

Extension 1 Assessment 2005.

① a) $P(x) = x^3 - 2x^2 - 2x$
 $P'(x) = 3x^2 - 4x - 2$
 $P(2.7) = (2.7)^3 - 2(2.7)^2 - 2(2.7)$
 $= -0.297$
 $P'(2.7) = 3(2.7)^2 - 4(2.7) - 2$
 $= 9.07$

2nd approximation = $2.7 + \frac{0.297}{9.07}$
 $= 2.73$ (3 significant figures). ✓

b) (i) $y = \frac{x^2}{4a}$
 $\frac{dy}{dx} = \frac{2x}{4a} = \frac{x}{2a}$

at P, $m = \frac{2ap}{2a} = p$. ✓

tangent: $y - ap^2 = p(x - 2ap)$
 $= px - 2ap^2$

∴ $px - y - ap^2 = 0$ is tangent at P. ✓

(ii) if $x=0$, $-y = ap^2$
 $\therefore y = -ap^2$
 $\therefore T$ is $(0, -ap^2)$ ✓
 But M is $(0, ap^2)$ ✓
 ∴ O is the midpoint of MT .

c) (i) no. of arrangements = $\frac{9!}{2! \cdot 2! \cdot 2!} = 45360$. ✓

(ii) no. with E's together = $\frac{8!}{2! \cdot 2!} = 10080$. ✓

Probability = $\frac{10080}{45360} = \frac{2}{9}$. ✓

d) $\angle BOD = \angle BCD$ (given)
 $\angle BOD = 2\angle BAD = 2x^\circ$ (angle at centre is twice angle at circumference if subtended by same arc) ✓
 $\angle BAD + \angle BCD = 180^\circ$ (opp. angles of a cyclic quad. are supp.) ✓

But $\angle BAD + \angle BCD = x + 2x = 3x^\circ$

∴ $x^\circ = 60^\circ$. ✓

② a) $P(x) = 2x^3 + 4x^2 - 3x - 6$
 $d + \beta + \gamma = -2$, $2\beta + \gamma + \beta\gamma = -\frac{3}{2}$, $d\beta\gamma = 3$. 10

(i) $d\beta\gamma = 3$. ✓
 (ii) $(d-1)(\beta-1)(\gamma-1) = (2\beta - d - \beta + 1)(\gamma - 1)$
 $= d\beta\gamma - d\gamma - \beta\gamma + \gamma = d\beta + d + \beta - 1$ ✓
 $= d\beta\gamma - (d\beta + d\gamma + \beta\gamma) + (d + \beta + \gamma) - 1$ ✓
 $= 3 + \frac{3}{2} - 2 - 1 = 1\frac{1}{2}$.

b) (i) no. of committees = $\binom{11}{5} = 462$. ✓

(ii) 3 girls 2 boys = $\binom{6}{3} \binom{5}{2} = 20 \times 10 = 200$

4 girls 1 boy = $\binom{6}{4} \binom{5}{1} = 15 \times 5 = 75$ concept ✓

5 girls = $\binom{6}{5} = 6$

Total no. is 281. ✓

c) $\angle CAB = 90^\circ$ (angle in a semi circle is 90°) ✓
 $\angle BAT = \angle ABC = 2$ (alt \angle 's equal, $BC \parallel PT$) ✓
 $\angle BAT = \angle ACB = 2$ (angle between tangent and chord equals angle in alternate segment) ✓
 $\angle CAB + \angle ABC + \angle ACB = 180^\circ$ (angle sum of $\triangle ACB$ is 180°)
 $\therefore 90 + 2 + 2 = 180^\circ$
 $d = 45^\circ$.

③ a) (i) if $x=0$, $py = 2ap + ap^3$ ✓

$y = 2a + ap^2$
 $\therefore L$ is $(0, 2a + ap^2)$ ✓

(ii) $J: \left(\frac{2ap}{2}, \frac{ap^2 + 2a + ap^2}{2} \right)$
 $= (ap, a + ap^2)$ ✓

(iii) let $x = ap$, $y = ap^2 + a$
 $p = \frac{x}{a}$, $y = a + \frac{x^2}{a^2} + a$
 $y = \frac{x^2}{a} + a$ ✓✓

This equation is of the form of a parabola
 $y = ax^2 + bx + c$

vertex is $(0, a)$ ✓

b) (i) $T_{k+1} = \binom{7}{k} x^{7-k} (x^{-1})^k$
 $= \binom{7}{k} x^{7-2k}$ ✓

if $7-2k = 3$ (where term is in x^3)
 $2k = 4$
 $k = 2$

$C_3 = \binom{7}{2} = 21$ ✓

(ii) she multiplied 3 by 21.
 she should have found the coefficient of x in the expansion of $(x + \frac{1}{x})^7$, multiplied this coefficient by 2 then added 21×3

④ a) 10 altogether ✓ ✓
 no. of arrangements $8! \times 2! = 80640$

b) (i) $\angle ANB = \angle AMB$ (given)
 $\therefore MNAB$ is cyclic (if angles subtended by the same arc are equal the 4 points must be concyclic) ✓

(ii) $\angle FBP = \angle FAP$ (angles subtended by the same arc in the larger circle are equal) ✓
 $\angle NAM = \angle NBM$ (angles subtended by the same arc in the smaller circle are equal) ✓

$\therefore \angle FBP = \angle NBM$

i.e. PB bisects $\angle NBF$

c) $P(x) = (x-2)(x+3)Q(x) + Ax - 1$

$P(-3) = 0 - 3A - 1 = 8$

$-3A = 9$

$A = -3$ ✓

$\therefore P(x) = (x-2)(x+3)Q(x) - 3x - 1$ ✓

remainder is $-3x - 1$

d) $(2+x)^n = \binom{n}{0} 2^n + \binom{n}{1} 2^{n-1} x + \binom{n}{2} 2^{n-2} x^2 + \dots + \binom{n}{n} x^n$ ✓

if $x=2$, $4^n = \binom{n}{0} 2^n + \binom{n}{1} 2^n + \binom{n}{2} 2^n + \dots + \binom{n}{n} 2^n$ ✓

$2^{2n} = 2^n \left[\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \dots + \binom{n}{n} \right]$

$\therefore 2^{2n} \div 2^n = \binom{n}{0} + \binom{n}{1} + \dots + \binom{n}{n}$

$2^n = \dots$ ✓