THE SCOTS COLLEGE

Extension 1 Mathematics

Pre-Trial Examination

17th March 2011

Time Allowed: 90 minutes + 5 minutes

Instructions:

- Show all necessary workings
- Approved non-programmable calculators may be used
- Begin a new sheet of paper for each question
- A removable page with standard integrals is located at the back

Outcomes to be assessed:

Preliminary	Q1	/10
Functions	Q2, Q3, Q4	/34
Calculus	Q5, Q6	/22
	TOTAL	/66

QUESTION ONE (10 MARKS) BEGIN A NEW SHEET OF PAPER

a) Solve $\frac{2x+6}{x-3} < -2$ *(2)*

b) P and Q are the end points of a focal chord of the parabola $x^2 = 4ay$ with focus *S*.
If the co-ordinates of P and Q are $(2ap, ap^2)$ and $(2aq, aq^2)$ respectively,

i) Write down the gradients of PS and QS. *(1)*

ii) Show that $pq = -1$ *(1)*

iii) Find the coordinates of the midpoint R of PQ in terms of *p* *(2)*

iv) Show that the equation of the locus of R is $x^2 = 2a(y - a)$ *(2)*

c) Let $f(x) = \log_e[(x-2)(1+x)]$, what is the domain of $f(x)$ *(2)*

QUESTION TWO (10 MARKS) BEGIN A NEW SHEET OF PAPER

a) Given that $(x+2)$ and $(x-3)$ are factors of $P(x) = x^3 - 6x^2 + px + q$, find the values of *p* and *q*. *(3)*

b) Without the use of calculus, sketch the graph of $f(x) = x(2x-1)(x-1)^3$, clearly showing any axis intercepts. *(3)*

c) Solve the equation $3x^3 - 7x^2 - 70x + 24 = 0$, given that the product of two of the roots is 2. *(4)*

QUESTION THREE (6 MARKS) BEGIN A NEW SHEET OF PAPER

Given the function $y = \frac{\sin x}{x-3}$

a) Show that a root of $\frac{\sin x}{x-3} = 0$ lies between $x = 6$ and $x = 6.5$. *(2)*

b) Use the method of halving the interval to show that this root lies between $x = 6.25$ and $x = 6.375$. *(2)*

c) Use one application of Newton's method to find an approximation of this root correct to 3 decimal places using a first approximation of *6.25* *(2)*

Q<u>UESTION</u> F<u>OUR</u> (18 MARKS) BEGIN A NEW SHEET OF PAPER

a) Given the function $f(x) = x^2 - 4x$

i) Graph the function $y = x^2 - 4x$. *(2)*

ii) State the largest positive domain for *f(x)* that will allow you to define the inverse function. *(1)*

iii) Find the inverse function and state the domain and range of the inverse function *(3)*

iv) Calculate the exact value of $f^{-1}(2)$ *(1)*

v) Graph the inverse function from *(iii)* on the same axes used in part *(i)* *(2)*

vi) Calculate the point of intersection of $f(x)$ and $f^{-1}(x)$ *(2)*

b) Evaluate $tan^{-1}(-\sqrt{3})$ *(1)*

c) Prove that $\sin^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{4}{5}\right) = \frac{\pi}{2}$ *(2)*

d) Given $y = acos^{-1}(bx)$ is a function. (where *a, b > 0*)

i) State the domain and range *(2)*

ii) Sketch this function *(2)*

Q<u>UESTION</u> F<u>IVE</u> (8 MARKS) BEGIN A NEW SHEET OF PAPER

a) Given $f(x) = e^{5x+1}$

i) Find $f'(x)$ *(1)*

ii) Find $f^{-1}(x)$ *(2)*

iii) Differentiate $f^{-1}(x)$ *(1)*

iv) Deduce that $\frac{dy}{dx} \times \frac{dx}{dy} = 1$ *(1)*

b) Find the equation of the normal to the curve $y = tan^{-1}x$ at the point $x = \sqrt{3}$. *(3)*

<u>QUESTION SIX</u> (14 MARKS) BEGIN A NEW SHEET OF PAPER

a) Differentiate, $e^{(cos^{-1}x)}$ *(2)*

b) $\int \frac{2}{\sqrt{25-4x^2}}dx$ *(3)*

c) Find the area enclosed between the curve $y = cos^{-1}x$, the y-axis and the lines $y = 0$ and $y = \frac{\pi}{4}$ *(2)*

d) For the curve $y = \frac{1}{\sqrt{1+x^2}}$, find the volume of the solid formed by rotating the curve about the *x-axis* from $x = \frac{1}{\sqrt{3}}$ to $x = 1$ *(4)*

e) Find $\int \frac{-x^2}{\sqrt{1-x^6}}dx$ by using the substitution $u = x^3$ *(3)*

END OF EXAM

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}}\,dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}}\,dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$