THE SCOTS COLLEGE

MATHEMATICS EXTENSION I

YEAR 12 PRETRIAL

19TH MARCH 2012

GENERAL INSTRUCTIONS

- Reading time – 5 minutes
- Working time - 1.5 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided
- Show all necessary working in Section II

WEIGHTING

30%

TOTAL MARKS

70

SECTION I (10 MARKS)

- Answers to be recorded on the multiple choice answer sheet provided
- Clearly label your answer sheet with your student number and class teacher's name
- Allow about 15 minutes for this section

SECTION II (60 MARKS)

- Questions 11- 14
- Answers to be recorded in the answer booklets provided
- *Each question must be completed in a new answer booklet.*
- Label each answer booklet with your student number, class teacher's name and the question number attempted. Clearly indicate the booklet order if more than one booklet is used for a question. Eg) Book 1 of 2 and 2 of 2.

SECTION I

QUESTION 1

Find the Cartesian equation of the following parametric equations $x+5t=3$ and $ty=1$

(A) $y=-\frac{1}{5}(x-3)$

(B) $y=\frac{5}{(3-x)}$

(C) $y=\frac{1}{5}(3-x)$

(D) $y=\frac{5}{(x-3)}$

QUESTION 2

Point E divides the interval joining C(4,0) and D(6,1) externally in the ratio 2:5. Find the coordinates of Point E.

(A) $\left(\frac{18}{3},-\frac{5}{3}\right)$

(B) $\left(\frac{8}{3},-\frac{2}{3}\right)$

(C) $\left(\frac{32}{3},-\frac{2}{3}\right)$

(D) $\left(\frac{22}{3},\frac{5}{3}\right)$

QUESTION 3

$\sqrt{3}\tan\theta=1$ where $-\frac{3\pi}{2}\le\theta\le\frac{\pi}{4}$. Identify the correct solution.

(A) $\theta=\frac{\pi}{6}-n\pi$ where n = 1 and n = 0

(B) $\theta=\frac{\pi}{6}+n\pi$ where n = 1 and n = 0

(C) $\theta=\frac{\pi}{6}-n\pi$ where n = 1

(D) $\theta=\frac{\pi}{6}+n\pi$ where n = 0

QUESTION 4

State the range of $y = -2\tan^{-1} 4x$

(A) $-1 < y < 1$

(B) $-\frac{p}{4} < y < \frac{p}{4}$

(C) $-p < y < p$

(D) All real *y*

QUESTION 5

Determine the angle between the lines $5x - 6y + 1 = 0$ and $8x + 5y - 3 = 0$ to the nearest degree

(A) 46^{o}

(B) 82^{o}

(C) 18^{o}

(D) 66^{o}

QUESTION 6

If 1, -1, *A* are non-zero roots of the equation $x^3 - Ax^2 - x + 3 = 0$, then the value of A is

(A) 3

(B) 0

(C) 1

(D) -1

QUESTION 7

Differentiate $y = x^2 + 5x$ by first principles

(A) $\lim\limits_{h \to 0} \dfrac{(x+h)^2 + 5(x+h) - x^2 + 5x}{h}$

(B) $\lim\limits_{h \to \infty} \dfrac{(x+h)^2 + 5h - x^2}{h}$

(C) $\lim\limits_{h \to 0} \dfrac{(x+h)^2 + 5h - x^2}{h}$

(D) $\lim\limits_{h \to \infty} \dfrac{(x+h)^2 + 5(x+h) - x^2 + 5x}{h}$

QUESTION 8

The period of the function $\cos\left(\dfrac{\pi x}{2}\right)$ is:

(A) 2

(B) 4π

(C) 2π

(D) 4

QUESTION 9

Use the table of 'Standard Integrals' to find $\int \sec 3x \tan 3x \, dx$

(A) $3\sec 3x + C$

(B) $\sec 3x + C$

(C) $3\sec \dfrac{1}{3}x + C$

(D) $\dfrac{1}{3}\sec 3x + C$

QUESTION 10

$\int \frac{1}{e^2 x + 1} dx$. Choose the best answer

(A) No solution

(B) Solve using reverse chain rule

(C) The solution is $e^2 \ln\left(e^2 x + 1\right)$

(D) The solution is $\frac{1}{e^2} \ln\left(e^2 x + 1\right)$

END OF MULTIPLE CHOICE SECTION

SECTION II

QUESTION 11 (START A NEW ANSWER BOOKLET) 15 MARKS

a) Evaluate $\sin^{-1} 0.6$ correct to three significant figures **1 mark**

b) Solve $\frac{1}{2} \le \frac{5x+3}{4x}$ **2 marks**

c) True or false: $(x-2)$ is a factor of $3x^3+2x^2-11x+10$ **2 marks**

Give reasons for your answer.

d) Find $\frac{d}{dx} x^2 \tan^{-1}\left(\frac{2x}{3}\right)$ **2 marks**

e) Evaluate $\lim_{x \to 0} \frac{\sin 3x}{5x}$ **1 mark**

f) Find

i. $\int \frac{1}{2} \tan x \sec^2 x \, dx$ using $u = \tan x$ **2 marks**

ii. $\int_{-\frac{1}{2}}^{0} \frac{3}{\sqrt{1-4x^2}} dx$ **2 marks**

iii. $\int 5\sin^2\left(\frac{x}{5}\right) dx$ **3 marks**

QUESTION 12 (START A NEW ANSWER BOOKLET) 15 MARKS

a) If $y = \frac{1}{3\left(9+x^2\right)}$ where $y > 0$ between $x = 3\sqrt{3}$ and $x = 3$ **3 marks**

Find the area of the region bounded by the curve $y = \frac{1}{3\left(9+x^2\right)}$, the x-axis and between $x = 3\sqrt{3}$ and $x = 3$

b) Find $\frac{d}{dx}\frac{\ln e^x + 1}{e^{x^2}}$ **3 marks**

c) $f(x) = \tan^{-1} x$

i. Sketch $f(x)$ on a Cartesian plane **2 marks**

ii. Hence, or otherwise, sketch $\frac{1}{f(x)}$ on a new Cartesian plane **1 mark**

iii. Based on your sketch, would an inverse function exist for $\frac{1}{f(x)}$? **1 mark**

Give reasons for your answer

d)

i. Sketch $y = 3\sin^{-1} x$ on a Cartesian plane **2 marks**

ii. On your sketch, shade the region that satisfies $\int_0^1 \left(3\sin^{-1} x\right) dx$ **1 mark**

iii. Hence evaluate the exact value of $\int_0^1 \left(3\sin^{-1} x\right) dx$ **2 marks**

QUESTION 13 (START A NEW ANSWER BOOKLET) 15 MARKS

a) $y = \cos^{-1} x + 1$ **3 marks**

Find the equation of the tangent at $x = -\frac{1}{2}$

b) Given $2(2^{2x}) - 5(2^x) \leq -2$

i. Solve $2\left(2^{2x}\right) - 5\left(2^x\right) + 2 = 0$ **2 marks**

ii. Show that $\frac{1}{2} \leq 2^x \leq 2$ **1 mark**

iii. Hence solve for the values of x **1 mark**

c)

i. $-\frac{1}{\sqrt{2}}\cos 2\theta + \frac{1}{\sqrt{2}}\sin 2\theta = R\cos\left(2\theta - \beta\right)$ **2 marks**

Find the values of R and β if $R > 0$ and $\pi < \beta < \frac{\pi}{2}$

ii. Hence, or otherwise, solve $-\frac{1}{\sqrt{2}}\cos 2\theta + \frac{1}{\sqrt{2}}\sin 2\theta = \frac{1}{2}$ for $-\pi \leq \theta \leq \pi$ **2 marks**

d)

i. Write an expression for *u* given $x = \frac{12}{5}\tan u$ **1 mark**

ii. Show that $\frac{dx}{du} = \frac{12}{5}\left(1 + \tan^2 u\right)$ **1 mark**

iii. Hence using the substitution $x = \frac{12}{5}\tan u$, show that **2 marks**

$$\int \frac{3}{144 + 25x^2}\,dx = \frac{1}{20}\tan^{-1}\left(\frac{5x}{12}\right) + C$$

QUESTION 14 (START A NEW ANSWER BOOKLET) 15 MARKS

a) The points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ lie on the parabola $x^2 = 4ay$

i. Show that the equation of the normal to the parabola at P is **2 marks**
$x + py - 2ap - ap^3 = 0$

ii. The equation of the normal at Q is $x + qy - 2aq - aq^3 = 0$

Hence show that the coordinates of R are
$\left(-apq(p+q), a(p^2 + pq + q^2 + 2)\right)$ **3 marks**

Where R is the intersection of the normals at P and Q.

iii.
If $pq = -1$, find the equation of the locus R in terms of a and x **2 marks**

b) O_1 and O_2 are t' rcle centres of the small and large circle respectively.

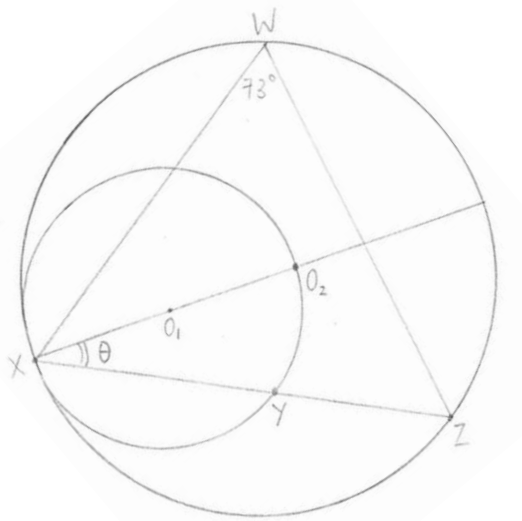

DIAGRAM NOT TO SCALE

COPY THE DIAGRAM INTO YOUR ANSWER BOOKLET

i. Dra e from O_2 to Y **2 marks**
State the size of $\angle XYO_2$
Give a reason for your answer

ii. Show that XY = YZ **3 marks**

iii. Hence find the angle θ if $\angle XWZ = 73^\circ$ **3 marks**

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$