THE SCOTS COLLEGE

MATHEMATICS EXTENSION I

YEAR 12 PRETRIAL

5TH APRIL 2013

GENERAL INSTRUCTIONS

- Reading time – 5 minutes
- Working time - 1.5 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided
- Show all necessary working in Section II

WEIGHTING

30%

TOTAL MARKS

55

SECTION I (7 MARKS)

- Answers to be recorded on the multiple choice answer sheet provided
- Clearly label your answer sheet with your student number
- Allow about 10 minutes for this section

SECTION II (48 MARKS)

- Questions 8 - 11
- Answers to be recorded in the answer booklets provided
- *Each question must be completed in a new answer booklet.*
- Label each answer booklet with your student number and the question number attempted. Clearly indicate the booklet order if more than one booklet is used for a question. Eg) Book 1 of 2 and 2 of 2.

SECTION I

QUESTION 1

Which of the following is the list of all the factors of $P(x) = 3x^3 + 4x^2 - 5x - 2$

(A) $(x+1)(x-2)(x-3)$

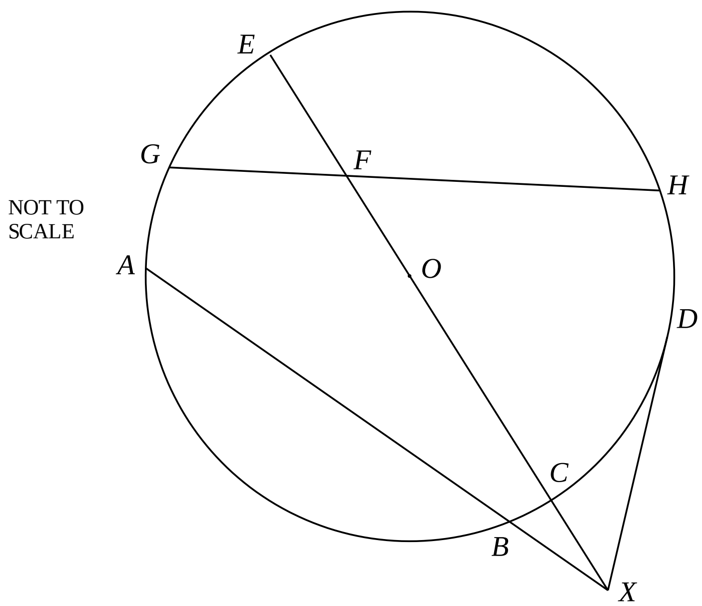
(B) $(x+1)(x+2)(x+3)$

(C) $3x(x-1)(x+1)$

(D) $(x-1)(x+2)(3x+1)$

QUESTION 2

A circle of centre O has a radius of 6 cm. From an external point X, a tangent is drawn with a point of contact D. From X the secants XA and XE are also drawn.



If DX = 8 cm calculate the distance CX.

(A) 4 cm

(B) -16 cm

(C) 8 cm

(D) 6 cm

QUESTION 3

The exact value of $\sin 75^{\circ}$ is:

(A) $\frac{4-\sqrt{6}}{4}$

(B) $\frac{2+\sqrt{2}}{2\sqrt{2}-1}$

(C) $\frac{\sqrt{6}+\sqrt{2}}{4}$

(D) $\frac{\sqrt{2}-\sqrt{6}}{4}$

QUESTION 4

Evaluate: $\int \frac{dx}{\sqrt{9-2x^2}}$

(A) $y = \sin^{-1}\sqrt{2}x + c$

(B) $y = \frac{1}{2}\cos\frac{2x}{3} + c$

(C) $y = \frac{1}{\sqrt{2}}\sin^{-1}\frac{\sqrt{2}x}{3} + c$

(D) $y = \frac{1}{\sqrt{2}}\log_e \sqrt{2}x + c$

QUESTION 5

Calculate the acute angle between the lines $l_1: 2y - 3x = 7$ and $l_2:\ 2x - 5y + 1 = 0$, to the nearest degree.

(A) $35°$

(B) $70°$

(C) $78°$

(D) $282°$

QUESTION 6

Given $\frac{d}{dx}\left(\frac{2x}{4+x^2}+\tan^{-1}\frac{x}{2}\right)=\frac{16}{(4+x^2)^2}$, evaluate: $\int_0^2 \frac{dx}{(4+x^2)^2}$

(A) $\frac{\pi}{16}$

(B) $\pi+4$

(C) $\frac{\pi+2}{64}$

(D) $2\pi-1$

QUESTION 7

For the curve $y=\frac{3x^2+1}{x^2+2x}$, the vertical and horizontal asymptotes are:

(A) $x=0,\ x=-2,\ y=3$

(B) $x=\frac{1}{3},\ y=0,\ y=-1$

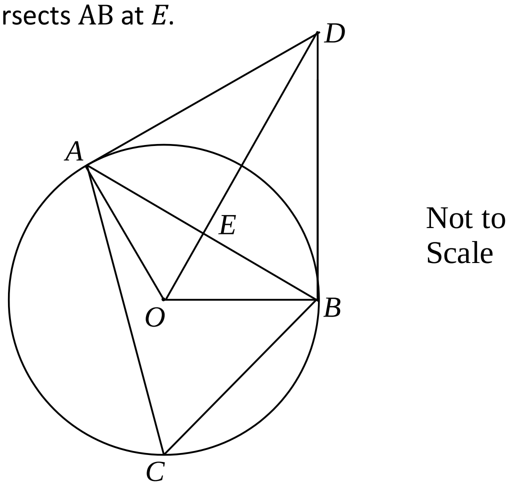
(C) $x=0,\ x=\sqrt{2},\ y=1$

(D) $x=6,\ y=-2,\ y=-1$

END OF MULTIPLE CHOICE SECTION

QUESTION 8 (START A NEW ANSWER BOOKLET) 12 MARKS

a) The diagram below shows the points A, B, and C on a circle with centre O. Tangents are drawn from A and B which meet at D. O is joined to D and the interval OD intersects AB at E.



i) Prove that $\angle AOB = 2 \times \angle DAB$. **2 marks**

ii) Prove that $AOBD$. Is a cyclic quadrilateral. **1 mark**

iii) Prove that E is the midpoint of AB. **2 marks**

b) Find the coordinates of the point $P(x, y)$ which divides the interval AB internally in the ratio 4 : 9 with $A(2, 3)$ and $B(5, -7)$. **2 marks**

c) Solve the inequality $\frac{x(x-3)}{x-2} > 2$. **3 marks**

d) Differentiate: $y = \sin^{-1}\frac{1}{4}(2x - 3)$ **2 marks**

END OF QUESTION 8

QUESTION 9 (START A NEW ANSWER BOOKLET) 12 MARKS

a) Given the parametric coordinates: $x = 2t$ and $y = t^2$.

i) Show that the Cartesian equation of the parabola is: $x^2 = 4y$ **1 mark**

ii) Given the parameter $t = 2$, show that the equation of the normal at that point is $x + 2y - 12 = 0$ **2 marks**

iii) Find the point of intersection of the normal and the $x -$ axis **1 mark**

b) Given the inverse trigonometric function: $y = 3\cos^{-1}(2x)$.

i) State the domain and range of $y = 3\cos^{-1}(2x)$ **2 marks**

ii) Find the gradient function of $y = 3\cos^{-1}(2x)$ **2 marks**

iii) Find the equation of the tangent to $y = 3\cos^{-1}(2x)$ at $x = 0$ **2 marks**

c) Find the general solution of $\sin\theta = \frac{\sqrt{2}}{2}$ **2 marks**

END OF QUESTION 9

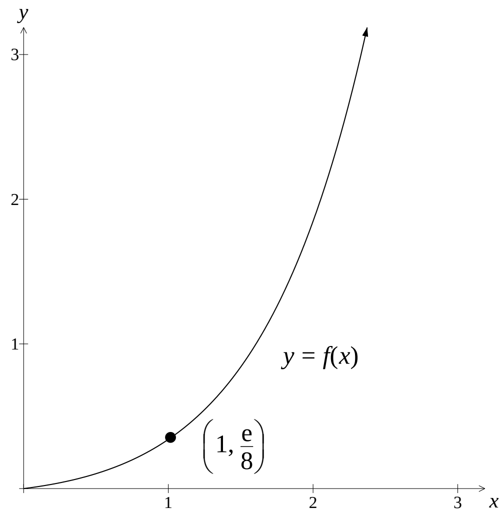
QUESTION 10 (START A NEW ANSWER BOOKLET) 12 MARKS

a) The polynomial $P(x) = x^4 - 3x^3 + ax^2 + bx - 6$ leaves a remainder of 8 when divided by $(x+1)$. If $x - 3$ is a factor of $P(x)$, find a and b. **3 marks**

b) Evaluate: $\int_0^{\frac{\pi}{4}} \sin^2 x \; dx$ **2 marks**

c) **i)** Show by differentiation that $y = \frac{xe^x}{2}$ is increasing for $x \geq 0$. **2 marks**

ii) A sketch of $y = f(x) = \frac{xe^x}{2};\ x \geq 0$ is shown below. Explain why $y = f(x)$ has an inverse function. **1 mark**



iii) Copy the graph above and add a sketch of the inverse function $y = f^{-1}(x)$. **1 mark**

d) **i)** Write $\sqrt{3}\cos x - \sin x$ in the form $A\cos(x + \phi);\ 0 < \phi < \frac{\pi}{2}$. **2 marks**

ii) Hence, or otherwise solve the equation $\sqrt{3}\cos x - \sin x = 1;$ $0 \leq x \leq 2\pi$. **1 mark**

END OF QUESTION 10

QUESTION 11 (START A NEW ANSWER BOOKLET) 12 MARKS

a) Find $\int_1^6 x\sqrt{x+3}\,dx$, given the substitution $u = \sqrt{x+3}$. **4 marks**

b) For the function $y = \frac{\log_e x}{x}$;

i) State the domain of the function **1 mark**

ii) Find any stationary points and determine their nature **2 marks**

iii) Find the x - intercept **1 mark**

iv) Hence, sketch the function including the above information and showing the property of the curve as $x \to \infty$ **2 marks**

c) Find the Cartesian equation of the curve represented by the following parametric equations: **2 marks**

$$x = \sin 2t, \quad y = \cos t$$

END OF SECTION II. END OF EXAM.

THE SCOTS COLLEGE – MATHEMATICS 2013
EXTENSION 1 MATHEMATICS PRE-TRIAL HSC

CANDIDATE NUMBER: ______________________

SECTION I – MULTIPLE CHOICE ANSWER SHEET (7 MARKS)

Mark the correct answer by filling in the circle. To make a correction, neatly place a cross over the circle and then fill in the correct circle.

EXAMPLE:	A	B	C	D
	○	☒	●	○

	A	B	C	D
Question 1	○	○	○	○
Question 2	○	○	○	○
Question 3	○	○	○	○
Question 4	○	○	○	○
Question 5	○	○	○	○
Question 6	○	○	○	○
Question 7	○	○	○	○

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

SECTION 1. MULTIPLE CHOICE.

Summary:

Q1. D

Q2. A

Q3. C

Q4. C

Q5. A

Q6. C

Q7. A

Solutions:

Q1. $P(x) = 3x^3 + 4x^2 - 5x - 2$

$P(1) = 3(1)^3 + 4(1)^2 - 5(1) - 2$

$= 0$

$\therefore (x-1)$ is a factor

$$
\begin{array}{r}
3x^2 + 7x + 2 \\
x - 1 \overline{) \, 3x^3 + 4x^2 - 5x - 2} \\
\underline{3x^3 - 3x^2} \\
7x^2 - 5x \\
\underline{7x^2 - 7x} \\
2x - 2 \\
\underline{2x - 2} \\
0
\end{array}
$$

$\therefore P(x) = (x-1)(3x^2 + 7x + 2)$

$= (x-1)(3x^2 + 6x + x + 2)$

$= (x-1)[3x(x+2) + 1(x+2)]$

$= (x-1)(3x+1)(x+2)$

(D)

Q2. $DX^2 = (CX)(XE)$

ratio of secant = square of tangent

$8^2 = (CX)(CX + 12)$

$64 = CX^2 + 12CX$

$CX^2 + 12CX - 64 = 0$

$(CX + 16)(CX - 4) = 0$

$\therefore CX = -16$ and $CX = 4$

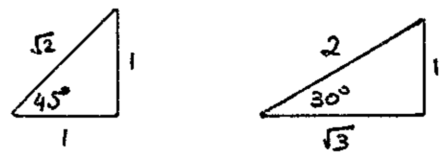
$CX \neq -ve$

$\therefore CX = 4$ cm

(A)

Q3. $\sin 75° = \sin(45° + 30°)$

$= \sin 45° \cos 30° + \cos 45° \sin 30°$



$\sin 45° = \frac{1}{\sqrt{2}}$ $\quad$ $\sin 30° = \frac{1}{2}$

$\cos 45° = \frac{1}{\sqrt{2}}$ $\quad$ $\cos 30° = \frac{\sqrt{3}}{2}$

$\therefore \sin 75° = \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$

$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$

$= \frac{\sqrt{6}+\sqrt{2}}{4}$

(C)

Q4. $\int \frac{dx}{\sqrt{9-2x^2}} = \int \frac{dx}{\sqrt{2(9/2 - x^2)}}$

$= \int \frac{dx}{\sqrt{2}\sqrt{(\frac{3}{\sqrt{2}})^2 - x^2}}$

$= \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{(\frac{3}{\sqrt{2}})^2 - x^2}}$

$= \frac{1}{\sqrt{2}} \sin^{-1} \frac{x}{3/\sqrt{2}} + C$

$= \frac{1}{\sqrt{2}} \sin^{-1} \frac{\sqrt{2}x}{3} + C$

(C)

Q5. $l_1: 2y - 3x = 7$

$\therefore y = \frac{3}{2}x + \frac{7}{2}$

$l_2: 2x - 5y + 1 = 0$

$\therefore y = \frac{2}{5}x + \frac{1}{5}$

$m_1 = \frac{3}{2} \quad m_2 = \frac{2}{5}$

$\therefore \tan\theta = \frac{3/2 - 2/5}{1 + (\frac{3}{2})(\frac{2}{5})}$

$= \frac{11/10}{8/5}$

$\theta = 35^\circ$ (A)

Q6. $\int_0^2 \frac{dx}{(4+x^2)^2} = \frac{1}{16}\int_0^2 \frac{16}{(4+x^2)^2}\,dx$

$= \frac{1}{16}\left[\frac{2x}{4+x^2} + \tan^{-1}\frac{x}{2}\right]\Big|_0^2$

$= \frac{1}{16}\left[\left(\frac{2(2)}{4+(2)^2} + \tan^{-1}\frac{(2)}{2}\right) - 0\right]$

$= \frac{1}{16}\left[\frac{4}{8} + \frac{\pi}{4} - 0\right]$

$= \frac{2+\pi}{64}$

(C)

Q7. Vertical:

$y = \frac{3x^2+1}{x(x+2)}$

i.e. $x \neq 0$ and $x \neq -2$.

Horizontal:

$\lim_{x\to\infty} \frac{3x^2+1}{x(x+2)} = \lim_{x\to\infty} \frac{\frac{3x^2}{x^2} + \frac{1}{x^2}}{\frac{x^2}{x^2} + \frac{2x}{x^2}}$

$= \lim_{x\to\infty} \frac{3 + 1/x^2}{1 + 1/x}$

$= \frac{3+0}{1+0}$

$= 3.$

$\therefore$ Assymptotes:

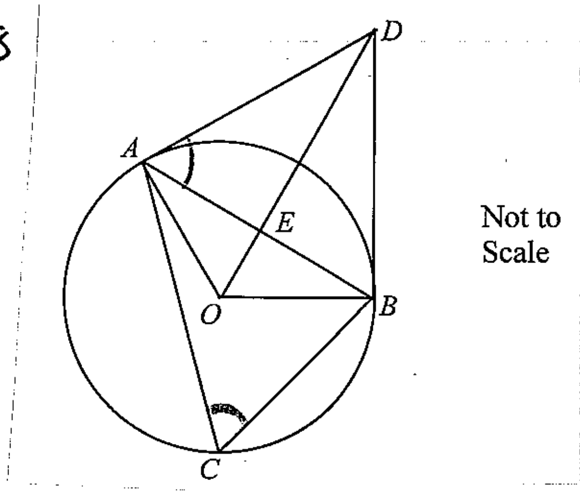
$x = 0,\ x = -2,\ y = 3$

(A)

END SECTION I.

SECTION II.

Q8



ai. Prove $\angle AOB = 2 \times \angle DAB$

$\angle DAB = \angle ACB$

($\angle$ between tangent and chord $= \angle$ in alternate segment)

$\angle AOB = 2 \times \angle ACB$

($\angle$ at centre is twice the $\angle$ at circumference on same arc)

$\therefore \angle AOB = 2 \times \angle DAB$

(since $\angle DAB = \angle ACB$)

*2 marks for a complete proof

*1 mark if some relevant facts are stated or proof is incomplete.

aii. Prove that AOBD is a cyclic quadrilateral.

$\angle DAO = \angle DBO = 90°$

(tangent perpendicular to radius)

$\therefore \angle DAO + \angle DBO = 180°$

(sum of two right angles)

$\therefore$ Opposite angles of AOBD are supplementary.

$\therefore$ AOBD is a cyclic quadrilateral

* 1 mark as long as statement that opposite angle are supplementary and why.

aiii. Prove: E is the midpoint of AB.

$AO = BO$ (equal radii)

$AD = BD$ (tangents from an external point are equal in length)

$\therefore$ AOBD is a kite

$\therefore$ OD bisects AB (symmetry of a kite)

$\therefore$ E is midpoint of AB.

8b. $A(2,3)$, $B(5,-7)$ $\frac{m}{n} = \frac{4}{9}$

$x = \frac{mx_2 + nx_1}{m+n}$ $\quad y = \frac{my_2 + ny_1}{m+n}$

$= \frac{20+18}{13}$ $\quad = \frac{-28+27}{13}$

$= \frac{38}{13}$ $\quad = -\frac{1}{13}$

$\therefore$ P is $\left(\frac{38}{13}, -\frac{1}{13}\right)$

* 2 marks for correct coordinates

* 1 mark for one of the coordinates correct or for a satisfactory amount of working correct and shown.

8c. $\frac{x(x-3)}{x-2} > 2$

$\frac{x^2-3x}{x-2} > 2$

$\frac{x^2-3x}{x-2} - 2 > 0 \quad , x \neq 2$

$\frac{x^2-3x}{x-2} - \frac{2x-4}{x-2} > 0$

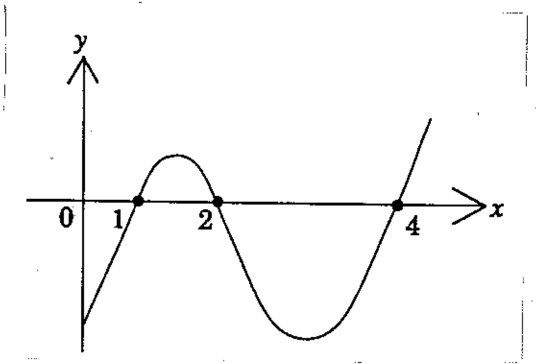
$\frac{x^2-5x+4}{x-2} > 0$

Multiply by $(x-2)^2$ (always >0)

$\therefore (x-4)(x-1)(x-2) > 0$

$\therefore y = (x-1)(x-2)(x-4)$

$y = 0$ when $x = 1, 2, 4$



$\therefore \frac{x(x-3)}{x-2} > 2$ for $1 < x < 2$ and $x > 4$

8d. $\frac{d}{dx} \sin^{-1} \frac{1}{4}(2x-3) = \frac{d}{dx} \sin^{-1}\left(\frac{1}{2}x - \frac{3}{4}\right)$

$= \frac{1/2}{\sqrt{1-\left(\frac{1}{2}x-\frac{3}{4}\right)^2}}$

$= \frac{1/2}{\sqrt{1-\left(\frac{1}{2}x-3/4\right)^2}} \times \frac{\sqrt{4}}{\sqrt{4}}$

$= \frac{1}{\sqrt{4-4\left(\frac{x^2}{4}-3x+\frac{9}{16}\right)}}$

$= \frac{1}{\sqrt{4-x^2+3x-9/4}}$

$= \frac{1}{\sqrt{7/4+3x-x^2}}$

we don't like fraction in the denominator.

$= \frac{\sqrt{4}}{\sqrt{4}} \times \frac{1}{\sqrt{7/4+3x-x^2}}$

$= \frac{2}{\sqrt{7+12x-4x^2}}$

9a. $x = 2t \quad y = t^2$

ai. $x = 2t$

$x^2 = 2t^2$

$\therefore x^2 = 4y$

aii. $t = 2$, $y = \frac{x^2}{4}$ $P(4, 4)$

$\therefore y' = m = \frac{2x}{4} = \frac{x}{2} = \frac{2t}{2} = t$

$\therefore m = 2 \rightarrow$ tangent

$\therefore$ Normal $m = -\frac{1}{2}$

$y - y_1 = m(x - x_1)$

$y - 4 = -\frac{1}{2}(x-4)$

$-2y + 8 = x - 4$

$-x - 2y + 12 = 0$

$x + 2y - 12 = 0$ as required.

OR.

$t=2 \quad x + 2y = 2(1)(2) + (1)(2)^3$

$a=1. \quad x + 2y = 12$

$x + 2y - 12 = 0$ as required.

aiii. $y = 0$

$\therefore x + 0 - 12 = 0$

$x = 12$

$\therefore$ Point of intersection : $(12, 0)$

9b. $y = 3\cos^{-1}(2x)$

bi. D: $\cos^{-1}x \quad -1 \leq x \leq 1$

D: $\cos^{-1}2x \quad -1 \leq 2x \leq 1$

$-\frac{1}{2} \leq x \leq \frac{1}{2}$

R: $\cos^{-1}x \quad 0 \leq x \leq \pi$

R: $3\cos^{-1}(2x) \quad 0 \leq x \leq 3\pi$

bii. $y = 3u$ where $u = \cos^{-1}(2x)$

$\therefore \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= 3 \times \frac{-2}{\sqrt{1-(2x)^2}}$

$= \frac{-6}{\sqrt{1-4x^2}}$

biii. when $x = 0$, $y = 3\cos^{-1}(2 \times 0)$

$y = 3 \times \frac{\pi}{2}$

$y = \frac{3\pi}{2}$

$m = y' = \frac{-6}{\sqrt{1-0}}$

$= -6$

$\therefore y - y_1 = m(x - x_1)$

$y - \frac{3\pi}{2} = -6x$

$6x + y - \frac{3\pi}{2} = 0$

$6x + y - \frac{3\pi}{2} = 0$

9c. $x = \sin^{-1}a + 2n\pi$

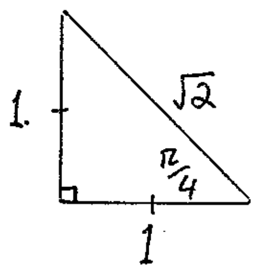
OR.

$x = (\pi - \sin^{-1}a) + 2n\pi$

where n is an integer.

If $x = \sin\frac{\sqrt{2}}{2}$ then

$x = \sin^{-1}\frac{\sqrt{2}}{2} + 2n\pi$ or $x = (\pi - \sin^{-1}\frac{\sqrt{2}}{2}) + 2n\pi$



$\therefore \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\therefore \sin^{-1}\frac{\sqrt{2}}{2} = \frac{\pi}{4}$

Hence.

$x = \frac{\pi}{4} + 2n\pi$ or $x = \frac{3\pi}{4} + 2n\pi$

10a. $P(-1) = 8$

$\therefore (-1)^4 - 3(-1)^3 + a(-1)^2 + b(-1) - 6 = 8$

$1 + 3 + a - b - 6 = 8$

$a - b - 2 = 8$

$a - b = 10$

$P(3) = 0$

$3^4 - 3(3)^3 + a(3)^2 + b(3) - 6 = 0$

$81 - 81 + 9a + 3b - 6 = 0$

$9a + 3b = 6$

$3a + b = 2$

①... $a - b = 10$

②... $3a + b = 2$

① + ②

$$4a = 12$$
$$a = 3$$

sub $a = 3$ into ①

$$3 - b = 10$$
$$-b = 7$$
$$b = -7$$

10b. $\int_0^{\pi/4} \sin^2 x \, dx$

Now $\cos 2x = 1 - 2\sin^2 x$

$\therefore 2\sin^2 x = 1 - \cos 2x$

$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

$\therefore \int_0^{\pi/4} \sin^2 x \, dx =$

$$= \int_0^{\pi/4} \frac{1}{2}(1 - \cos 2x)\, dx$$
$$= \frac{1}{2}\int_0^{\pi/4} (1 - \cos 2x)\, dx$$
$$= \frac{1}{2}\left[x - \frac{1}{2}\sin 2x\right]_0^{\pi/4}$$
$$= \frac{1}{2}\left[\frac{\pi}{4} - \frac{1}{2}\sin\frac{\pi}{2} - 0\right]$$
$$= \frac{\pi}{8} - \frac{1}{4}\sin\frac{\pi}{2} = \frac{\pi}{8} - \frac{1}{4}$$

10ci. $y = \frac{xe^x}{2}$

$$y' = \frac{x}{2} \times \frac{d}{dx}(e^x) + e^x \times \frac{d}{dx}\left(\frac{x}{2}\right)$$
$$y' = \frac{x}{2} \times e^x + e^x \times \frac{1}{2}$$
$$y' = \frac{xe^x + e^x}{2}$$

For $x > 0$, $e^x > 1$

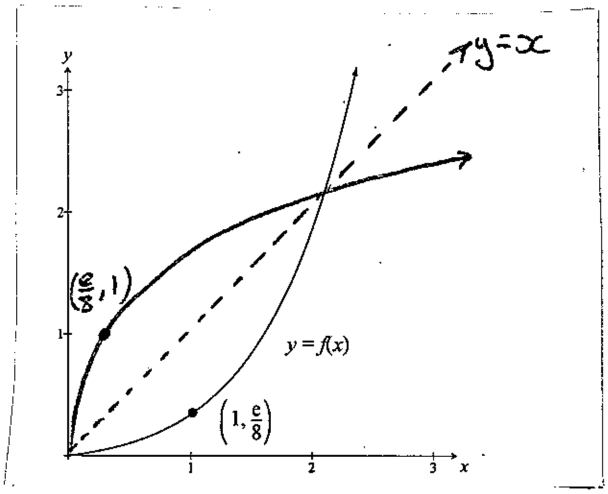
$\therefore xe^x > 0$ (positive × positive)

$\therefore xe^x + e^x > 0$ (positive + positive)

$\therefore \frac{xe^x + e^x}{2} > 0$ (positive ÷ positive)

10cii. For a given function $f(x)$, it must be one-to-one or pass a horizontal line test for the inverse of the function $f^{-1}(x)$ to likewise be one-to-one and pass a vertical line test in the given domain.

10ciii.



* As long as the graph appears to be a reflection of the original curve in $y = x$, mark awarded.

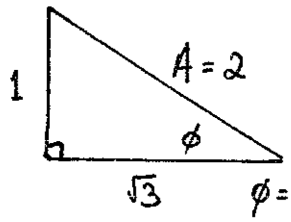
10di. Let $\sqrt{3}\cos x - \sin x = A\cos(x+\phi)$

$\therefore \sqrt{3}\cos x - \sin x \equiv A\cos x\cos\phi - A\sin x\sin\phi$

$\therefore \sqrt{3}\cos x = A\cos x\cos\phi$ | $-\sin x = A\sin x\sin\phi$

$\sqrt{3} = A\cos\phi$ | $1 = A\sin\phi$

$\cos\phi = \frac{\sqrt{3}}{A}$ | $\sin\phi = \frac{1}{A}$



$\phi = \frac{\pi}{6}$

$\therefore \sqrt{3}\cos x - \sin x = 2\cos\left(x + \frac{\pi}{6}\right)$

dii $\sqrt{3}\cos x - \sin x = 1.$

$\therefore 2\cos\left(x+\frac{\pi}{6}\right) = 1.$ from part i.

$\cos\left(x + \frac{\pi}{6}\right) = \frac{1}{2}$

$\therefore x + \frac{\pi}{6} = \frac{\pi}{3}$ or $\left(2\pi - \frac{\pi}{3}\right)$

$\therefore x = \frac{\pi}{6}$ or $x = \frac{3\pi}{2}$

11a. $\int_1^6 x\sqrt{x+3}\,dx$ given

$u = \sqrt{x+3}$ $\qquad \frac{du}{dx} = \frac{1}{2\sqrt{x+3}}$

$\therefore u^2 = x+3$

$\$ \; x = u^2 - 3$ $\qquad = \frac{1}{2u}$

$\therefore dx = 2u\,du$

when $x = 1$

$u = \sqrt{1+3} = 2$

$x = 6$

$u = \sqrt{6+3} = 3$

$\therefore \int_1^6 x\sqrt{x+3}\,dx =$

$= \int_2^3 (u^2-3)(u)(2u)\,du$

$= 2\int_2^3 (u^2-3)(u^2)\,du$

$= 2\int_2^3 u^4 - 3u^2\,du$

$= 2\left[\frac{u^5}{5} - u^3\right]_2^3$

$= 2\left[\frac{243}{5} - 27 - \left(\frac{32}{5} - 8\right)\right]$

$= 46\frac{2}{5}$

11b. $x = \sin 2t$, $y = \cos t$

$\therefore x = 2\sin t\cos t$

$\therefore x^2 = 4\sin^2 t\cos^2 t$

$x^2 = 4\cos^2 t\,(1 - \cos^2 t)$

$x^2 = 4y^2(1-y^2)$

11ci. $x \in R$ where $x > 0$.

cii. $y = \frac{\log_e x}{x}$

$y' = \frac{x \times \frac{1}{x} - \log_e x \times 1}{x^2}$

$y' = \frac{1 - \log_e x}{x^2}$

Stationary points at $y' = 0$

$\frac{1 - \log_e x}{x^2} = 0$

$\therefore\ 1 - \log_e x = 0$

$\log_e x = 1$

$\therefore\ x = e$

when $x = e$, $y = \frac{\log_e e}{e}$

$y = \frac{1}{e}$

$\therefore$ Stationary point : $(e, \frac{1}{e})$

x	2	e	3
y'	+ve	0	−ve
Slope	/	—	\

NATURE:

$\therefore$ MAXIMUM at $(e, \frac{1}{e})$

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ciii. x-intercept at $y = 0$

$\frac{\log_e x}{x} = 0$

$\log_e x = 0$

$x = e^0$

$x = 1.$

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civ.