St George Girls High School

Year 12

Mid-HSC Course Examination

2015

Mathematics
Extension 1

General Instructions

- Working time – 90 minutes
- Reading time – 5 minutes
- Write using blue or black pen.
- Board-approved calculators may be used.
- A table of standard integrals is provided.

Total marks – 65

Section I: 5 marks

- Attempt Questions 1 – 5
- All questions are of equal value
- Use the multiple choice answer sheet provided

Section II : 60 marks

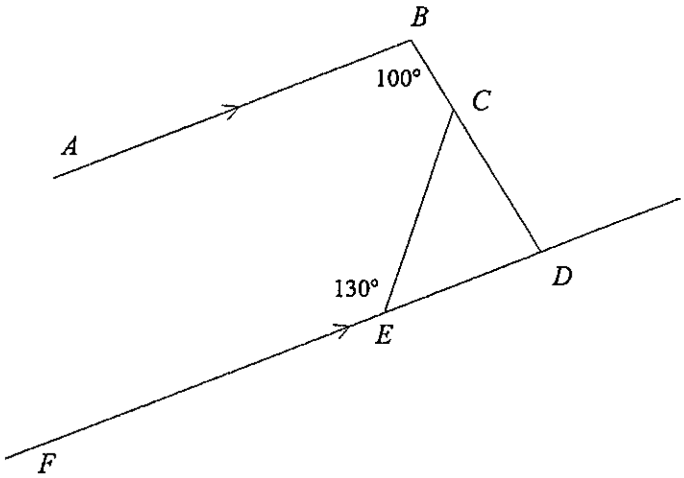
- Attempt Questions 6 – 10
- All questions are of equal value
- In Questions 6 – 10, show relevant mathematical reasoning and/or calculations

Section I:

5 Marks
Attempt Questions 1 – 5

Use the multiple choice answer sheet provided for Questions 1 – 5.

1. In the diagram below, AB is parallel to FD, $\angle ABC = 100°$ and $\angle CEF = 130°$



What is the value of $\angle BCE$?

(A) 100°

(B) 110°

(C) 120°

(D) 130°

2. Which of the following expressions is correct?

(A) $\tan^{-1} x = \cos^{-1} \frac{1}{\sqrt{1-x^2}}$

(B) $\tan^{-1} x = \cos^{-1} \frac{1}{\sqrt{1+x^2}}$

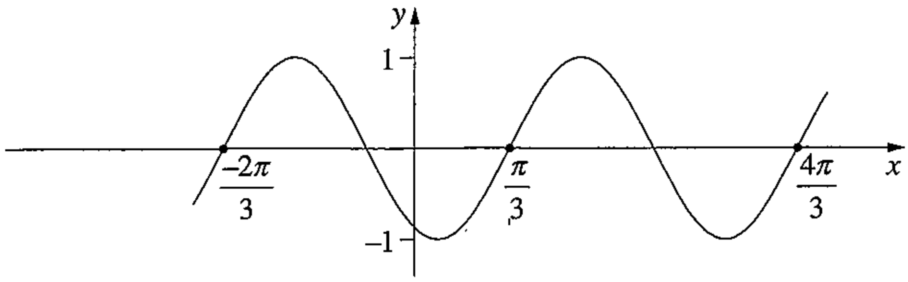
(C) $\tan^{-1} x = \cos^{-1} \frac{x}{\sqrt{1-x^2}}$

(D) $\tan^{-1} x = \cos^{-1} \frac{x}{\sqrt{1+x^2}}$

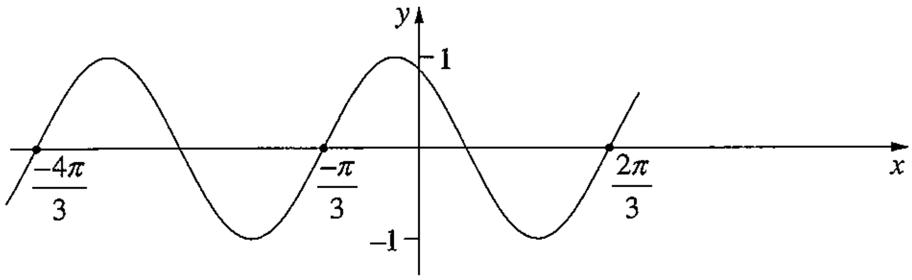
Section I (cont'd)

3. Which diagram shows the graph $y = \sin\left(2x + \frac{\pi}{3}\right)$?

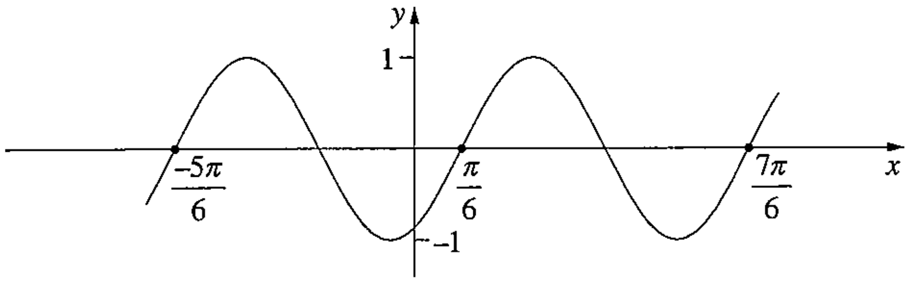
(A)



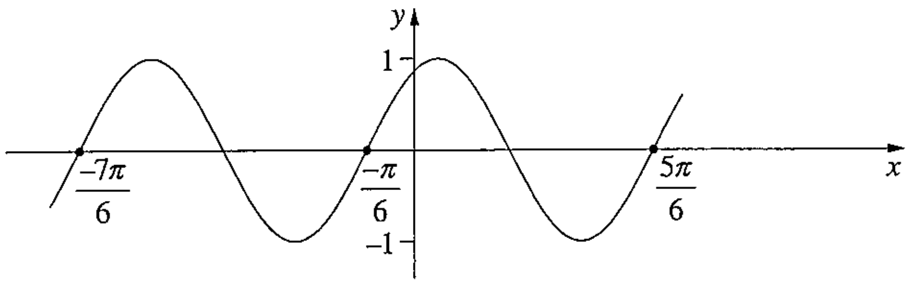
(B)



(C)

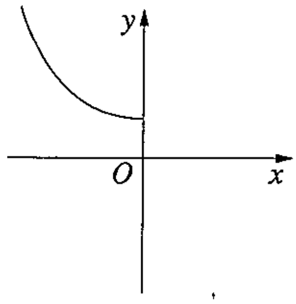


(D)



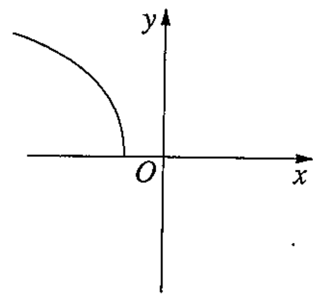
Section I (cont'd)

4. The diagram shows the graph $y = f(x)$.

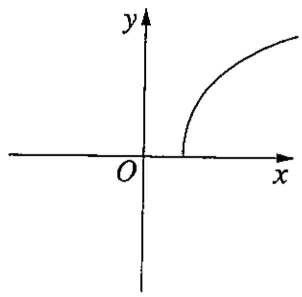


Which diagram shows the graph $y = f^{-1}(x)$?

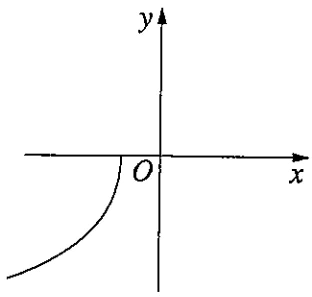
(A)



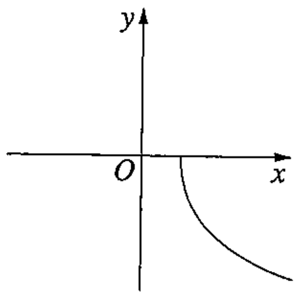
(B)



(C)



(D)



5. What is the primitive function of $3\sin^2 \frac{2x}{5}$?

(A) $\frac{3x}{2} - \frac{6}{5}\sin\frac{4x}{5} + c$

(B) $\frac{3x}{2} - \frac{15}{8}\sin\frac{4x}{5} + c$

(C) $3x - \frac{15}{4}\sin\frac{4x}{5} + c$

(D) $3x - \frac{6}{5}\sin\frac{4x}{5} + c$

End of Section I

Section II

60 marks
Attempt Questions 6 – 10
Start each question in a new booklet.

In Questions 6, 7, 8, 9 and 10 your responses should include relevant mathematical reasoning and/or calculations.

Question 6 (12 Marks) Use a SEPARATE writing booklet. **Marks**

a) Let $f(x) = \sqrt{4x - 3}$.

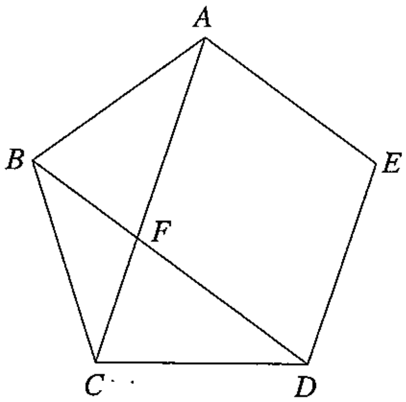
(i) Find the domain of $f(x)$. 1

(ii) Find an expression for the inverse function $f^{-1}(x)$. 2

(iii) Find the points where the graphs $y = f(x)$ and $y = x$ intersect. 2

(iv) On the same set of axes, sketch the graphs $y = f(x)$ and $y = f^{-1}(x)$ showing the information found in part (iii). 2

b)



NOT
TO
SCALE

In the diagram, $ABCDE$ is a regular pentagon. The diagonals AC and BD intersect at F .

Copy or trace this diagram into your writing booklet.

(i) Show that the size of $\angle ABC$ is $108°$. 1

(ii) Find the size of $\angle BAC$. Give reasons for your answer. 2

(iii) By considering the sizes of angles, show that ΔABF is isosceles. 2

Question 7 (12 Marks) Use a SEPARATE writing booklet. **Marks**

a) Show that $\sin\left(2\tan^{-1}\frac{1}{2}\right)=\frac{4}{5}$. 2

b) Given that $f(x)=3\cos^{-1}\left(\frac{x-1}{2}\right)$.

(i) Write down the domain and range of $y=f(x)$. 2

(ii) Draw a neat sketch of $y=f(x)$, showing all important features. 2

c) The region bounded by $y=\cos 4x$ and the x-axis, between $x=0$ and $x=\frac{\pi}{8}$, is rotated about the x-axis to form a solid.

Find the volume of the solid. 3

d) Prove that $\frac{\sin x-\cos 2x+1}{\sin 2x+\cos x}=\tan x$. 3

Question 8 (12 Marks) Use a SEPARATE writing booklet. **Marks**

a) ABC is an isosceles triangle with a right-angle at B. The sides AB, BC are each of length 2 cm. An arc, centre A, radius 2 cm cuts the side AC at D.

(i) Draw a diagram to represent all this information. 1

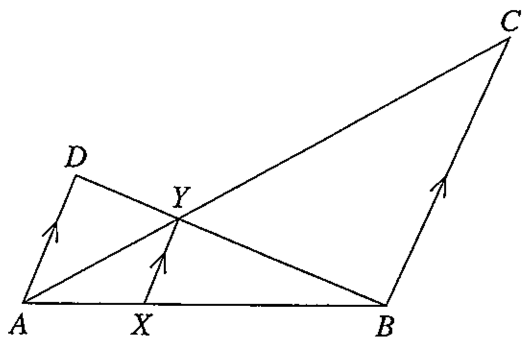
(ii) If BDC is the part of ΔABC outside the circle, show that the area of BDC is $\left(2-\frac{\pi}{2}\right)$ cm². 2

b) Find $\lim_{x \to 0} \dfrac{\tan 5x}{\sin \frac{1}{3}x}$. 2

You must justify your answer.

c) Find all solutions of $\sin(\pi + x) = \tan x$. 3

d) The diagram shows triangles ABC and ABD with AD parallel to BC. The sides AC and BD intersect at at Y. The point X lies on AB such that XY is parallel to AD and BC.



(i) Prove that ΔABC is similar to ΔAXY. 2

(ii) Hence, or otherwise, prove that $\dfrac{1}{XY} = \dfrac{1}{AD} + \dfrac{1}{BC}$. 2

Question 9 (12 Marks) Use a SEPARATE writing booklet. **Marks**

a) Differentiate $x^2 \sin^{-1} 5x$. 2

b) Use the t-formula to solve $2\sin\theta + \cos\theta = 1$ for $0 \le \theta \le 2\pi$. 3

c) (i) Show that $\dfrac{\sec^2\theta}{\tan\theta} = \dfrac{1}{\sin\theta\cos\theta}$. 1

(ii) Find the exact value of $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{\sin\theta\cos\theta}\,d\theta$. 2

d) (i) Show that $\cos\theta - \sqrt{3}\sin\theta = 2\cos\left(\theta + \frac{\pi}{3}\right)$. 2

(ii) Hence, solve $\cos\theta - \sqrt{3}\sin\theta = 1$ for $0 \le \theta \le 2\pi$. 2

Question 10 (12 Marks) Use a SEPARATE writing booklet. **Marks**

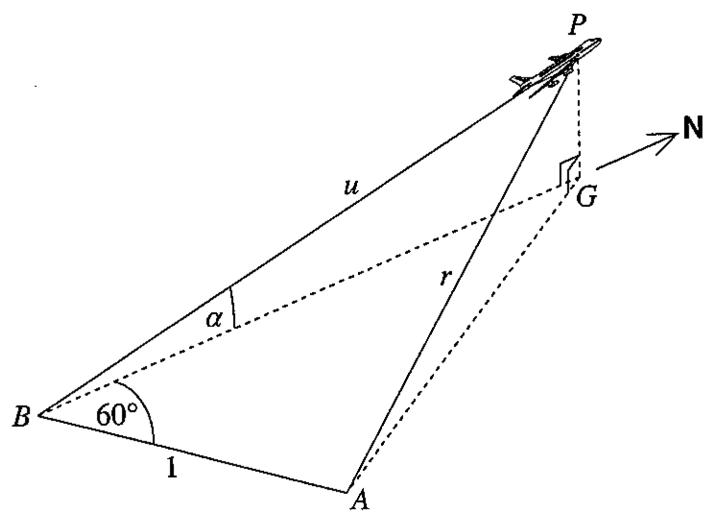
a) If $\sin\alpha = \frac{3}{4}$ and $\sin\beta = \frac{2}{3}$ where $0 < \alpha < \frac{\pi}{2}$, and $\frac{\pi}{2} < \beta < \pi$, find the exact value of $\cos(\alpha - \beta)$. 4

b) If $f(x) = 2\cos^{-1}\left(\frac{x}{\sqrt{2}}\right) - \sin^{-1}(1 - x^2)$

(i) Find $f'(x)$. 3

(ii) Hence, prove that $f(x) = \frac{\pi}{2}$. 2

c) A plane P takes off from a point B. It flies due north at a constant angle α to the horizontal. An observer is located at A, 1 km from B, at a bearing 060° from B. Let u km be the distance from B to the plane and let r km be the distance from the observer to the plane. The point G is on the ground directly below the plane.



Show that $r = \sqrt{1 + u^2 - u\cos\alpha}$. 3

End of Paper

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}}\,dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}}\,dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Student Name: ______________________________

Class Teacher: ______________________________

Section I

Year 12 Mid-HSC Course

Mathematics Extension 1

Multiple-choice Answer Sheet - Questions 1 – 5

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ☒ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows:

A ☒ B ☒ ← *correct* C ○ D ○

1.	A ○	B ○	C ○	D ○
2.	A ○	B ○	C ○	D ○
3.	A ○	B ○	C ○	D ○
4.	A ○	B ○	C ○	D ○
5.	A ○	B ○	C ○	D ○

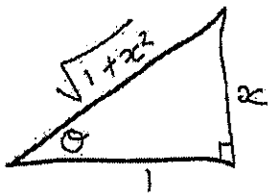
2015 HSC MATHEMATICS EXTENSION 1
MID-COURSE EXAMINATION SOLUTIONS

1

$\angle CED = 50°$
$\angle CDE = 80°$
$\angle ECD = 50°$
$\therefore \angle BCE = 130°$ $\qquad \therefore$ D

2

let $\theta = \tan^{-1} x$
$x = \tan\theta$



$\therefore \cos\theta = \dfrac{1}{\sqrt{1+x^2}}$

$\theta = \cos^{-1}\left(\dfrac{1}{\sqrt{1+x^2}}\right)$

$\therefore \tan^{-1}(x) = \cos^{-1}\left(\dfrac{1}{\sqrt{1+x^2}}\right)$ $\qquad \therefore$ B

3

x-intercepts when $\sin\left(2x+\frac{\pi}{3}\right) = 0$
$2x + \frac{\pi}{3} = 0$
$2x = -\frac{\pi}{3}$
$x = \dfrac{-\pi}{6}$ $\qquad \therefore$ D

4

$y = f^{-1}(x)$ is a reflection of $y = f(x)$ in the line $y = x$ $\quad \therefore$ D

5

$\int 3\sin\left(\frac{2x}{5}\right)^2 dx = 3\int\left(\frac{1}{2} - \frac{1}{2}\cos\frac{4x}{5}\right)dx$

$= 3\left[\frac{1}{2}x - \frac{5}{8}\sin\frac{4x}{5}\right] + C$

$= \frac{3}{2}x - \frac{15}{8}\sin\frac{4x}{5} + C$ $\qquad \therefore$ B

Question 6

a) i

$4x - 3 \geq 0$
$4x \geq 3$
$x \geq \frac{3}{4}$ $\qquad \therefore$ Domain: all real x, $x \geq \frac{3}{4}$

ii

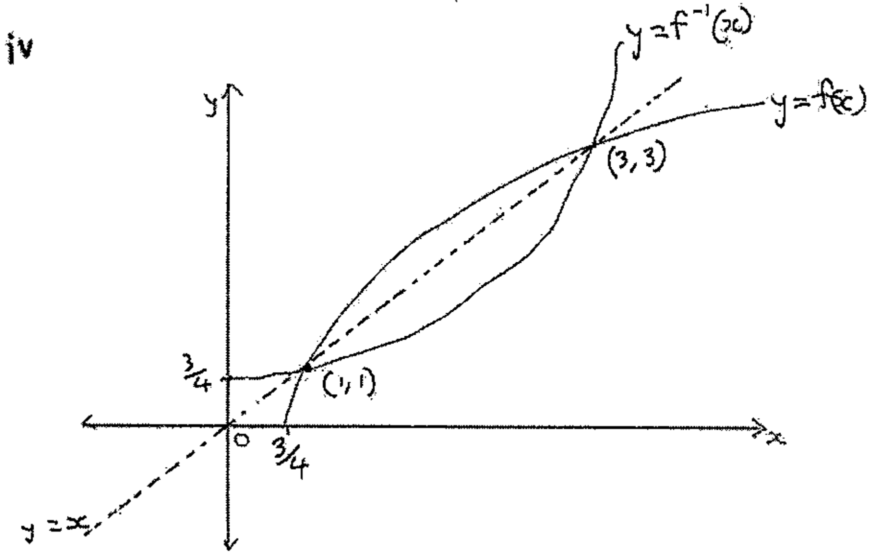
let $y = \sqrt{4x-3}$
inverse $x = \sqrt{4y-3}$ $\qquad x^2 = 4y - 3$
$4y = x^2 + 3$
$y = \dfrac{x^2+3}{4}$
$\therefore f^{-1}(x) = \dfrac{x^2+3}{4}$

iii

Points of intersection where $\sqrt{4x-3} = x$
$4x - 3 = x^2$
$x^2 - 4x + 3 = 0$
$(x-3)(x-1) = 0$
$\therefore x = 3,\ x = 1$

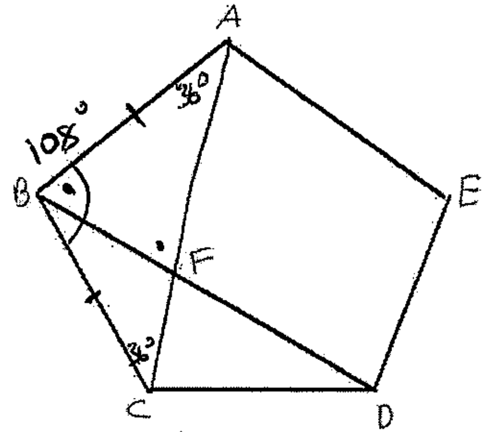
$\therefore$ points are $(1,1)$ and $(3,3)$

iv



Question 6 continued

b



i $\angle ABC = \frac{(5-2)\times 180}{5}$

$= \frac{540}{5}$

$= 108°$

ii $AB = BC$ (equal sides of regular pentagon)

$\therefore \angle BAC = \angle BCA$ (equal angles opposite equal sides)

$2\angle BAC + 108° = 180°$ (angle sum of $\triangle BAC$)

$2\angle BAC = 72°$

$\angle BAC = 36°$

iii Similarly, $\triangle BCD$ is isosceles, and $\angle CBD = 36°$

$\angle ABF + \angle CBD = 108°$

$\angle ABF = 108 - 36$

$= 72°$

Also $\angle AFB + 72° + 36° = 180°$ (angle sum of $\triangle ABF$)

$\therefore \angle AFB = 72°$

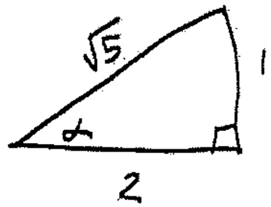
So $\angle AFB = \angle ABF = 72°$

$\therefore \triangle ABF$ is isosceles

Question 7

a) let $\alpha = \tan^{-1}\frac{1}{2}$

$\therefore \tan\alpha = \frac{1}{2}$



$\sin\left(2\tan^{-1}\frac{1}{2}\right) = \sin 2\alpha$

$= 2\sin\alpha\cos\alpha$

$= 2 \times \frac{1}{\sqrt{5}} \times \frac{2}{\sqrt{5}}$

$= \frac{4}{5}$

b) i Domain of $\cos^{-1}\alpha$ $\quad -1 \leq \alpha \leq 1$,

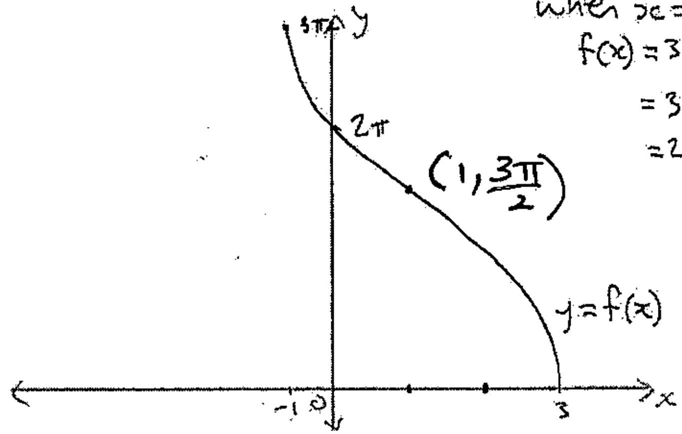
so for $\cos^{-1}\left(\frac{x-1}{2}\right)$ $\quad -1 \leq \frac{x-1}{2} \leq 1$

$-2 \leq x-1 \leq 2$

$-1 \leq x \leq 3$

Range : $0 \leq y \leq 3\pi$

ii



when $x = 0$

$f(x) = 3\cos^{-1}\left(\frac{-1}{2}\right)$

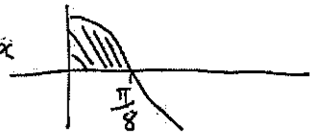
$= 3 \times \frac{2\pi}{3}$

$= 2\pi$

Question 7 continued

c) $V = \pi \int y^2 \, dx$

$y^2 = \cos^2 4x$
$= \frac{1}{2} + \frac{1}{2}\cos 8x$



$= \pi \int \frac{1}{2} + \frac{1}{2}\cos 8x$

$= \frac{\pi}{2}\left[x + \frac{\sin 8x}{8}\right]_0^{\frac{\pi}{8}}$

$= \frac{\pi}{2}\left[\left(\frac{\pi}{8} + 0\right) - (0 + 0)\right]$

$= \frac{\pi^2}{16}$ units^3

d) $LHS = \dfrac{\sin x - \cos 2x + 1}{\sin 2x + \cos x}$

$= \dfrac{\sin x - (\cos^2 x - \sin^2 x) + 1}{2\sin x \cos x + \cos x}$

$= \dfrac{\sin x - (1 - \sin^2 x) + \sin^2 x + 1}{\cos x \; (2\sin x + 1)}$

$= \dfrac{2\sin^2 x + \sin x}{\cos x \; (2\sin x + 1)}$

$= \dfrac{\sin x \, (2\sin x + 1)}{\cos x \; (2\sin x + 1)}$

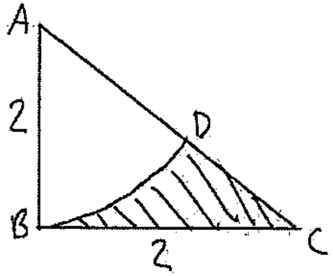
$= \dfrac{\sin x}{\cos x}$

$= \tan x$

$= RHS$

Question 8

a) i



$\angle BAC = 45°$, as $AB = BC$

ii $Area_{BDC} = Area_{ABC} - Area_{ABD}$

$= \frac{1}{2} \times 2 \times 2 - \pi \times 2^2 \times \frac{45}{360}$

$= 2 - \frac{4\pi}{8}$

$= 2 - \frac{\pi}{2}$ cm^2

b) $\lim_{x \to 0} \dfrac{\tan 5x}{\sin \frac{1}{3}x} = \lim_{x \to 0} \dfrac{\tan 5x}{x} \times \dfrac{x}{\sin \frac{1}{3}x}$

$= \lim_{x \to 0} 5 \times \dfrac{\tan 5x}{5x} \times 3 \times \dfrac{\frac{1}{3}x}{\sin \frac{1}{3}x}$

$= 5 \times 1 \times 3 \times 1$

$= 15$

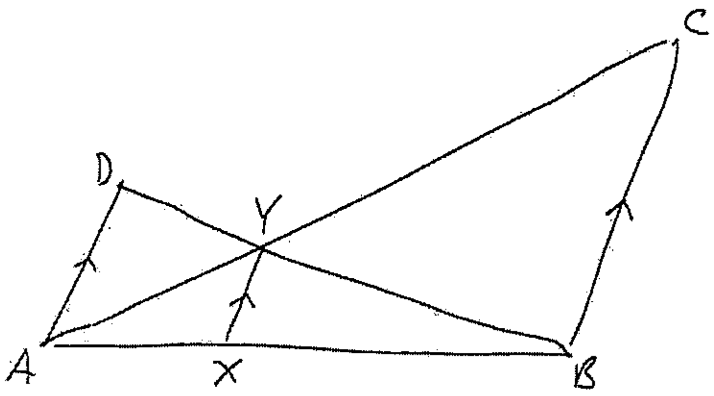
c) $\sin(\pi + x) = \tan x$
$-\sin x = \tan x$
$-\sin x \cos x = \sin x$
$\sin x + \sin x \cos x = 0$
$\sin x \, (1 + \cos x) = 0$
$\therefore \sin x = 0, \quad x = 0, \pi, 2\pi ..$

or $\cos x = -1$
$x = \pi, 3\pi, 5\pi, \ldots$

$\therefore x = n\pi, \; n \in \mathbb{Z}$

Question 8 continued

d)



i In triangles AXY and ABC:

$\angle AXY = \angle ABC$ (corresponding angles, $XY \parallel BC$)

$\angle YAX$ is common

$\therefore \triangle ABC \parallel\!| \triangle AXY$ (equiangular)

ii Similarly, $\triangle BAD \parallel\!| \triangle BXY$

$\frac{XY}{DA} = \frac{XB}{AB}$ (corresponding sides of similar triangles)

$\frac{YX}{CB} = \frac{AX}{AB}$ (similarly)

so $\frac{XY}{AD} + \frac{XY}{BC} = \frac{AX}{AB} + \frac{XB}{AB}$

$XY\left(\frac{1}{AD} + \frac{1}{BC}\right) = \frac{AX + XB}{AB}$

$= \frac{AB}{AB}$

$\therefore \frac{1}{AD} + \frac{1}{BC} = \frac{1}{XY}$

Question 9

a) let $y = x^2 \sin^{-1} 5x$

let $u = x^2$ $\quad v = \sin^{-1} 5x$

$u' = 2x$ $\quad v' = \frac{1}{\sqrt{\frac{1}{25} - x^2}}$

$y' = vu' + uv'$

$= 2x \sin^{-1} 5x + \frac{x^2}{\sqrt{\frac{1}{25} - x^2}}$

b) $2\sin\theta + \cos\theta = 1$ $\quad 0 \le \theta < 2\pi$

let $t = \tan\frac{1}{2}\theta$ $\quad 0 \le \frac{1}{2}\theta \le \pi$

$\therefore 2\left(\frac{2t}{1+t^2}\right) + \left(\frac{1-t^2}{1+t^2}\right) = 1$

$4t + 1 - t^2 = 1 + t^2$

$2t^2 - 4t = 0$

$2t(t-2) = 0$

$\therefore t = 0$ $\quad t - 2 = 0$

$\tan\frac{1}{2}\theta = 0$ $\quad t = 2$

$\frac{1}{2}\theta = 0, \pi$ $\quad \tan\frac{1}{2}\theta = 2$

$\theta = 0, 2\pi$ $\quad \frac{1}{2}\theta = \tan^{-1} 2$

$\theta = 2\tan^{-1} 2$

$= 2.214297$

$\doteqdot 2.2$ (1 d.p.)

$\therefore \theta = 0, 2.2, 2\pi$

Check $\theta = \pi$

$2\sin\pi + \cos\pi = 1$

$-1 \ne 1$

$\therefore \theta = \pi$ is <u>not</u> a solution.

c) i $LHS = \frac{\sec^2\theta}{\tan\theta}$

$= \frac{1}{\cos^2\theta} \times \frac{\cos\theta}{\sin\theta}$

$= \frac{1}{\sin\theta\cos\theta}$

$= RHS$

Question 9 continued

c) ii

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{\sin\theta\cos\theta}\,d\theta = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sec^2\theta}{\tan\theta}$$

$$= \left[\ln(\tan\theta)\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$$

$$= \ln\sqrt{3} - \ln\frac{1}{\sqrt{3}}$$

$$= \ln\sqrt{3} - \ln\sqrt{3}^{-1}$$

$$= \ln\sqrt{3} + \ln\sqrt{3}$$

$$= \ln 3$$

d) i $RHS = 2\cos\left(\theta + \frac{\pi}{3}\right)$

$$= 2\cos\theta\cos\frac{\pi}{3} - 2\sin\theta\sin\frac{\pi}{3}$$

$$= 2\times\cos\theta\times\frac{1}{2} - 2\times\sin\theta\times\frac{\sqrt{3}}{2}$$

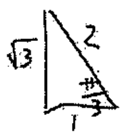
$$= \cos\theta - \sqrt{3}\sin\theta$$

$$= RHS$$

ii $\cos\theta - \sqrt{3}\sin\theta = 1 \qquad 0 \leq \theta \leq 2\pi$

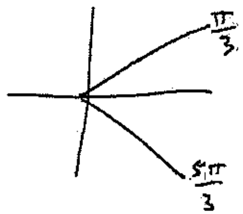
$2\cos\left(\theta + \frac{\pi}{3}\right) = 1 \qquad \frac{\pi}{3} \leq \theta + \frac{\pi}{3} \leq \frac{2\pi}{3}$

$\cos\left(\theta + \frac{\pi}{3}\right) = \frac{1}{2}$



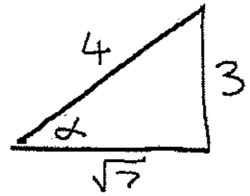
$\therefore \theta + \frac{\pi}{3} = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}$

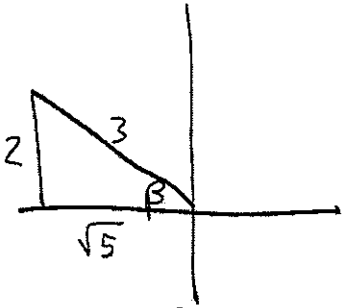
$\therefore \theta = 0, \frac{4\pi}{3}, 2\pi$



Question 10

a)





$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$= \frac{\sqrt{7}}{4}\times -\frac{\sqrt{5}}{3} + \frac{3}{4}\times\frac{2}{3}$$

$$= -\frac{\sqrt{35}}{12} + \frac{6}{12}$$

$$= \frac{6 - \sqrt{35}}{12}$$

b) i $f(x) = 2\cos^{-1}\left(\frac{x}{\sqrt{2}}\right) - \sin^{-1}\left(1 - x^2\right)$

$$f'(x) = 2\times\frac{-1}{\sqrt{2 - x^2}} + \frac{-2}{\sqrt{2 - x^2}}$$

$$= 0$$

let $u = 1 - x^2$

$\frac{du}{dx} = -2x$

$y = \sin^{-1}(u)$

$y' = \frac{1}{\sqrt{1 - u^2}}$

$= \frac{1}{\sqrt{1 - (1 - x^2)^2}}$

$= \frac{1}{\sqrt{(1 - 1 + x^2)(1 + 1 - x^2)}}$

$= \frac{1}{\sqrt{x^2(2 - x^2)}}$

$\therefore \frac{dy}{dx} = \frac{-2}{\sqrt{2 - x^2}}$

ii Since $f'(x) = 0$, $f(x)$ is a constant function, so $f(x) = k$, for some constant K.

$f(0) = 2\cos^{-1}\left(\frac{0}{\sqrt{2}}\right) - \sin^{-1}\left(1 - 0^2\right)$

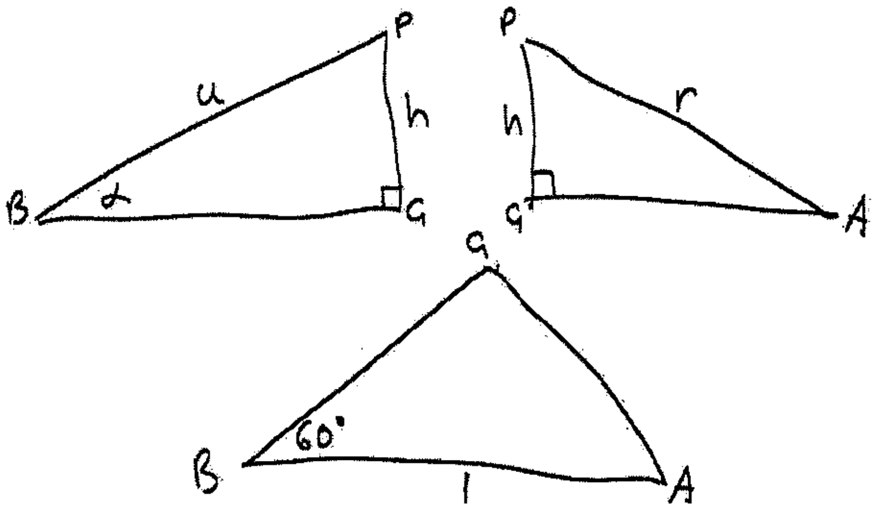
$$= 2\times\frac{\pi}{2} - \frac{\pi}{2}$$

$$= \frac{\pi}{2}$$

$\therefore f(x) = \frac{\pi}{2}$

Question 10 continued

c)



From $\triangle PGB$ $\quad \cos\alpha = \frac{BG}{u}$

$BG = u\cos\alpha$

also $BG^2 = u^2 - h^2$ (Pythagoras' Theorem)

From $\triangle PGA$ $\quad r^2 = h^2 + AG^2$ (similarly)

$AG^2 = r^2 - h^2$

From $\triangle ABG$

$AG^2 = BG^2 + 1^2 - 2 \times BG \times 1 \times \cos 60$

$r^2 - h^2 = u^2 - h^2 + 1 - 2 \times BG \times \frac{1}{2}$

$r^2 = 1 + u^2 - u\cos\alpha$

$r = \sqrt{1 + u^2 - u\cos\alpha}$

Question. 6. (12 marks)

(a) (i) $4x - 3 \geq 0$

$4x \geq 3$

$x \geq \frac{3}{4}$

D: $x \geq \frac{3}{4}$

(ii) let $y = \sqrt{4x-3}$

$x = \sqrt{4y-3}$

$x^2 = 4y - 3$

$4y = x^2 + 3$

$y = \frac{x^2+3}{4}$

$\therefore f^{-1}(x) = \frac{x^2+3}{4} = \frac{x^2}{4} + \frac{3}{4}$

(iii) $y = \sqrt{4x-3}$

$y = x$

ie $x = \sqrt{4x-3}$

$x^2 = 4x - 3$

$x^2 - 4x + 3 = 0$

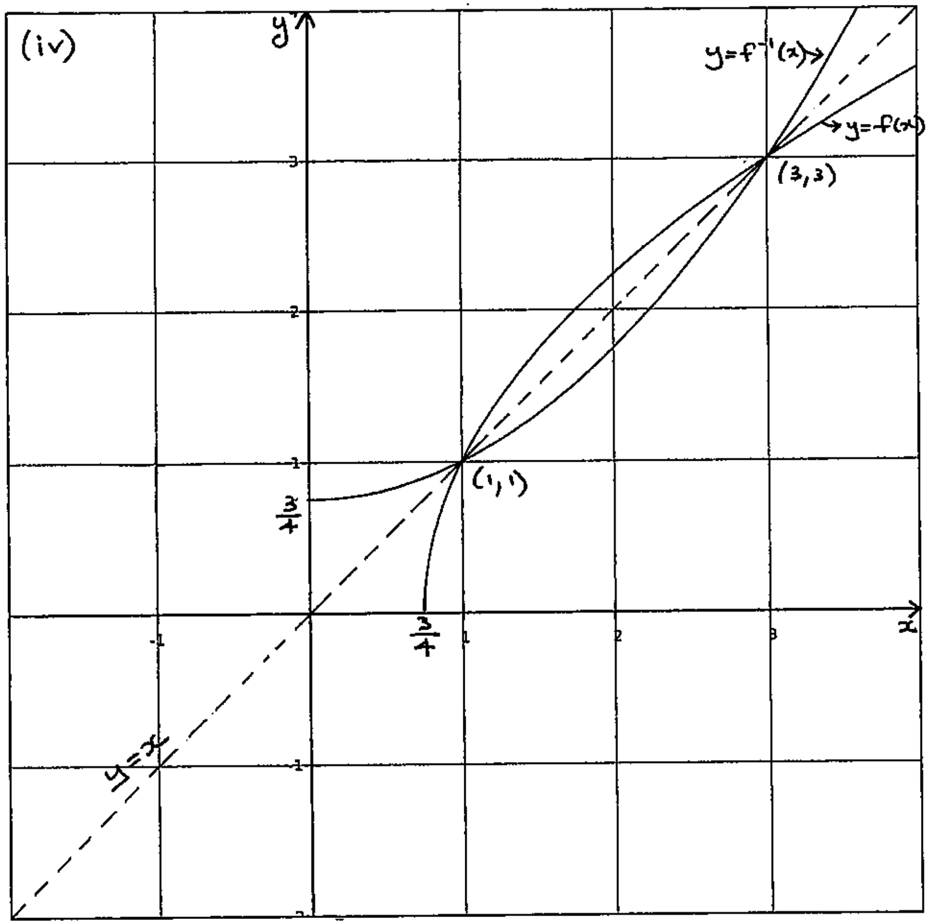
$(x-3)(x-1) = 0$

$x = 3 \quad x = 1$

$y = 3 \quad y = 1$

$\therefore (1,1)$ & $(3,3)$

(iv)



(b) (i) $A\hat{B}C = \frac{(5-2) \times 180^\circ}{5}$ (interior angles of a regular pentagon are equal).

$= \frac{540^\circ}{5}$

$\therefore A\hat{B}C = 108^\circ$

(ii) $AB = BC$ (equal sides of a regular polygon)

$B\hat{A}C = B\hat{C}A$ (angles opposite equal sides are equal)

$2B\hat{A}C + 108^\circ = 180^\circ$ (angle sum of a triangle is 180°)

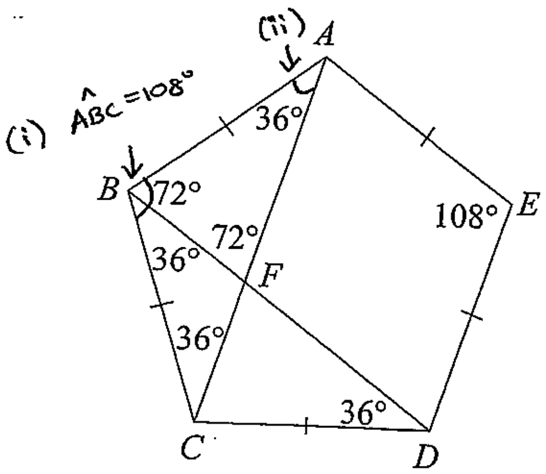
$2B\hat{A}C = 72^\circ$

$\therefore B\hat{A}C = 36^\circ$

OR

$\therefore B\hat{A}C = \frac{180^\circ - 108^\circ}{2}$

$= 36^\circ$



(iii) Similarly, $\triangle BDC$ is isosceles and $C\hat{B}D = C\hat{D}B = 36^\circ$

$A\hat{B}C = A\hat{B}F + C\hat{B}D = 108^\circ$

$A\hat{B}F = 108^\circ - 36^\circ$

$= 72^\circ$

$A\hat{F}B + 72^\circ + 36^\circ = 180^\circ$ (angle sum of triangle ABF is 180°)

$A\hat{F}B = 180^\circ - 108^\circ$

$A\hat{F}B = 72^\circ$

$\therefore A\hat{B}F = A\hat{F}B$ (both equal to 72°)

$\therefore AB = AF$ (sides opposite equal angles are equal).

$\therefore \triangle ABF$ is isosceles

OR

(ii) In Δ BAC,

AB = BC (equal sides of a regular pentagon)

$\therefore \Delta$ BAC is isosceles

$B\hat{A}C = B\hat{C}A$ (equal angles of isosceles triangle)

$2B\hat{A}C + 108^\circ = 180^\circ$ (angle sum of a triangle is 180°)

$2B\hat{A}C = 72^\circ$

$\therefore B\hat{A}C = 36^\circ$ #.

OR

(iii) In Δ's ABC & BCD

BA = BC (equal sides of a regular pentagon)

$A\hat{B}C = B\hat{C}D$ (interior angles of regular pentagon are equal)
$= 108^\circ$

BC = CD (equal sides of a regular pentagon)

$\therefore \Delta ABC \equiv \Delta BCD$ (SAS)

$\therefore B\hat{A}C = C\hat{B}D$ (corresponding angles in congruent triangles are equal)
$= 36^\circ$

$B\hat{A}C = B\hat{C}A$ (equal angles in isosceles Δ BAC)
$= 36^\circ$

$B\hat{F}A = C\hat{B}D + B\hat{C}A$ (exterior angle equals the sum of the two interior opposite angles)
$= 36^\circ + 36^\circ$
$= 72^\circ$

$A\hat{B}F = A\hat{B}C - C\hat{B}D$
$= 108^\circ - 36^\circ$
$= 72^\circ$

$\therefore B\hat{F}A = A\hat{B}F$ (both equal to 72°)

$\therefore$ AB = AF (sides opposite equal angles are equal)

$\therefore \Delta$ ABF is isosceles