Sydney Boys High School.

ASSESSMENT TASK # 2 - April 2000

MATHEMATICS

3/4 UNIT COMMON

Time allowed — One and a half hours
(Plus 5 minutes reading time)

Examiners: F. Jordan, P. Bigelow

DIRECTIONS TO CANDIDATES

- *All* questions may be attempted.
- All necessary working should be shown in every question. Full marks may not be awarded for careless or badly arranged work.
- Approved calculators may be used.
- There are **6** questions and *each question* is to be returned in a *separate* booklet, clearly marked Question 1, etc. Each booklet must also show your name and teacher.
- Start each question in a new booklet.
- If required, additional booklets may be obtained from the Examination Supervisor upon request.

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Question 1 **Marks**

(a) Evaluate $\lim_{x \to 0} \frac{\sin 7x}{6x}$ **2**

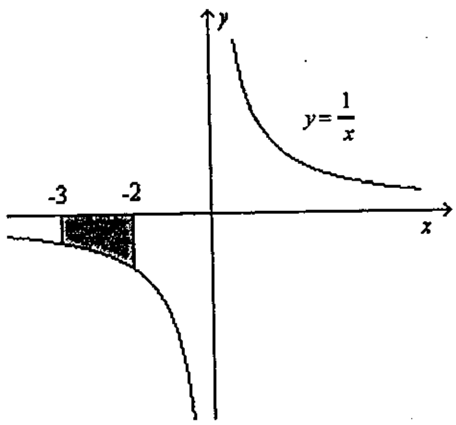
(b) Differentiate: **3**

(i) $\cos 2x$

(ii) $x \tan x$

(c) Find $\int_0^{\frac{\pi}{2}} \sin \frac{x}{2} \, dx$ **2**

(d) **2**



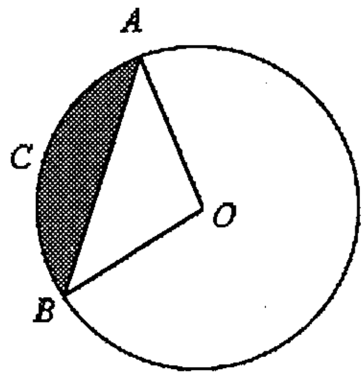
Find the value of the shaded area correct to 2 decimal places

Question 2 (Start a new booklet) **Marks**

(a) Differentiate $\dfrac{x}{\ln x}$ **2**

(b) Find a primitive of $\dfrac{2}{4+3x}$ **2**

(c) **3**



A circle has centre O and radius 10 cm. $\angle AOB = \frac{5\pi}{6}$.

Find the exact value of the shaded region.

(d) If $y = x^n e^{ax}$ show that $\dfrac{dy}{dx} - ay = \dfrac{ny}{x}$ **2**

Question 3 (Start a new booklet) **Marks**

(a) (i) For the curve $y = 2\sin(\frac{\pi}{2} + 4x)$ state the period and amplitude **4**

(ii) Sketch $y = 2\sin(\frac{\pi}{2} + 4x)$ for $-0 \cdot 5\pi \le x \le 0 \cdot 5\pi$

$4x = \frac{\pi}{2}$

$x = -\frac{\pi}{8}$

(b) Find $\frac{d}{dx}\{\log_e(\cos x)\}$ and hence find the area under the curve $y = \tan x$ from $x = 0$ to $x = \frac{\pi}{4}$ **3**

(c) The curve $y = 3 + 2\sin x$ is rotated about the x axis between $x = 0$ and $x = \pi$. Find the volume of the solid generated. **3**

Question 4 (Start a new booklet) **Marks**

(a) State the domain and range of $y = \sqrt{1 + \ln x}$ **2**

$\sqrt{\ } = +ve$

(b) Find $\int_0^{\frac{\pi}{4}} (\cos^2 x - \sin^2 x)dx$ **2**

(c) Use one step of Newton's Method to find an improved value of that root of $f(x) = e^{4x} - \sqrt{x + 0\cdot 81}$ which lies close to $x = 0$ **3**

(d) The planet Hollywood is in a far, far away galaxy and has a moon with diameter of 148 000 km. If the distance between the centre of Hollywood and the centre of the moon is $1\cdot 28 \times 10^7$ km, find the angle subtended by the moon at the centre of the planet in seconds. **2**

Question 5 (Start a new booklet) **Marks**

(a) Find the equation of the normal to $y = 3\cos\frac{x}{2}$ at the point where $x = \frac{\pi}{3}$ **3**

$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

(b) The *error function* is given by $\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,dt$. **3**

Use the trapezoidal rule with 5 function values to estimate erf(2)

(c) If $f(x) = 2e^{-x^2}$ then **3**

(i) Find $f(\sqrt{\ln x})$

(ii) Find a, where $f''(a) = 0$

(d) Find the area enclosed by $f(x) = 3(x-2)$ and $g(x) = (x-1)(x-2)(x-3)$. **3**

[HINT: Make a sketch]

$3x - 6 = 0$ $y = -6$

$3x = 6$

$x = 2$

Question 6 (Start a new booklet) **Marks**

(a) Sketch $y = \tan x$ for $|x| < \frac{\pi}{2}$. **2**

By sketching the appropriate line find the number of roots of the equation $\tan x - x = \dfrac{\pi}{2}$ within this domain.

(b) (i) Show that $\sum_{r=1}^{n} r = \dfrac{n(n+1)}{2}$ **5**

(ii) Hence, using induction or otherwise prove that $\sum_{r=1}^{n} r^3 = \left(\sum_{r=1}^{n} r\right)^2$

(c) (i) Find any stationary point and points of inflexion on the curve defined below for $x > 0$. **4**

$$y = x^2 \ln\left(\frac{1}{x^3}\right)$$ – separate.

(ii) Sketch the curve.

END OF THE PAPER