SYDNEY TECHNICAL HIGH SCHOOL

File

YEAR 12 ASSESSMENT TASK 2

MARCH 2002

EXTENSION 1 MATHEMATICS

Time Allowed: 70 minutes

Instructions:
Attempt all questions.
Answers to be written on the paper provided.
Start each question on a new page.
All necessary working should be shown.
Marks may not be awarded for careless or badly arranged working.
This question paper must be stapled on top of your answers.
Marks shown are for guidance and may be changed slightly if needed.

Name:______________________________ Teacher:__________________________

Question 1	Question 2	Question 3	Question 4	Question 5	Question 6
/10	/10	/10	/10	/8	/10

Total
/58

Question 1 (10 Marks)

a) A circle has radius of 10m. Find the length of the arc that subtends an angle of 55° at the centre (correct to 1 decimal place). *(2)*

b) A function is defined by

$$f(x) = \begin{cases} 1 \ \ for \ x \leq 0 \\ 1 + x^2 \ \ for \ x > 0 \end{cases}$$

i) sketch the function **(1)**

ii) evaluate $\int_{-2}^{4} f(x)dx$ **(3)**

c) i) Sketch $y = \sin \pi \ x \ for \ -2 \leq x \leq 2$ **(3)**

ii) Hence find how many solutions there are to the equation $\sin \pi \ x = x$ **(1)**

Question 2 (10 marks) **<u>Start a new page</u>**

a) A circle has centre C and radius of 15cm. An arc AB of this circle has length 20cm. Find the area of the sector ACB. **(2)**

b) Solve $\sin 2\theta = \cos\theta \ for \ 0 \leq \theta \leq 2\pi$ **(4)**

c) Find $\int (x^2 + 1)^3 \ .2x \ dx$ by using the substitution $u = x^2 + 1$ or otherwise. **(4)**

Question 3 **(10 marks)** **(Start a new page)**

(a) The curve $f(x) = \dfrac{1}{x+1}$ is rotated around the x axis between $x=1$ and $x=3$. Copy and complete the table below **(3)**

x	1	1.5	2	2.5	3
$f(x)$					
$[f(x)]^2$					

Use Simpsons Rule to find the volume of the solid formed (answer correct to 3 significant figures).

b) i) Show the equation of the tangent to the curve $y = x^2 + 2$ at the point of Q (2,6) is is $4x - y - 2 = 0$ **(2)**

ii) Sketch the curve and its tangent. On your sketch indicate where the tangent cuts the x *and* y axes. **(1)**

iii) Find the volume of the solid formed when the area enclosed by the **curve**, its **tangent** and the ***y* axis** is rotated around the ***y* axis.** **(4)**

Question 4 **(10 marks)** **Start a new page**

a) Consider the curve $y = x + \dfrac{1}{x^2}$

i) Show the curve has one stationary point and determine its nature. **(2)**

ii) By considering $\dfrac{d^2y}{dx^2}$ explain why there are no points of inflexion on this curve **(1)**

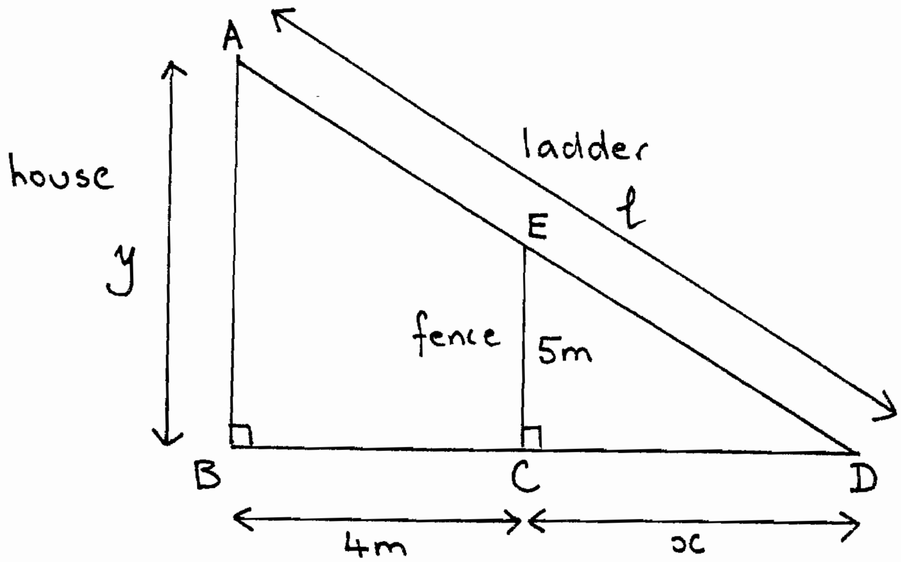
iii) Find any asymptotes **(2)**

b) Use the substitution $u = x^2 + 1$ to find the area bounded by the curve $y = \dfrac{x}{(x^2+1)^2}$, the x axis and the lines $x=1$ and $x=2$. **(5)**

Question 5 **(8 marks)** **start a new page**

A 5m fence stands 4m from the wall of a house

A ladder is placed to touch the wall at A, the fence at E and the ground at D.



as in diagram let $AB = y$ and $CD = x$ and ladder ℓ

a) Use the similar triangles to show that $y = \dfrac{5(x+4)}{x}$ **(2)**

b) Hence show $\ell^2 = (x+4)^2(1+\dfrac{25}{x^2})$ **(2)**

c) Given $\dfrac{d}{dx}\left(\ell^2\right) = 2(x+4)(\dfrac{25}{x^2}+1) - \dfrac{50}{x^3}(x+4)^2$

Find the length of the shortest ladder that can reach from the ground outside the fence to the wall. **(4)**

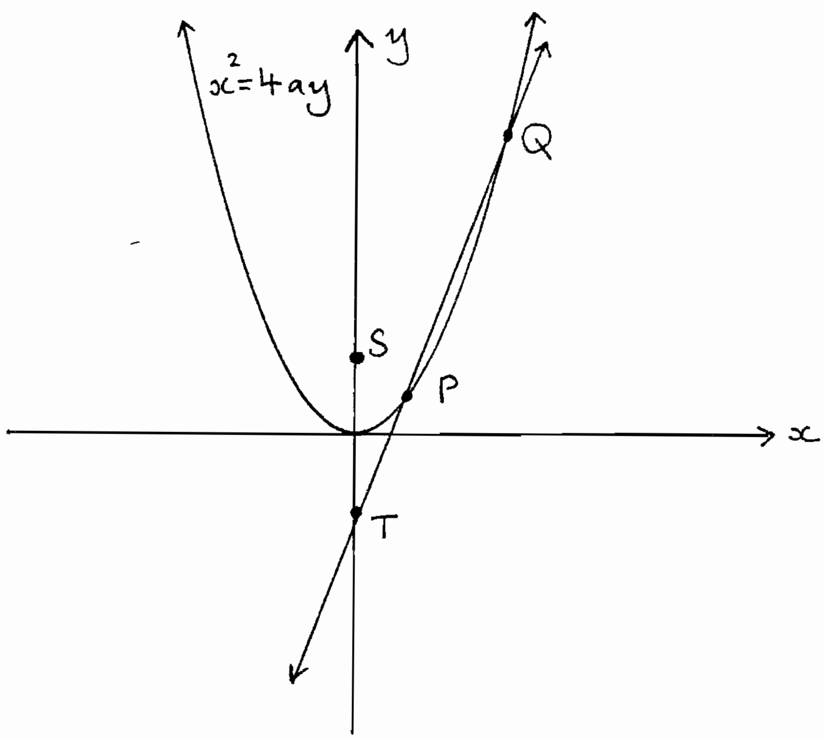
Question 6 (10 marks) Start a new page

a) Form a cartesian equation from the following parametric equations

$x = 1 + t$

$y = 4t^2 + 4t$ **(2)**

b) A straight line through T (0, -a) cuts the parabola $x^2 = 4ay$ at P$(2ap, ap^2)$ and $Q(2aq, aq^2)$. *S (O,a)* is the focus.



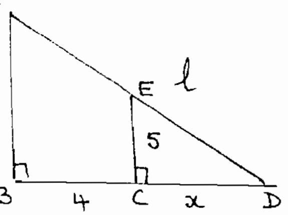
i) Show the equation of TP is $2py = x(p^2 + 1) - 2ap$ **(2)**

ii) Prove that for TP to pass through Q then *pq=1* **(2)**

iii) Show $SP = a(p^2 + 1)$ **(2)**

iv) Hence prove $\dfrac{1}{SP} + \dfrac{1}{SQ} = \dfrac{1}{a}$ **(2)**

STION 5



$\dfrac{y}{5} = \dfrac{4+x}{x}$ — (1)

$\therefore y = \dfrac{5(4+x)}{x}$ — (1)

$l^2 = y^2 + (x+4)^2$

$= \left(\dfrac{5(4+x)}{x}\right)^2 + (x+4)^2$ — (1)

$= \dfrac{25}{x^2}(x+4)^2 + (x+4)^2$

$\therefore l^2 = (x+4)^2\left[\dfrac{25}{x^2} + 1\right]$ — (1)

ven:

$\dfrac{d(l^2)}{dx} = 2(x+4)\left(\dfrac{25}{x^2}+1\right) - \dfrac{50}{x^3}(x+4)^2$

$\dfrac{d(l^2)}{dx} = 0$

$2(x+4)\left(\dfrac{25}{x^2}+1\right) = \dfrac{50}{x^3}(x+4)^2$ — (1)

$\dfrac{25}{x^2} + 1 = \dfrac{25}{x^3}(x+4)$

$x^3\left(\dfrac{25}{x^2}+1\right) = 25x + 100$

$25x + x^3 = 25x + 100$

$x^3 = 100$

$x = \sqrt[3]{100} \doteqdot 4.64$ — (1)

test max/min using table

x	4	4.6	5
$\dfrac{d(l^2)}{dx}$	-ve	0	+ve

— (1)

$\therefore$ min l^2 when $x = \sqrt[3]{100}$

$\therefore l \doteqdot 12.7\text{ m}$ — (1)

QUESTION 6

a) $x = 1 + t \quad \therefore t = x - 1$ (1)

$y = 4t^2 + 4t$

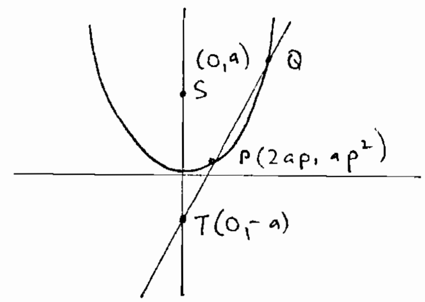
$y = 4(x-1)^2 + 4(x-1)$ — (1)

$y = 4(x^2 - 2x + 1) + 4x - 4$

$y = 4x^2 - 8x + 4 + 4x - 4$

$y = 4x^2 - 4x$

b)



i) $m_{TP} = \dfrac{ap^2 + a}{2ap} = \dfrac{a(p^2+1)}{2ap}$

$\therefore m_{TP} = \dfrac{p^2+1}{2p}$ — (1)

eqn TP: $y = \dfrac{(p^2+1)}{2p}x - a$

(1) $2py = x(p^2+1) - 2ap$

ii) sub Q into line TP — (1)

$2p.aq^2 = 2aq(p^2+1) - 2ap$

$2apq^2 = 2aqp^2 + 2aq - 2ap$

$pq^2 = p^2q + q - p$

$pq^2 - p^2q = q - p$

$pq(q-p) = (q-p)$

$\therefore pq = 1$ — (1)

iii) $SP = \sqrt{(2ap-0)^2 + (ap^2-a)^2}$ — (1)

$= \sqrt{4a^2p^2 + a^2(p^2-1)^2}$

$= \sqrt{4a^2p^2 + a^2p^4 - 2a^2p^2 + a^2}$

$= \sqrt{a^2(p^4 + 2p^2 + 1)}$

$= a\sqrt{(p^2+1)^2}$

$SP = a(p^2+1)$ — (1)

iii) $\therefore SQ = a(q^2+1)$

$\dfrac{1}{SP} + \dfrac{1}{SQ} = \dfrac{1}{a(p^2+1)} + \dfrac{1}{a(q^2+1)}$

(1) $= \dfrac{q^2+1 + p^2+1}{a(p^2+1)(q^2+1)}$

$= \dfrac{p^2+q^2+2}{a(p^2q^2 + p^2 + q^2 + 1)}$

(since $pq = 1$)

(1) $= \dfrac{(p^2+q^2+2)}{a(p^2+q^2+2)}$

$\therefore \dfrac{1}{SP} + \dfrac{1}{SQ} = \dfrac{1}{a}$

YEAR 12 — ASSES TASK 2 — MARCH 2002 EXT 1

QUESTION 1

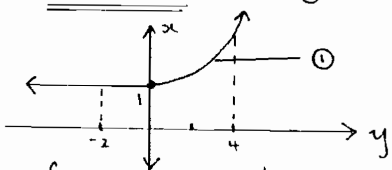
a) $55° = \frac{\pi}{180} \times 55$ radians

$= \frac{11\pi}{36}$ —— (1)

$l = 10 \times \frac{11\pi}{36}$

$= 9.6$ m —— (1)

b) i)



—— (1)

ii) $\int_{-2}^{4} f(x)\,dx = 2 + \int_{0}^{4} (1+x^2)\,dx$

(1) $= 2 + \left[x + \frac{x^3}{3}\right]_0^4$

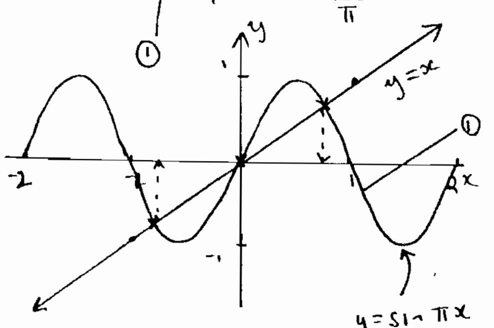
(1) $= 2 + \left[4 + \frac{64}{3}\right]$

(1) —— $= 27\frac{1}{3}$

c)

i) $y = \sin \pi x$ $\quad -2 \le x \le 2$

amp $= 1$ (1) $\quad$ period $= \frac{2\pi}{\pi} = 2$ —— (1)



—— (1)

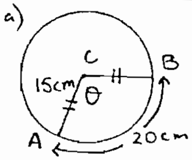
ii) $y = x$ (sketched above) has 3 points of intersection with $y = \sin \pi x$

$\therefore \sin \pi x = x$

has 3 solutions —— (1)

QUESTION 2

a)



$\theta = \frac{l}{r} = \frac{20}{15} = \frac{4}{3}$ —— (1)

Area $= \frac{1}{2} \times 15^2 \times \frac{4}{3}$

Area $= 150$ cm^2 —— (1)

b) $\sin 2\theta = \cos\theta$

$2\sin\theta\cos\theta - \cos\theta = 0$ —— (1)

$\cos\theta\,(2\sin\theta - 1) = 0$

$\therefore \cos\theta = 0 \qquad \sin\theta = \frac{1}{2}$ —— (1)

$\theta = \frac{\pi}{2},\ \frac{3\pi}{2},\ \frac{\pi}{6},\ \frac{5\pi}{6}$ —— (2)

c) $u = x^2 + 1$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$\therefore dx = \frac{du}{2x}$ —— (1)

$\int (x^2+1)^3 . 2x\,dx$

$\int u^3 . 2x . \frac{du}{2x}$

$\int u^3\,du$ —— (1)

$\frac{u^4}{4} + c$ —— (1)

$\therefore \int (x^2+1)^3 . 2x\,dx = \frac{(x^2+1)^4}{4} + c$ (1)

QUESTION 3

a)

x	1	1·5	2	2·5	3
$f(x)$	·5	·4	$0\cdot\dot{3}$	·286	·25
$[f(x)]^2$	·25	·16	$0\cdot\dot{1}$	·082	·063

(1)

$V = \pi\,\frac{1/2}{3}\left[\cdot 25 + \cdot 063 + 4(\cdot 16 + \cdot 082) + 2(0\cdot\dot{1})\right]$ (1)

$V = \cdot 787$ unit^3 —— (1)

b) i) $y = x^2 + 2$

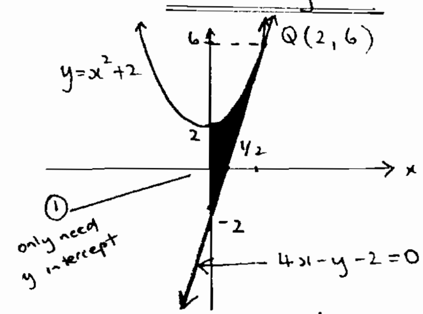
$\frac{dy}{dx} = 2x \qquad m_T = 4$ at $(2,6)$ (1)

$\therefore$ tang. $y - 6 = 4(x - 2)$

$y - 6 = 4x - 8$ —— (1)

$0 = 4x - y - 2$

ii)



(1)

(2) $V_y = \pi\left[\int_{-2}^{6} \left(\frac{y+2}{4}\right)^2 dy - \int_{2}^{6} (y-2)\,dy\right]$

(1) $= \pi\left\{\left[\frac{1}{16}\,\frac{(y+2)^3}{3}\right]_{-2}^{6} - \left[\frac{(y-2)^2}{2}\right]_2^6\right\}$

$= \pi\left\{\frac{1}{48}(8^3) - 8\right\}$

(1) —— $= \frac{8\pi}{3}$ unit^3

QUESTION 4

a) i) $y = x + x^{-2}$

$\frac{dy}{dx} = 1 - 2x^{-3}$

$\frac{d^2y}{dx^2} = 6x^{-4}$

st pts $\frac{dy}{dx} = 0 \qquad \therefore 1 - \frac{2}{x^3} = …$

(1)

$1 = \frac{2}{x^3}$

at $\therefore x = \sqrt[3]{2} \qquad \frac{d^2y}{dx^2} > 0$ …

ii) $\frac{d^2y}{dx^2} = 0$ for pts of …

$\frac{6}{x^4} \ne 0$ —— …

$\therefore$ no pts of inflexion …

iii) asymptotes at (…

$x = 0$ and $y = x$ …

b) Let $u = x^2 + 1$

$\frac{du}{dx} = 2x$

$\therefore dx = \frac{du}{2x}$ —— (1)

if $x = 1 \quad u = 2$
$x = 2 \quad u = 5$ —— (1)

$\therefore \int_1^2 \frac{x}{(x^2+1)^2}\,dx = \int_2^5 \frac{x}{u^2}$ …

$= \frac{1}{2}\int_2^5 u^{-2}\,du$ ——

$= \frac{1}{2}\left[\frac{u^{-1}}{-1}\right]_2^5$

$= \frac{1}{2}\left[-\frac{1}{u}\right]_2^5$ —— (…

$= \frac{1}{2}\left[-\frac{1}{5} - -\frac{1}{2}\right] = …$

(1)